SpeedLabs

MATHS

CBSE 10th

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Q.1 Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

(i) $2x^3 + x^2 - 5x + 2$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$

$= 0$

$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2 = 0$

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2 = 0$

Therefore, $\frac{1}{2}, 1$ and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, w eobtain $a = 2, b = 1, c = -5, d = 2$

we can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficient is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes of the polynomial are 2, 1, 1.

$p(2) = 2^3 - 4(2^2) + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 0$

$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1, b = -4, c = 5, d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes $= 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

Multiplication of zeroes taking two at a time $= (2)(1) + (1)(1) + (2)(1)$

$= 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes $= 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Q.2 Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Sol: Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-7}{1} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$

If $a = 1$, then $b = -2, c = -7, d = 14$.

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q.3 If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Sol: $p(x) = x^3 - 3x^2 + x + 1$

Zeroes are $a - b, a, a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + 1$, we obtain

$p = 1, q = -3, r = 1, t = 1$

Sum of zeroes $= a - b + a + a + b$

$\frac{-q}{p} = 3a$

$\frac{-(-3)}{1} = 3a$

$a = 1$

The zeroes are $1 - b, 1, 1 + b$

Multiplication of zeroes $= 1(1-b)(1+b)$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$1 - b^2 = -1$

$1 + 1 = b^2$

$b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Q.4 It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138 - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Sol: Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = x^2 + 4 - 4x - 3 = x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138 - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138 - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Q.5 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial$x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Sol: By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 25x + 10 - a$ by $x^2 - 2x + k$.

$$\text{Quotient: } x^2 - 4x + (8 - k)$$

$$x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a$$

$$x^4 - 2x^3 + kx^2$$

$$- \quad + \quad -$$

$$-4x^3 + (16 - k)x^2 - 26x$$

$$-4x^3 + 8x^2 - 4kx$$

$$+ \quad - \quad +$$

$$(8 - k)x^2 - (26 - 4k)x + 10 - a$$

$$(8 - k)x^2 - (16 - 2k)x + (8k - k^2)$$

$$- \quad + \quad -$$

$$(-10 + 2k)x + (10 - a - 8k + k^2)$$

It can be observed that $(-10 + 2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$2k = 10$

And thus, $k = 5$

For $(10 - a - 8k + k^2) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$