SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

Polynomials

Exercise-2.4

SpeedLabs
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1. Determine which of the following polynomials has $(x-1)$ a factoe:

(i) x^3+x^2+x+1 **(ii)** $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3+3x^2+x+1$ **(iv)** $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Ans - **(i)** x^3+x^2+x+1

While applying the factor theorem, we get

$p(x)=x^3+x^2+x+1$

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

We conclude that on dividing the polynomial x^3+x^2+x+1 by (x-1), we get the remainder as0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+x^3+x^2+x+1$

$p(-1)=(-1)^4+(-1)^1+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1$

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we get the remainder as0.

Therefore, we conclude that $(x+1)$ is a factor of $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+3x^3+3x^2+x+1$

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$=1-3+3-1+1$

$=1$

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

While applying the factor theorem, we get

$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2}$

We conclude that on dividing the polynomial $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ by $(x + 1)$, we will get the remainder as $2\sqrt{2}$ which is not 0.

Therefore, we conclude that $(x + 1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 + 4x^2 + x + 6, g(x) = x - 3$

Ans - (**i**) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

We know that according to the factor theorem, (x – a) is a factor of p (x), if p(a) = 0.

We can conclude that g(x) is a factor of p(x), if p(-1)=0.

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2 + 1 - 1 - 2$

$= 0$

Therefore, we conclude that the g(x) is a factor of p(x).

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

We know that according to the factor theorem, (x – a) is a factor of p (x), if p(a) = 0.

We can conclude that g(x) is a factor of p(x), if p(-2)=0.

$p(-2) = (-2)^3 + 3(-2)^2 - 3(-2) + 1$

$= -8 + 12 - 6 - 1$

$= -1$

Therefore, we conclude that the g(x) is a factor of p(x).

(iii) $p(x) = x^3 + 4x^2 + x + 6, g(x) = x - 3$

We know that according to the factor theorem, (x – a) is a factor of p (x), if p(a) = 0.

We can conclude that g(x) is a factor of p(x), if p(-3)=0.

$p(3) = (3)^3 - 4(-3)^2 - (3) + 6$

$= 27 - 36 + 3 + 6 = 0$

Therefore, we conclude that the g(x) is a factor of p(x).

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(**i**) $p(x) = x^2 + x + k$ (**ii**) $p(x)2x^2 + kx + \sqrt{2}$

(**iii**) $p(x) = kx^2 - \sqrt{2}x + 1$ (**iv**) $p(x) = kx^2 - 3x + k$

Ans - (**i**) $p(x) = x^2 + x + k$

$p(a) = 0$, if $x - a$ is a factor of p (x).

We conclude that if (x – 1) is a factor of $p(x) = x^2 + x + k$, then p(1) =0

$p(1) = (1)^2 + (1) + k = 0$, or

$k + 2 = 0$

$k = -2$

Therefore, we can conclude that the value of k is -2.

(**ii**) $p(x)2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of p (x).

We conclude that if (x – 1) is a factor of $p(x)2x^2 + kx + \sqrt{2}$ then p(1) =0

$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0$, or

$2 + k + \sqrt{2} = 0$

$k = -(2 + \sqrt{2})$

Therefore, we can conclude that the value of k is $-(2 + \sqrt{2})$

(**iii**) $p(x) = kx^2 - \sqrt{2}x + 1$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of p (x).

We conclude that if (x – 1) is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$ then p(1) =0

$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0$, or

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

Therefore, we can conclude that the value of k is $\sqrt{2}) - 1$

(**iv**) $p(x) = kx^2 - 3x + k$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of p (x).

We conclude that if (x – 1) is a factor of $p(x) = 3kx^2 - 3x + k$ then p(1) =0

$p(1) = k(1)^2 - 3(1) + k$, or $2k - 3 = 0 \Rightarrow k\frac{3}{2}$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

4. Factorize:

(i) $13x^2 - 7x + 1$ **(ii)** $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ **(iv)** $3x^2 - x - 4$

Ans - **(i)** $13x^2 - 7x + 1$

$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$3x(4x - 1) - 1(4x - 1)$

$= (3x - 1)(4x - 1).$

Therefore, we conclude that on factorizing the polynomial $13x^2 - 7x + 1$ we get $(3x - 1)(4x - 1)$.

(ii) $2x^2 + 7x + 3$

$= 2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3).$

Therefore, we conclude that on factorizing the polynomial $2x^2 + 7x + 3$ we get, $(2x + 1)(x + 3)$.

(iii) $6x^2 + 5x - 6$

$6x^2 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2\ (2x + 3)$

$= (3x - 2)(2x + 3).$

Therefore, we conclude that on factorizing the polynomial$6x^2 + 5x - 6$, we get $(3x - 2)(2x + 3)$.

(iv) $3x^2 - x - 4$

$3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$

$= 3x(x + 1) - 4(x + 1)$

$= (3x - 4)(x + 1)$

Therefore, we conclude that on factorizing the polynomial $3x^2 - x - 4$, we get $(3x - 4)(x + 1)$.

5. Factorize:

(i) $x^3 - 2x^2 - x + 2$ **(ii)** $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ **(iv)** $2y^3 - y^2 - 2y + 1$

Ans - **(i)** $x^2 - 2x^2 - x + 2$

We need to consider the factors of 2, which are $\pm 1, \pm 2$

Let us substitute 1 in the polynomial $x^2 - 2x^2 - x + 2$ to get

$(1)^3 - 2(1)^2 - (1) + 2 = 1 - 1 - 2 + 2 = 0$

Thus, according to factor theorem, we can conclude that (x – 1) is a factor of the polynomial

$x^3 - 2x^2 - x + 2$

Let us divide the polynomial $x^3 - 2x^2 - x + 2$ by (x -1) , to get

$$\begin{array}{r} x^2 - x - 2 \\ x-1\overline{)x^3 - 2x^2 - x + 2} \\ \underline{x^3 - x^2} \\ -x^2 - x \\ \underline{-x^2 + x} \\ -2x + 2 \\ \underline{-2x + 2} \\ 0 \end{array}$$

$x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$

$x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$.

$= (x - 1)(x^2 + x - 2x - 2)$

$= (x - 1)[x(x + 1) - 2(x + 1)]$

$= (x - 1)(x - 2)(x + 1)$.

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$, we get $(x - 1)(x - 2)(x + 1)$.

(ii) $x^3 - 3x^2 - 9x - 5$

We need to consider the factors of -5, which are $\pm 1, \pm 5$.

Let us substitute 1 in the polynomial $x^3 - 3x^2 - 9x - 5$, to get

$(-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$

Thus, according to factor theorem, we can conclude that (x -1) is a factor of the polynomial $x^3 - 3x^2 - 9x - 5$.

Let us divide the polynomial $x^3 - 3x^2 - 9x - 5$ By $(x + 1)$, to get

$$\begin{array}{r} x^2 - 4x - 5 \\ x+1\overline{)x^3 - 3x^2 - 9x - 5} \\ \underline{x^3 + x^2} \\ -4x^2 - 9x \\ \underline{-4x^2 - 4x} \\ -5x - 5 \\ \underline{-5x - 5} \\ 0 \end{array}$$

$x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$

$= (x - 1)(x^2 + x - 5x - 5)$

$= (x + 1)[x(x + 1) - 5(x + 1)]$

$= (x + 1)(x - 5)(x + 1).$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 3x^2 - 9x - 5$ we get

$(x + 1)(x - 5)(x + 1).$

(iii) $x^3 + 13x^2 + 32x + 20$

We need to consider the factors of 20, which are $\pm 5, \pm 4, \pm 2, \pm 1$.

Let us substitute -1 in the polynomial $x^3 + 13x^2 + 32x + 20$, to get

$(-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = -20 + 20 = 0$

Thus, according to factor theorem, we can conclude that $(x + 1)$ is a factor of the polynomial

$x^3 + 13x^2 + 32x + 20$

Let us divide the polynomial $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$, to get

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & \underline{x^3 + x^2} \\
 & \quad - \quad - \\
 & 12x^2 + 32x \\
 & \underline{12x^2 + 12x} \\
 & \quad - \quad - \\
 & 20x + 20 \\
 & \underline{20x + 20} \\
 & \quad - \quad - \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^2 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 2x + 10x + 20)$

$= (x + 1)[x(x + 2) + 10(x + 2)]$

$= (x + 1)(x + 10)(x + 2).$

Therefore, we can conclude that on factorizing the polynomial $x^3 + 13x^2 + 32x + 20$, we get $(x + 1)(x + 10)(x + 2)$.

(iv) $2y^3 - y^2 - 2y + 1$

We need to consider the factors of -1 which are ± 1

Let us substitute 1 in the polynomial $2y^3 - y^2 - 2y + 1$,to get

$$2(1)^3 + (1)^2 - 2y - 1 = 2 + 1 - 2 - 1 = 3 - 3 = 0$$

Thus, according to factor theorem, we can conclude that (y – 1) is a factor of the polynomial

$2y^3 - y^2 - 2y + 1$

Let us divide the polynomial $2y^3 - y^2 - 2y + 1$ by (y – 1), to get

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & \underline{2y^3 - 2y^2} \\
 & \quad\; - \qquad + \\
 & 3y^2 - 2y \\
 & \underline{3y^2 - 3y} \\
 & \quad\; - \qquad + \\
 & y - 1 \\
 & \underline{y - 1} \\
 & - \quad - \\
 & \underline{\quad 0} \\
\end{array}
$$

$2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1$

$= (y - 1)[2y(y + 1) + 1(y + 1)$

$= (y - 1)(2y + 1)(y + 1).$

Therefore, we can conclude that on factorizing the polynomial $2y^3 + y^2 - 2y - 1$ we get $(y - 1)(2y + 1)(y + 1)$.