SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

Polynomials

Exercise-2.5

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"An Innovative Practice Methodology by IITians."

1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ **(ii)** $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ **(iv)** $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans - **(i)** $(x+4)(x+10)$

We know that $(x+a)(x+b) = x^2 + (a+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$(x+4)(x+10) = x^2 + (4+10)x + (4 \times 10)$

$= x^2 + 14x + 40$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2 + 14x + 40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b) = x^2 + (a+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$(x+8)(x-10) = x^2 + [8+(-10)]x + [8 \times (-10)]$

$= x^2 - 2x - 80$.

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2 - 2x - 80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b) = x^2 + (a+b) = x^2 + (a+b)x + ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$(3x+4)(3x-5) = (3x)^2 + [4+(-5)3x + [4 \times (-5)]$

$= 9x^2 - 3x - 20$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2 - 3x - 20$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

We know that $(x+y)(x-y) = x^2 - y^2$

We need to apply the above identity to find the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$= (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$

(v) $(3 + 2x)(3 - 2x)$

We know that $(x + y)(x - y) = x^2 - y^2$

We need to apply the above identity to find the product $(3 + 2x)(3 - 2x)$

$(3 + 2x)(3 - 2x) = (3)^2 - (2x)^2$

$= 9 - 4x^2$.

Therefore, we conclude that the product $(3 + 2x)(3 - 2x)$ to $(9 - 4x^2)$.

2. Evaluate the following products without multiplying directly:

(i) 103×107 **(ii)** 95×96 **(iii)** 104×96

Ans - **(i)** 103×107

103×107 can also be written as $(100 + 3)(100 + 7)$

We can observe that, we can apply the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(100 + 3)(100 + 7) = (100)^2 + (3 + 7)(100) + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

Therefore, we conclude that the value of the product 103×107 is 11021

(ii) 95×96

95×96 can also be written as $(100 + 5)(100 + 4)$

We can observe that, we can apply the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(100 - 5)(100 - 4) = (100)^2 + [(-5) + (-4)](100) + (-5) \times (-4)$

$= 10000 – 9000 + 20$

$= 9120$

Therefore, we conclude that the value of the product 95×96 is 9120

(iii) 104×96

104×96 can also be written as $(100 + 4)(100 - 4)$

We can observe that, we can apply the identity $(x + y)(x - y) = x^2 - Y^2$ with respect to the expression (100 + 4) (100 - 4), to get

$(100 + 4)(100 - 4) = (100)^2 - (4)^2$

$= 10000 – 16$

$= 9984$

Therefore, we conclude that the value of the product 104×96 is 9984.

3. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$ **(ii)** $4Y^2 - 4y + 1$

(iii) $x^2 - \frac{Y^2}{100}$

Ans - **(i)** $9x^2 + 6xy + y^2$

$9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$

We can observe that, we can apply the identity $(x + y)^2 = x^2 + 2xy + y^2$

$\Rightarrow (3x)^2 + 2 \times 3x \times y \times (y)^2 = (3x + y)^2$

(ii) $4Y^2 - 4y + 1$

$4y^2 - 4y + 1 = (2\,y)^2 - 2 \times 2y \times 1 + (1)^2$

We can observe that, we can apply the identity $(x - y)^2 = x^2 - 2xy + y^2$

$\Rightarrow (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y + 1)^2$

(iii) $x^2 - \frac{Y^2}{100}$

We can observe that, we can apply the identity $(x)^2 - (y)^2 = (x + y)(x - y)$

$\Rightarrow (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$ **(ii)** $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ **(iv)** $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ **(vi)** $\left(\frac{1}{4}a - \frac{1}{2}\,b + 1\right)^2$

Ans - **(i)** $(x + 2y + 4z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(x + 2y + 4z)^2$

$(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(2x - y + z)^2$

$(2x - y + z)^2 = [2x + (-y) + z]^{-2}$

$= (2x)^2 + (-y) + (z)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$

$(-2+3y+2z)^2 = [(-2x)+3y+2z]^{-2}$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2xy + 2yz + 2zx$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a-7b-c)^2$

We know that $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$

$(3a-7b-c)^2 = [3a+(-7b)+(-c)]^{-2}$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2\times 3a\times(-7b) + 2\times(-7b)\times(-c) + 2\times(-c)\times 3a$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.$

(v) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$

$(-2x+5y-3z)^2 = [(-2x)+5y+(-3z)]^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2\times(-2x)\times 5y + 2\times 5y\times(-3z) + 2\times(-3z)\times(-2)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx.$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

We know that $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

$\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left[\frac{a}{4} + \left(-\frac{b}{2}\right) + 1\right]^{-2}$

$= \left(\frac{a}{4}\right)^2 + \left(-\frac{b}{2}\right)^2 + (1) + 2\times\frac{a}{4}\times\left(-\frac{b}{2}\right) + 2\times\left(-\frac{b}{2}\right)\times 1 + 2\times 1\times\frac{a}{4}$

$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}.$

5. Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy$

Ans - **(i)** $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 24yz - 16xz$

The expression $x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 24yz - 16xz$ can also be written as

$(2x)^2 + (3y)^2 + (-4z)^2 + 2\times 2x\times 3y\times(-4z) + 2\times(-4z)\times 2x.$

We can observe that, we can apply the identity $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with respect to the expression $(2x)^2 + (3y)^2 + (-4z)^2 + 2\times 2x\times 3y + 2\times 3y(-4z) + 2\times(-4z)\times 2x$,to get

$(2x+3y-4z)^2$

Therefore, we conclude that after factorizing the expression $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$,we get $(2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy$

The expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy$ can also be written as

$\left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2 \times \left(-\sqrt{2}x\right) \times y \times \left(2\sqrt{2}x\right) \times y + 2 \times y \times \left(2\sqrt{2}z\right) + 2 \times \left(2\sqrt{2}z\right) \times \left(-\sqrt{2}x\right)$

We can observe that, we can apply the identity $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

with respect to the expression

$\left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2 \times \left(-\sqrt{2}x\right) \times y \times \left(2\sqrt{2}x\right) \times y + 2 \times y \times \left(2\sqrt{2}z\right) + 2 \times \left(2\sqrt{2}z\right) \times \left(-\sqrt{2}x\right)$

to get $\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xy$, we get $\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$ **(ii)** $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$ **(iv)** $\left(x - \frac{2}{3}y\right)^3$

Ans - **(i)** $(2x + 1)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\therefore (2x + 1)^3 = (2x)^3 + (1)^3 + 3 \times 2x \times 1(2x + 1)$

$= 8x^2 + 1 + 6x(2x + 1)$

$8x^2 + 12x^2 + 6x + 1.$

Therefore, the expansion of the expression $(2x + 1)^3$ is $8x^2 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\therefore (2a + 3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a - 3b)$

$= 8x^3 - 27b^3 - 18ab(2a + 3b)$

$= 8x^3 + 36a^2b + 54ab^2 - 27b^3.$

Therefore, the expansion of the expression $8x^3 + 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x + 1\right)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\therefore \left(\frac{3}{2}x + 1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \frac{3}{2}x \times 1\left(\frac{3}{2}x + 1\right)$

$= \frac{3}{2}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x + 1\right)$

$\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$

Therefore, the expansion of the expression $\left(\frac{3}{2}x + 1\right)^3$ is $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x - \frac{2}{3}y\right)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\therefore \left(x - \frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3 \times x \times 3 \times \frac{2}{3}y\left(x - \frac{2}{3}y\right)$

$= x^2 - \frac{8}{27}y^2 - 2xy\left(x - \frac{2}{3}y\right)$

$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^2.$

Therefore, the expansion of the expression $\left(x - \frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^2.$

7. Evaluate the following using suitable identities:

(i) $(99)^3$ **(ii)** $(102)^3$ **(iii)** $(998)^3$

Ans - **(i)** $(99)^3$

$(99)^3$ can also be written as $(100 - 1)^3$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(100 - 1)^3 + (100)^3 - (1)^3 - 3 \times 100 \times 1(100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 999999 - 29700$

$= 970299$

(ii) $(102)^3$

$(102)^3$ Can also be written as $(100 + 2)^3$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(100 + 2)^3 = (100)^3 + (2)^3 + 3 \times 100 \times 2(100 + 2)$

$= 1000000 + 8 + 6000(102)$

$= 1000008 + 61200$

$= 1061208$

(iii) $(998)^3$

$(998)^3$ can also be written as $(100 - 2)^3$

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(1000 - 2)^3 = (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000 - 2)$

$= 1000000000 - 8 - 6000(998)$

$= 999999992 - 5988000$

$= 994011992$

8. Factorize each of the following:

(**i**) $8a^3 + b^3 + 12a^2b + 6ab^2$ (**ii**) $8a^3 - b^3 - 12a^2b + 6ab^2$

(**iii**) $27 - 125a^3 - 135a + 225a^2$ (**iv**) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(**v**) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans - (**i**) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 + (b)^3 + 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b),$

Using identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ With respect to the expression

$(2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b)$, we get $(2a + b)^3$

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a + b)^3$

(**ii**) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$an also be written as

$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b),$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ With respect to the expression

$(2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b)$, we get $(2a - b)^3$

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b - 6ab^2$, we get $(2a - b)^3$

(**iii**) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ an also be written as

$= (3)^3 - (5a)^3 - 3 \times 3 \times 3 \times 5a + 3 \times 3 \times 5a \times 5a$

$= (3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a).$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ With respect to the expression

$(3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a)$, we get $(3 - 5a)^3$

Therefore, after factorizing the expression

$27 - 125a^3 - 135a + 225a^2$, we get $(3 - 5a)^3$

(**iv**) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$

$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ With respect to the expression

$(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$. We get $(4a - 3b)^3$.

Therefore, after factorizing the expression

$64a^3 - 27b^3 - 144a^2b + 108ab^2$, we get $(4a - 3b)^3$.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times 3p \times \frac{1}{6} \times \frac{1}{6}$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ With respect to the expression

$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$, to get $\left(3p - \frac{1}{6}\right)^3$

Therefore, after factorizing the expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ we get $\left(3p - \frac{1}{6}\right)^3$

9. Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ **(ii)** $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Ans - **(i)** $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that $(x-y)^3 = x^3 + Y^3 + 3xy(x+y)$

$\Rightarrow (x-y)^3 = (x+y)^3 - 3xy(x+y)$

$= (x+y)[(x+y)^2 - 3xy]$

$(x+y)[(x+y)^2 - 3xy]$

$\because$ We know that $(x+y)^2 = x^2 + 2xy + y^2$

$\therefore x^3 + y^3 = (x+y)(x^2 + 2xy + y^2 - 3xy)$

$= (x-y)(x^2 - xy + y^2)$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$= (x-y)[(x-y)^2 + 3xy]$

$\because$ We know that $(x-y)^2 = x^2 - 2xy + y^2$

$\therefore x^3 - y^3 = (x-y)(x^2 - 2xy + y^2 + 3xy)$

$= (x-y)(x^2 + xy + y^2)$

Therefore, the desired result has been verified.

10. Factorize:

(**i**) $27y^3 + 125z^3$ (**ii**) $64m^3 - 34n^3$

Ans - (**i**) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$(3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - 3y \times 5z + (5z)^2]$

$(3y + 5z)(9y^2 - 15yz + 25z^2)$.

(**ii**) $64m^3 - 34n^3$

The expression $64m^3 - 34n^3$ can also be written as $(4m)^3 + (7n)^3$.

We know that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$(4m)^3 - (7n)^3 = (4m - 7n)[(4m)^2 + 4m \times 7n + (7n)^2]$

$(4m - 7n)(16m^2 - 28mn + 49n^2)$.

Therefore, we conclude that after factorizing the expression

$64m^3 - 34n^3$ we get $(4m - 7n)(16m^2 - 28mn + 49n^2)$

11. Factorize: $27x^3 + y^3 + z^3 - 9zyz$

Ans - The expression $27x^3 + y^3 + z^3 - 9zyz$ can also be written as

$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

$\therefore (3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z = (3x + y + z)[(3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$.

Therefore, we conclude that after factorizing the expression

$27x^3 + y^3 + z^3 - 9zyz$, we get $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Ans - LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

And also, we know that $(x - y)^2 = x^2 - 2xy + y^2$.

$$\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2(z - x)^2]$$

$$\frac{1}{2}(x + y + z)[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2xy + x^2)]$$

$$\frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2zx)$$

$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

Therefore, we can conclude that the desired result is verified.

13. If$x + y + z = 0$, show that $x^3 + y^2 + z^3 = 0$.

Ans - We know that $x^3 + y^2 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

We need to substitute $x^3 + y^2 + z^3 = 0$ in

$x^3 + y^2 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$. To get

$x^3 + y^2 + z^3 - 3xyz = (0) = (x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$.

Therefore, the desired result is verified.

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$ **(ii)** $(28)^3(-15)^3 + (-13)^3$

Ans - **(i)** $(-12)^3 + (7)^3 + (5)^3$

Let a = -12, b = 7 and c = 5

We know that, if $a + b + c = 0$ then $a^2 + b^2 + c^2 = 3abc$

Here, $a + b + c = 12 + 7 + 5 = 0$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3(-15)^3 + (-13)^3$

Let a = 28, b = - 15 and c = - 13

We know that, if $a + b + c = 0$ then $a^2 + b^2 + c^2 = 3abc$

Here, $a + b + c = 28 – 15 – 13 = 0$

$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Are: $25a^2 - 35a + 12$ **(ii)** Are: $35y^2 + 13a + 12$

Ans - **(i)** Are: $25a^2 - 35a + 12$

The expression $25a^2 - 35a + 12$ can also be written as $25a^2 - 15a - 20a + 12$

$25a^2 - 15a - 20a + 12 = 5a(5a - 3) - 4(5a - 3)$

$= (5a - 4)(5a - 3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2 - 35a + 12$ is Lengh $= (5a - 4)$ andBreadth $= (5a - 3)$.

(ii) Are: $35y^2 + 13a + 12$

The expression $35y^2 + 13a + 12$ can also be written as $35a^2 + 28y - 15y - 12$

$35a^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4)$

$= (7y – 3)(5y + 4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13a + 12$ is Lengh $= (7y – 3)$ and Breadth $=)(5y + 4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2 - 12x$ **(ii)** Volume: $12ky^2 + 8ky - 20k$

Ans - **(i)** Volume: $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x - 4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is $3, x$ and $(x - 4)$.

(ii) Volume: $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$k(12y^2 + 8y - 20) = k(12y^2 - 12y + 20y - 20)$

$= k[12y(y - 1) + 20(y - 1)]$

$k(12y + 20)(y - 1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k, (3y + 5)$ and $(y - 1)$.