SpeedLabs

MATHS

CBSE 10th

TEEVRA EDUTECH PVT. LTD.

Polynomials

Exercise-2.3

Q.1 Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ \ g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ \ g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6,\ \ \ \ g(x) = 2 - x^2$

Sol:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,$

$q(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x - 3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \quad\quad - 2x \\
 & - \quad\quad\quad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \quad\quad + 6 \\
 & + \quad\quad\quad - \\
\hline
 & 7x - 9
\end{array}
$$

Quotient $= x - 3$

Remainder $= 7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
x^2 - x + 1 & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \ + \ - \\
\hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \ + \ - \\
\hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \ - \ + \\
\hline
 & 8
\end{array}
$$

$q(x) = x^2 + 1 - x = x^2 - x + 1$

Quotient $= x^2 + x - 3$

Remainder $= 8$

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$g(x) = 2 - x^2 = x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 \quad - 4 \\
 & - \quad + \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

Quotient $= -x^2 - 2$

Remainder $= -5x + 10$

Q.2 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Sol:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0.t - 3$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 + 0.t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & - \quad - \quad + \\
\hline
 & 3t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 + 0.t^2 - 9t \\
 & - \quad - \quad + \\
\hline
 & 4t^2 + 0.t - 12 \\
 & 4t^2 + 0.t - 12 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0.

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
\hline
 & 0
\end{array}
$$

Since the remainder is 0.

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 \quad + 3x + 1 \\
 & -x^3 \quad + 3x - 1 \\
 & + \quad - \quad + \\
\hline
 & 2
\end{array}
$$

Since the remainder $\neq 0$.

Hence, $x^3 - 3x + 1$, is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q.3 Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol: $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, w edivide the given polynomial by $x^2 - \frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
\hline
x^2 + 0.x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$3x^4 + 6x^3 - 2x^2 - 10 - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

We factorize $x^2 + 2x + 1 = (x + 1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x + 1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, -1 and -1.

Q.4 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol: $p(x) = x^3 - 3x^2 + x + 2$ (Dividend)

$g(x) = ?$ (Divisor)

Quotient $= (x - 2)$

Remainder $= (-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$g(x)$ is the quotient when we divide by $(x^3 - 3x^2 + x + 2)$ by $(x - 2)$.

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$\therefore g(x) = (x^2 - x + 1)$

Q.5 Give examples of polynomial p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Sol: According to the division algorithm, if p(x) and g(x) are two polynomials with

$g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that

$p(x) = g(x) \times q(x) + r(x)$,

where r(x) = 0 or degree of r(x) < degree of g(x)

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg p(x) = deg q(x)

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$ and $r(x) = 0$

Degree of p(x) and q(x) is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2(3x^2 + x + 1) = 6x^2 + 2x + 2$

Thus, the division algorithm is satisfied.

(ii) deg q(x) = deg r(x)

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii) deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.