SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Q.1 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1+3+3^2+\text{.........}+3^{n-1}=\frac{\left(3^n-1\right)}{2}$$

Ans. Let the given statement be P(*n*), i. e

P(n): 1 + 3 + 32 + ...+ 3n–1 $\frac{\left(3^n-1\right)}{2}$

For n = 1, we have

P (1): $1=\frac{\left(3^1-1\right)}{2}=\frac{3-1}{2}=\frac{2}{2}=1$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$1+3+3^2+\text{.........}\frac{3^k-1}{2}$...............(i)

We shall now prove that P(k + 1) is true.

Consider

$1 + 3 + 3^2 + ... + 3^{k}–1 + 3^{(k+1)-1}$

= (1 + 3 + 32 +... + 3^{k}–1) + 3^k

$$=\frac{\left(3^k-1\right)}{2}+3^k \qquad [\text{Using(i)}]$$

$$=\frac{\left(3^k-1\right)}{2}+2.3^k$$

$$=\frac{\left(1+2\right)3^k-1}{2}$$

$$=\frac{3.3^k-1}{2}$$

$$=\frac{3^{k+1}-1}{2}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.2 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1^3+2^3+3^3+\text{....}+n^3=\left(\frac{n\left(n+1\right)}{2}\right)^2$$

Ans. Let the given statement be P(n), i.e.,

$$P(n):\ 1^3+2^3+3^3+\ldots.+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

For n = 1, we have

$$P(1):\ 13 = 1 = =\left(\frac{1(1+1)}{2}\right)^2=\left(\frac{1.2}{2}\right)^2=1^2=1,\text{ which is true.}$$

Let P(k) be true for some positive integer k, i.e.,

$$^3+2^3+3^3+\ldots..+k^3=\left(\frac{k(k+1)}{2}\right)^2 \quad \ldots\ldots..(i)$$

We shall now prove that P (k + 1) is true.

Consider

$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k + 1)^3$

$$=\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3\ (\text{Using(i)})$$

$$=\frac{k^2(k+1)^2}{4}+(k+1)^3$$

$$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$$

$$=\frac{(k+1)^2\left\{k^{2+4}(k+1)\right\}}{4}$$

$$=\frac{(k+1)^2(k+2)^2}{4}$$

$$=\frac{(k+1)^2(k+1+1)^2}{4}$$

$$=\left(\frac{(k+1)(k+1+1)}{2}\right)^2$$

= (13 + 23 + 33 + …. + k3) + (k + 1)3

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.3 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots..n)}=\frac{2n}{(n+1)}$$

Ans. Let the given statement be P(n), i.e.,

$$P(n):\ 1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots..n)}=\frac{2n}{(n+1)}$$

For n = 1, we have

$P(1): 1=\frac{2.1}{1+1}=\frac{2}{2}=1$ which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1+\frac{1}{1+2}+\ldots.+\frac{1}{1+2+3}+..+\frac{1}{1+2+3+\ldots.+k}=\frac{2k}{k+1}\ldots(i)$$

We shall now prove that P (k + 1) is true.

Consider

$$1+\frac{1}{1+2}+\ldots.+\frac{1}{1+2+3}+..+\frac{1}{1+2+3+\ldots.+k}=\frac{1}{1+2+3+..+k+(k+1)}\ldots(i)$$

$$=\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots..k}\right)+\frac{1}{1+2+3+\ldots.+k+(k+1)}$$

$$=\frac{2k}{k+1}+\frac{1}{1+2+3\ldots+k+(k+1)}\quad[\text{Using(i)}]$$

$$=\frac{2k}{k+1}+\frac{1}{\left(\frac{(k+1)(k+1+1)}{2}\right)}\quad\left[1+2+3+\ldots..n=\frac{n(n+1)}{2}\right]$$

$$=\frac{2k}{k+1}+\frac{2}{(k+1)(k+2)}$$

$$=\frac{2k}{k+1}\left(k+\frac{1}{(k+2)}\right)$$

Q.4 Prove the following by using the principle of mathematical induction for all n ∈ N: 1.2.3 + 2.3.4 + … + n (n + 1) (n + 2) = $\frac{n(n+1)(n+2)(n+3)}{4}$

Ans. Let the given statement be P(n), i.e.,

P(n): 1.2.3 + 2.3.4 + … + n (n + 1) (n + 2) = $\frac{n(n+1)(n+2)(n+3)}{4}$

For n = 1, we have

P (1): 1.2.3 = 6 = $\frac{1(1+1)(1+2)(1+3)}{4}=\frac{1.2.3.4}{4}=6$, which is true.

Let P(k) be true for some positive integer k, i.e.,

1.2.3 + 2.3.4 + ... + k (k + 1) (k + 2) = $\frac{k(k+1)(k+2)(k+3)}{4}$........(i)

We shall now prove that P (k + 1) is true.

Consider

1.2.3 + 2.3.4 + ... + k (k + 1) (k + 2) + (k + 1) (k + 2) (k + 3)

= {1.2.3 + 2.3.4 + ... + k (k + 1) (k + 2)} + (k + 1) (k + 2) (k + 3)

$=\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3)$ [Using(i)]

$=(k+1)(k+2)(k+3)\left(\frac{k}{4}+1\right)$

$=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$

$=\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$

Thus, P k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.5 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1.3+2.3^2+3.3^3+....+n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$$

Ans. Let the given statement be P(n), i.e.,

P(n): $1.3+2.3^2+3.3^3+....+n.3^n=\frac{(2n-1)3^{n+1}+3}{4}$

For n = 1, we have

P (1): 1.3 = 3, $=\frac{(2.1-1)^{1+1}+3}{4}=\frac{3^2+3}{4}=\frac{12}{4}=3$ which is true.

Let P(k) be true for some positive integer k, i.e.,

$1.3+2.3^2+3.3^3+....+k.3^n=\frac{(2k-1)3^{k+1}+3}{4}$....(i)

We shall now prove that P (k + 1) is true.

1.3 + 2.3^2 + 3.3^3 + ... + k3^k+ (k + 1) 3^{k+1}

= (1.3 + 2.3^2 + 3.3^3 + ...+ k.3^k) + (k + 1) 3^{k+1}

$$=\frac{(2k-1)3^{k+1}+3}{4}\ (k+1)3^{k+1} \quad (\text{Using}(i))$$

$$=\frac{(2k-1)3^{k+1}+3+4(k+1)3^{k+1}}{4}$$

$$=\frac{3^{k+1}\{2k-1+4(k+1)\}+3}{4}$$

$$=\frac{3^{k+1}\{6k+3\}+3}{4}$$

$$=\frac{3^{k+1}.3\{2k+1\}+3}{4}$$

$$=\frac{\{2(k+1)-1\}3^{(k+1)+1}+3}{4}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.6 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1.2+2.3+3.4+....+n(n+1)=\left[\frac{n(n+1)(n+2)}{3}\right]$$

Ans. Let the given statement be P(n), i.e.,

$$1.2+2.3+3.4+....+n(n+1)=\left[\frac{n(n+1)(n+2)}{3}\right]$$

P(n):

For n = 1, we have

P (1): $1.2=2=\frac{1(1+1)(1+2)}{3}=\frac{1.2.3}{3}=2$ which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1.2+2.3+3.4+.....+k.\frac{k(k+1)(k+2)}{3}........(i)$$

We shall now prove that P(k + 1) is true.

Consider

1.2 + 2.3 + 3.4 + … + k. (k + 1) + (k + 1). (k + 2)

= [1.2 + 2.3 + 3.4 + … + k. (k + 1)] + (k + 1). (k + 2)

$$=\frac{k(k+1)(k+2)}{3}+(k+1)(k+2) \quad [\text{Using}(i)]$$

$$=(k+1)(k+2)\left(\frac{k}{3}+1\right)$$

$$=\frac{(k+1)(k+2)(k+3)}{3}$$

$$=\frac{(k+1)(k+1+1)(k+1+2)}{3}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.7 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.3+3.5+5.7+\ldots+(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$$

Ans. Let the given statement be P(n), i.e.,

$$1.3+3.5+5.7+\ldots+(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$$

P(n):

For n = 1, we have

$p(1): 1.3=3=\frac{1\left(4.1^2+6.1-1\right)}{3}=\frac{4+6-1}{3}=\frac{9}{3}=3$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$1.3+3.5+5.7+\ldots.+(2k-1)(2k+1)=\frac{k\left(4k^2+6k-1\right)}{3}$$

We shall now prove that P (k + 1) is true.

Consider

(1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) + {2(k + 1) – 1} {2(k + 1) + 1}

$$=\frac{k\left(4k^2+6k-1\right)}{3}+(2k+2-1)(2k+2+1) \quad [\text{Using(i)}]$$

$$=\frac{k\left(4k^2+6k-1\right)}{3}+(2k+1)(2k+3)$$

$$= \frac{k\left(4k^2+6k-1\right)}{3}+\left(4k^2+8k+3\right)$$

$$= \frac{k\left(4k^2+6k-1\right)+\left(4k^2+8k+3\right)}{3}$$

$$= \frac{4k^2+6k^2-k+12k^2+24k+9}{3}$$

$$= \frac{4k^3+18k^2+23k+9}{3}$$

$$= \frac{4k^3+14k^2+9k+4k^2+14k+9}{3}$$

$$= \frac{k\left(4k^2+14k+9\right)+1\left(4k^2+14k+9\right)}{3}$$

$$= \frac{(k+1)\left(4k^2+14k+9\right)}{3}$$

$$= \frac{\left(k+1\left\{4k^2+8k+4+6k+6-1\right\}\right)}{3}$$

$$= \frac{(k+1)\left\{4\left(k^2+2k+1\right)+6(k+1)-1\right\}}{3}$$

$$= \frac{(k+1)\left\{4(k+1)^2+6(k+1)-1\right\}}{3}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.8 Prove the following by using the principle of mathematical induction for all $n \in N$: $1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n-1)\,2^{n+1} + 2$

Ans. Let the given statement be P(n), i.e.,

P(n): $1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n-1)\,2^{n+1} + 2$

For n = 1, we have

P (1): $1.2 = 2 = (1-1)\,2^{1+1} + 2 = 0 + 2 = 2$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$1.2 + 2.2^2 + 3.2^2 + \ldots + k.2^k = (k-1)\,2^{k+1} + 2$ ... (i)

We shall now prove that P (k + 1) is true.

Consider

$$\left\{1.2+2.2^2+3.2^3+\ldots.+k.2^k\right\}+(k+1).2^{k+1}$$
$$=(k-1)2^{k+1}+2+2(k+1)2^{k+1}$$
$$=2^{k+1}\left\{(k-1)+(k+1)\right\}+2$$
$$=2^{k+1}.2k+2$$
$$=k.2^{(k+1)+1}+2$$
$$=\left\{(k+1)-1\right\}2^{(k+1)+1}+2$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.9 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

Ans. Let the given statement be P(n), i.e.,

$$P(n):\ \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

For n = 1, we have

P (1): $\frac{1}{2}=1-\frac{1}{2^1}=\frac{1}{2}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots.+\frac{1}{2^k}=1-\frac{1}{2^k}\ \ldots\ldots(i)$$

We shall now prove that P (k + 1) is true.

Consider

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots.+\frac{1}{2^k}=1-\frac{1}{2^k}$$
$$=\left(1-\frac{1}{2^k}\right)+\frac{1}{2^{k+1}}\quad[\text{Using(i)}]$$
$$=1-\frac{1}{2^k}+\frac{1}{2.2^k}$$
$$=1-\frac{1}{2^k}\left(1-\frac{1}{2}\right)$$
$$=1-\frac{1}{2^k}\left(\frac{1}{2}\right)$$
$$=1-\frac{1}{2^{k+1}}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n

Q.10 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

Ans. Let the given statement be P(n), i.e.,

$$P\ (n):\ \frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

For n = 1, we have

$$p(1)=\frac{1}{2.5}=\frac{1}{10}=\frac{1}{6.1+4}=\frac{1}{10}$$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{2.5}+\frac{1}{5.8}=\frac{1}{8.11}+\ldots.+\frac{1}{(3k-1)(3k+2)}=\frac{k}{6k+4}\ \ldots\ldots\ldots\text{(i)}$$

We shall now prove that P (k + 1) is true.

Consider

$$\frac{1}{2.5}+\frac{1}{5.8}=\frac{1}{8.11}+\ldots.+\frac{1}{(3k-1)(3k+2)}=\frac{k}{\{3(k+1)-1\}\{3(k+1)+2\}}$$

$$=\frac{k}{6k+4}+\frac{1}{(3k+3-1)(3k+3+2)}$$

$$=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)}$$

$$=\frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+5)}$$

$$=\frac{1}{(3k+2)}+\left(\frac{k}{2}+\frac{1}{3k+5}\right)$$

$$=\frac{1}{(3k+2)}\left(\frac{k(3k+5)+2}{2(3k+5)}\right)$$

$$=\frac{1}{(3k+2)}\left(\frac{3k^2+5k+2}{2(3k+5)}\right)$$

$$=\frac{1}{(3k+2)}\left(\frac{(3k+2)(k+1)}{2(3k+5)}\right)$$

$$= \frac{(k+1)}{6k+10}$$

$$= \frac{(k+1)}{6(k+1)+4}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.11 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+.....+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

Ans. Let the given statement be P(n), i.e.,

$$P(n): \frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+.....+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

For n = 1, we have

$$p(1)\frac{1}{1.2.3}\frac{1.(1+3)}{4(1+1)(1+2)}=\frac{1.4}{4.2.3}=\frac{1}{1.2.3}$$, which is true.

Let P(k) be true for some positive integer k, i.e

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+.....+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)} \quad ....(i)$$

We shall now prove that P(k + 1) is true.

$$\left[\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{k(k+1)(k+2)}\right]+\frac{1}{(k+1)(k+2)(k+3)}$$

$$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)}{4}+\frac{1}{k+3}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)^2+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k^2+6k+9)+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k^3+6k^2+9k+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k\left(k^2+2k+1\right)+4\left(k^2+2k+1\right)}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+1)^2+4(k+1)^2}{4(k+3)}\right\}$$

$$=\frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}$$

$$=\frac{(k+1)\{(k+1)+3\}}{4\{(k+1)+1\}\{(k+1)+2\}}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n

Q.12 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$a+ar+ar^2+....+ar^{n-1}=\frac{a\left(r^n-1\right)}{r-1}$$

Ans. Let the given statement be P(n), i.e

$$a+ar+ar^2+....+ar^{n-1}=\frac{a\left(r^n-1\right)}{r-1}$$

For n = 1, we have

$p(1)=\frac{a\left(r^1-1\right)}{r-1}=a$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$a+ar+ar^2+....+ar^{k-1}=\frac{a\left(r^k-1\right)}{r-1}.....(1)$$

We shall now prove that P (k + 1) is true.

Consider

$$\left\{a+ar+ar^2+.....+ar^{k-1}\right\}+ar^{(k+1)-1}$$

$$=\frac{a\left(r^k-1\right)}{r-1}+ar^k$$

$$=\frac{a\left(r^k-1\right)+ar^k(r-1)}{r-1}$$

$$=\frac{a\left(r^k-1\right)+ar^{k+1}-ar^k}{r-1}$$

$$= \frac{ar^k - a + ar^{k+1} - ar^k}{r-1}$$

$$= \frac{ar^{k+1} - a}{r-1}$$

$$= \frac{a\left(r^{k+1}-1\right)}{r-1}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.13 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$P(n): \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)......\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$$

Ans. Let the given statement be P(n), i.e.,

$$P(n): \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)......\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$$

For n = 1, we have

$$P(1): \left(1+\frac{3}{1}\right) = 4 = (1+1)^2 = 2^2 = 4, \text{which is true}$$

Let P(k) be true for some positive integer k, i.e.,

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).....\left(1+\frac{(2k+1)}{k^2}\right) = (k+1)^2$$

We shall now prove that P (k + 1) is true.

Consider

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right).....\left(1+\frac{(2k+1)}{k^2}\right)\left\{1+\frac{\{2(k+1)+1\}}{(k+1)^2}\right\}$$

$$= (k+1)^2\left(1+\frac{2(k+1)+1}{(k+1)^2}\right)$$

$$= (k+1)^2\left[\frac{(k+1)^2 + 2(k+1)+1}{(k+1)^2}\right]$$

$$= (k+1)^2 + 2(k+1) + 1$$

$$= \{(k+1)+1\}^2$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.14 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right).....\left(1+\frac{1}{n}\right)=(n+1)$$

Ans. Let the given statement be P(n), i.e.,

$$P(n)\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right).....\left(1+\frac{1}{n}\right)=(n+1)$$

For n = 1, we have

$P(1):\left(1+\frac{1}{1}\right)=2=(1+1)$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$P(k)\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right).....\left(1+\frac{1}{k}\right)=(+1)\ ....(1)$$

We shall now prove that P (k + 1) is true.

Consider

$$\left[\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)....\left(1+\frac{1}{k}\right)\right]\left(1+\frac{1}{k+1}\right)$$

$$=(k+1)\left(1+\frac{1}{k+1}\right)............[\text{Using}(1)]$$

$$=(k+1)\left(\frac{(k+1)+1}{(k+1)}\right)$$

$$=(k+1)+1$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.15 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1^2+3^2+5^2+......+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$$

Ans. Let the given statement be P(n), i.e.,

$$1^2+3^2+5^2+......+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$$

For $n=1$, we have

$P(1)=1^2=1=\frac{1(2.1-1)(2.1+1)}{3}=\frac{1.1.3}{3}=1$, which is tru

Let P(k) be true for some positive integer k, i.e.,

$$P(k) = 1^2 + 3^2 + 5^2 + ... + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3} ....(1)$$

We shall now prove that P (k + 1) is true.

Consider

$$\left\{1^2 + 3^2 + 5^2 + ... + (2k-1)^2\right\} + \left\{2(k+1)-1\right\}^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+2-1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$= \frac{(2k+1)\left\{k(2k-1) + 3(2k+1)\right\}}{3}$$

$$= \frac{(2k+1)\left\{2k^2 - k + 6k + 3\right\}}{3}$$

$$= \frac{(2k+1)\left\{2k^2 + 5k + 3\right\}}{3}$$

$$= \frac{(2k+1)(2k^2 + 2k + 3k + 3)}{3}$$

$$= \frac{(2k+1)\left\{2k(k+1) + 3(k+1)\right\}}{3}$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= \frac{(k+1)\left\{2(k+1)-1\right\}\left\{2(k+1)+1\right\}}{3}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.16 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

Ans. Let the given statement be P(n), i.e.,

$$P(1): \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

For $n = 1$, we have

$$P(1) = \frac{1}{1.4} = \frac{1}{3.1+1} = \frac{1}{4} = \frac{1}{1.4}, \text{which is true}$$

Let P(k) be true for some positive integer k, i.e.,

$$P(k) = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \ldots..(1)$$

We shall now prove that P (k + 1) is true.

Consider

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{\{3(k+1)-2\}\{3(k+1)+1\}} \ldots..(1)$$

$$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} \qquad [\text{Using}(1)]$$

$$= \frac{1}{3k+1} + \left\{k + \frac{1}{(3k+4)}\right\}$$

$$= \frac{1}{3k+1}\left\{k\frac{(3k+4)+1}{(3k+4)}\right\}$$

$$= \frac{1}{3k+1}\left\{\frac{3k^2+4k+1}{3k+4}\right\}$$

$$= \frac{1}{3k+1}\left\{\frac{3k^2+3k+k+1}{(3k+4)}\right\}$$

$$= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)}{3(k+1)+1}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.17 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots.. + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Ans. Let the given statement be P(n), i.e.,

$$P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots.. + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

For n = 1, we have

$P(1): \frac{1}{3.5} = \frac{1}{3(2.1+3)} = \frac{1}{3.5}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$$P(k): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2k+1)(2k+3)} = \frac{n}{3(2k+3)} .....(1)$$

We shall now prove that P (k + 1) is true.

Consider

$$P(k): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2k+1)(2k+3)} + \frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}} .....(1)$$

$$= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \qquad [\text{Using}(1)]$$

$$= \frac{1}{(2k+3)}\left[\frac{k}{3} + \frac{1}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{3(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{3(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{3(2k+5)}\right]$$

$$= \frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$$

$$= \frac{(k+1)}{3\{2(k+1)+3\}}$$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n

Q.18 Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1+2+3+....+n < \frac{1}{8}(2n+1)^2$$

Ans. Let the given statement be P(n), i.e.,

$P(n): 1+2+3+....+n < \frac{1}{8}(2n+1)^2$

It can be noted that P(n) is true for n = 1 since $1 < \frac{1}{8}(2.1+1)^2 = \frac{9}{8}$.

Let P(k) be true for some positive integer k, i.e.,

$1+2+....+k < \frac{1}{8}(2k+1)^2$ ....(1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$(1+2+....+k)+(k+1) < \frac{1}{8}(2k+1)^2 + (k+1)$ [Using(1)]

$= < \frac{1}{8}\{(2k+1)^2 + 8(k+1)\}$

$= < \frac{1}{8}\{4k^2 + 4k + 1 + 8k + 8\}$

$= < \frac{1}{8}\{4k^2 + 12k + 9\}$

$= < \frac{1}{8}(2k+3)^2$

$= < \frac{1}{8}\{2(k+1)+1\}2$

Hence, $(1+2+3+..........+k)+(k+1) < \frac{1}{8}(2k+1)+(k+1)$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.19 Prove the following by using the principle of mathematical induction for all n ∈ N: n (n +1) (n + 5) is a multiple of 3.

Ans. Let the given statement be P(n), i.e.,

P(n): n (n + 1) (n + 5), which is a multiple of 3.

It can be noted that P(n) is true for n = 1 since 1 (1 + 1) (1 + 5) = 12, which is a multiple of 3.

Let P(k) be true for some positive integer k, i.e.,

k (k + 1) (k + 5) is a multiple of 3.

∴ k (k + 1) (k + 5) = 3m, where m ∈ N … (1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$(k+1)\{(k+1)+1\}\{(k+1)+5\}$

$= (k+1)(k+2)\{(k+5)+1\}$

$= (k+1)(k+2)(k+5)+(k+1)(k+2)$

$= \{k(k+1)(k+5)+2(k+1)(k+5)\}+(k+1)(k+2)$

$= 3m+(k+1)\{2(k+5)+(k+2)\}$

$= 3m+(k+1)\{2k+10+k+2\}$

$= 3m+(k+1)(3k+12)$

$= 3m+3(k+1)(k+4)$

$= 3\{m+(k+1)(k+4)\} = 3\times q$, where $q = \{m+(k+1)(k+4)\}$ is some natural numbers

= therefore, $(k+1)\{(k+1)+1\}\{(k+1)+5\}$ is a multiple of 3.

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.20 Prove the following by using the principle of mathematical induction for all n ∈ N: 102n – 1 + 1 is divisible by 11

Ans. Let the given statement be P(n), i.e.,

P(n): 102n – 1 + 1 is divisible by 11.

It can be observed that P(n) is true for n = 1 since P(1) = 102.1 – 1 + 1 = 11, which is divisible by 11.

Let P(k) be true for some positive integer k, i.e.,

102k – 1 + 1 is divisible by 11.

∴102k – 1 + 1 = 11m, where m ∈ N … (1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$10^{2(k+1)-1}+1$

$= 10^{2k+2-1}+1$

$= 10^{2k+1}+1$

$= 10^2\left(10^{2k-1}+1-1\right)+1$

$= 10^2\left(10^{2k-1}+1\right)-10^2+1$

$= 10^2.11m-100+1 \quad [\text{Using}(1)]$

$= 100\times 11m-99$

$= 11(100m - 9)$

$= 11r$, where $r = (100m - 9)$ is some natural number

there , $10^{2(k+1)-1} + 1$ is divisible by 11.

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.21 Prove the following by using the principle of mathematical induction for all n ∈ N: x2n –y2n is divisible by x + y.

Ans. Let the given statement be P(n), i.e.,

P(n): x2n – y2n is divisible by x + y.

It can be observed that P(n) is true for n = 1.

This is so because x2 × 1 – y2 × 1 = x2 – y2 = (x + y) (x – y) is divisible by (x + y).

Let P(k) be true for some positive integer k, i.e.,

x2k – y2k is divisible by x + y.

∴x2k – y2k = m (x + y), where m ∈ N ... (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$x^{2(k+1)} - y^{2(k+1)}$

$= x^{2k}.x^2 - y^{2k}.y^2$

$= x^2\left(x^{2k} - y^{2k} + y^{2k}\right) - y^{2k}.y^2$

$= x^2\left\{m(x+y) + y^{2k}\right\} - y^{2k}.y^2$

$= m(x+y)x^2 + y^{2k}\left(x^2 - y^2\right)$

$= m(x+y)x^2 + y^{2k}(x+y)(x-y)$

$= (x+y)\left\{mx^2 + y^{2k}(x-y)\right\}$, which ia a factore of $(x+y)$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.22 Prove the following by using the principle of mathematical induction for all n ∈ N: 32n + 2 – 8n – 9 is divisible by 8.

Ans. Let the given statement be P(n), i.e.,

P(n): 32n + 2 – 8n – 9 is divisible by 8.

It can be observed that P(n) is true for n = 1 since 32 × 1 + 2 – 8 × 1 – 9 = 64, which is divisible by 8.

Let P(k) be true for some positive integer k, i.e.,

32k + 2 – 8k – 9 is divisible by 8.

∴32k + 2 – 8k – 9 = 8m; where m ∈ N … (1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$$3^{2(k+1)+2}-8(k+1)-9$$

$$=3^{2(k+1)+2}.3^{2}-8k-8-9$$

$$=3^{2}\left(3^{2k+2}-8k-9+8k+9\right)-8k-17$$

$$=3^{2}\left(3^{2k+2}-8k-9\right)+3^{2}\left(8k+9\right)-8k-17$$

$$=9.8m+9(8k+9)-8k-17$$

$$=9.8m+72k+81-8k-17$$

$$=9.8m+64k+64$$

$$=8(9m+8k+8)$$

$$=8r,\text{where } r=(9m+8k+8)\text{is a natural numbers.}$$

therefore , $3^{2(k+1)+2}-8(k+1)-9$ is divisible by 8

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.23 Prove the following by using the principle of mathematical induction for all n ∈ N: 41n –14n is a multiple of 27.

Ans. Let the given statement be P(n), i.e.,

P(n):41^n – 14nis a multiple of 27.

It can be observed that P(n) is true for n = 1 since $41^1-41^1=27$, which is a multiple of 27.

Let P(k) be true for some positive integer k, i.e., 41^k – $14^k$is a multiple of 27

∴41^k – 14^k = 27m, where m ∈ N … (1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$$41^{k+1}-41^{k+1}$$

$$=41^{k}.41-14^{k}.14$$

$$=41\left(41^{k}-14^{k}+14^{k}\right)-14^{k}.14$$

$$=41\left(41^{k}-14^{k}\right)+41.14^{k}-14^{k}.14$$

$= 41.27m + 27.14^k$

$= 27\left(41m - 14^k\right)$

$= 27 \times r$, where $r = \left(41m - 14^k\right)$ is a natural number

therefore, $41^{k+1} - 14^{k+1}$ is a multiple of 27.

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

Q.24 Prove the following by using the principle of mathematical induction for all (2n +7) < (n + 3)2

Ans. Let the given statement be P(n), i.e.,

P(n): (2n +7) < (n + 3)2

It can be observed that P(n) is true for n = 1 since 2.1 + 7 = 9 < (1 + 3)2 = 16, which is true.

Let P(k) be true for some positive integer k, i.e.,

(2k + 7) < (k + 3)2 ... (1)

We shall now prove that P (k + 1) is true whenever P(k) is true.

Consider

$\left\{2\left(k+1\right)+7\right\} = \left(2k+7\right)+2$

$\therefore \left\{2\left(k+1\right)+7\right\} = \left(2k+7\right)+2 < \left(2k+7\right)^2+2$

$2\left(k+1\right)+7 < k^2+6k+9+2$

$2\left(k+1\right)+7 < k^2+6k+11$

Now, $k^2+6k+11 < k^2+8k+16$

$\therefore 2\left(k+1\right)+7 < \left(k+4\right)^2$

$2\left(k+1\right)+7 < \left\{\left(k+1\right)+3\right\}2$

Thus, P (k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.