SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	-0.1	0.3

(iii)

Y	-1	0	1
P (Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	-1
P (Z)	0.3	0.2	0.4	0.1	0.05

Sol: It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities $= 0.4 + 0.4 + 0.2 = 1$

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for $X = 3$, $P(X) = -0.1$ It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities $= 0.6 + 0.1 + 0.2 = 0.9 \neq 1$ Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities $= 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$

Therefore, the given table is not a probability distribution of random variables

Q.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

Sol: The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

∴X (BB) = 2

X (BR) = 1

X (RB) = 1

X (RR) = 0

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable.

Q.3 Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Sol: A coin is tossed six times and X represents the difference between the number of heads and the number of tails..

∴ X (6 H, 0T) =|6-0|= 6

X (5 H, 1 T) = |5-1| = 4

X (4 H, 2 T) = |4-2| = 2

X (3 H, 3 T) = |2-4| = 0

X (2 H, 4 T) = |2-4| = 2

X (1 H, 5 T) = |1-5| = 4

X (0H, 6 T) = |0-6| = 6

Thus, the possible values of X are 6, 4, 2, and 0.

Q.4 Find the probability distribution of

(i) number of heads in two tosses of a coin

(ii) number of tails in the simultaneous tosses of three coins

(iii) number of heads in four tosses of a coin

Sol: **(i)** When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads.

∴ X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

$$P\,(X = 0) = P\,(TT) = \frac{1}{4}$$

$P(X = 1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(X = 2) = P(HH) = \frac{1}{4}$

Thus, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

$P(X = 0) = P(HHH) = \frac{1}{8}$

$P(X = 1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X = 2) = P(HTT) + P(THT) + P(TTH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X = 3) = P(TTT) = \frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S = \left\{ \begin{matrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{matrix} \right\}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

$P(X = 0) = P(TTTT) = \frac{1}{16}$

$P(X = 1) = P(TTTH) + P(TTHT) + P(THTT) + P(HTTT)$

$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$

P (X = 2) = P (HHTT) + P (THHT) + P (TTHH) + P (HTTH) + P (HTHT) + P (THTH)

$$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$$

P (X = 3) = P (HHHT) + P (HHTH) + P (HTHH) P (THHH)

$$=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$$

P (X = 4) = P (HHHH) $=\frac{1}{16}$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Q.5 Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

Sol: When a die is tossed two times, we obtain (6 × 6) = 36 number of observations.

Let X be the random variable, which represents the number of successes.

(i) Here, success refers to the number greater than 4.

P (X = 0) = P (number less than or equal to 4 on both the tosses) $=\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$

P (X = 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$=\frac{4}{6}\times\frac{2}{6}\times\frac{4}{6}\times\frac{2}{6}=\frac{4}{9}$$

P (X = 2) = P (number greater than 4 on both the tosses)

$$=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$$

Thus, the probability distribution is as follows.

X	1	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

P (Y = 0) = P (six does not appear on any of the dice) $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P (Y = 1) = P (six appears on at least one of the dice) $= \frac{11}{36}$

Thus, the required probability distribution is as follows.

Y	0	1
P(Y)	$\frac{25}{36}$	$\frac{11}{36}$

Q.6 From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Sol: It is given that out of 30 bulbs, 6 are defective.

⇒ Number of non-defective bulbs = 30 − 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

∴P (X = 0) = P (4 non-defective and 0 defective) $= {}^4C_0 . \frac{4}{5} . \frac{4}{5} . \frac{4}{5} . \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non-defective and 1 defective) $= {}^4C_1 . \left(\frac{1}{5}\right) . \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non-defective and 2 defective) $= {}^4C_2 . \left(\frac{1}{5}\right)^2 . \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non-defective and 3 defective) $= {}^4C_3 . \left(\frac{1}{5}\right)^3 . \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non-defective and 4 defective) $= {}^4C_4 . \left(\frac{1}{5}\right)^4 . \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Q.7 A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Sol: Let the probability of getting a tail in the biased coin be x.

$\therefore$ P (T) = x

$\Rightarrow$ P (H) = 3x

For a biased coin, P (T) + P (H) = 1

$\Rightarrow x + 3x = 1$

$\Rightarrow 4x = 1$

$\Rightarrow x = \frac{1}{4}$

$\therefore P(T) = \frac{1}{4}$ and $P(H) = \frac{3}{4}$

When the coin is tossed twice, the sample space is {HH, TT, HT, TH}. Let X be the random variable representing the number of tails.

$\therefore$ P (X = 0) = P (no tail) = P (H) × P (H) $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P (X = 1) = P (one tail) = P (HT) + P (TH)

$= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4}$

$= \frac{3}{16} + \frac{3}{16}$

$= \frac{3}{8}$

P (X = 2) = P (two tails) = P (TT) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Q.8 A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P(X)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Determine

(i) k

(ii) P $(X < 3)$

(iii) P $(X > 6)$

(iv) P $(0 < X < 3)$

Sol: **(i)** It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore 0+k+2k+2k+3k+k^2+2k^2+\left(7k^2+k\right)=1$$

$$\Rightarrow 10k^2+9k-1=0$$

$$\Rightarrow (10k-1)(k+1)=0$$

$$\Rightarrow k=-1, \frac{1}{10}$$

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore k=\frac{1}{10}$$

(ii) P $(X < 3)$ = P $(X = 0)$ + P $(X = 1)$ + P $(X = 2)$

$$=0+k+2k$$

$$=3k$$

$$=3\times\frac{1}{10}$$

$$=\frac{3}{10}$$

(iii) P $(X > 6)$ = P $(X = 7)$

$$=7k^2+k$$

$$=7\times\left(\frac{1}{10}\right)^2+\frac{1}{10}$$

$$=\frac{7}{100}+\frac{1}{10}$$

$$=\frac{17}{100}$$

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k$

$= 3k$

$= 3 \times \frac{1}{10}$

$= \frac{3}{10}$

Q.9 The random variable X has probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, \text{if } x = 0 \\ 2k, \text{if } x = 1 \\ 3k, \text{if } x = 2 \\ 0, \text{otherwise} \end{cases}$$

(a) Determine the value of k.

(b) Find $P(X < 2)$, $P(X \geq 2)$, $P(X \geq 2)$.

Sol: **(a)** It is known that the sum of probabilities of a probability distribution of random variables is one.

$\therefore k + 2k + 3k + 0 = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$= k + 2k$

$= 3k$

$= \frac{3}{6}$

$= \frac{1}{2}$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k$

$= 6k$

$= \frac{6}{6}$

$= 1$

$P(X \geq 2) = P(X = 2) + P(X > 1)$

$= 3k + 0$

$= 3k$

$= \frac{3}{6}$

$= \frac{1}{2}$

Q.10 Find the mean number of heads in three tosses of a fair coin.

Sol: Let X denote the success of getting heads.

Therefore, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that X can take the value of 0, 1, 2, or 3.

∴P(X=0) = P(TTT)

= P(T).P(T).P(T)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

∴ P (X = 1) = P (HHT) + P (HTH) + P (THH)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

∴P(X = 2) = P (HHT) + P (HTH) + P (THH)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

∴P(X=3) = P(HHH)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X E(X), $\mu = \Sigma X_i P(X_i)$

$$= 0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$$

$$= \frac{3}{8}+\frac{3}{4}+\frac{3}{8}$$

$$= \frac{3}{2}$$

$= 1.5$

Q.11 Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Sol: Here, X represents the number of sixes obtained when two dice are thrown simultaneously.

Therefore, X can take the value of 0, 1, or 2.

$\therefore$ P (X = 0) = P (not getting six on any of the dice) $= \frac{25}{26}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$$= 2\left(\frac{1}{6}\times\frac{5}{6}\right) = \frac{10}{36}$$

P (X = 2) = P (six on both the dice) $= \frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) $= \Sigma X_i P(X_i)$

$$= 0\times\frac{25}{36}+1\times\frac{10}{36}+2\times\frac{1}{36}$$

$$= \frac{1}{3}$$

Q.12 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find E(X).

Sol: The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6. For X = 2, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For X = 4, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For X = 5, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For X = 6, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, $E(X) = \Sigma X_i P(X_i)$

$$=0.\frac{1}{15}+3.\frac{2}{15}+4.\frac{1}{5}+5.\frac{4}{15}+6.\frac{1}{3}$$

$$=\frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2$$

$$=\frac{70}{15}$$

$$=\frac{14}{3}$$

Q.13 Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Sol: When two fair dice are rolled, $6 \times 6 = 36$ observations are obtained.

$P(X = 2) = P(1, 1) = \frac{1}{36}$

$P(X = 3) = P(1, 2) + P(2, 1) = \frac{2}{36} = \frac{1}{18}$

$P(X = 4) = P(1, 3) + P(2, 2) + P(3, 1) = \frac{3}{36} = \frac{1}{12}$

$P(X = 5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1) = \frac{4}{36} = \frac{1}{9}$

$P(X = 6) = P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) = \frac{5}{36}$

$P(X = 7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) = \frac{6}{36} = \frac{1}{6}$

$P(X = 8) = P(2, 6) + P(3, 5) + P(4, 4) + P(5, 3) + P(6, 2) = \frac{5}{36}$

$P(X = 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3) = \frac{4}{36} = \frac{1}{9}$

$P(X = 10) = P(4, 6) + P(5, 5) + P(6, 4) = \frac{3}{36} = \frac{1}{12}$

$P(X = 11) = P(5, 6) + P(6, 5) = \frac{2}{36} = \frac{1}{18}$

$P(X = 12) = P(6, 6) = \frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Then, $E(X) = \Sigma X_i P(X_i)$

$$= 2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$

$$= \frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{5}{6}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$

$$= 7$$

$E(X^2) = \Sigma X_i^2.P(X_i)$

$$= 4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{6}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{987}{18} = \frac{329}{6} = 54.833$$

Then, $Var(X) = E(X^2)-[E(X)]^2$

$= 54.833-(7)^2$

$= 54.833-49$

$=5.833$

$\therefore$ Standard deviation $=\sqrt{Var(X)}$

$=\sqrt{5.833}$

$= 2.415$

Q.14 A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

Sol: There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$. The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$, $P(X = 16) = \frac{2}{15}$, $P(X = 16) = \frac{3}{15}$,

$P(X = 18) = \frac{1}{15}$, $P(X = 19) = \frac{2}{15}$, $P(X = 20) = \frac{3}{15}$, $P(X = 21) = \frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of X = E(X)

$= \Sigma X_i P(X_i)$

$$=14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$

$$=\frac{1}{15}\left(28+15+32+51+18+38+60+21\right)$$

$$=\frac{263}{15}$$

= 17.53

$E(X^2) = \Sigma X_i^2.P(X_i)$

$$=(14)^2.\frac{2}{15}+(15)^2.\frac{1}{15}+(16)^2.\frac{2}{15}+(17)^2.\frac{3}{15}+(18)^2.\frac{1}{15}+(19)^2.\frac{2}{15}+(20)^2.\frac{3}{15}+(21)^2.\frac{1}{15}$$

$$=\frac{1}{15}.\left(392+225+512+867+324+722+1200+441\right)$$

$$=\frac{4683}{15}$$

= 312.2

$\therefore$Variance(X) = $E(X^2)-[E(X]^2$

$$=312.2-\left(\frac{263}{15}\right)^2$$

= 312.2 – 307.4177

= 4.7823

≈ 4.78

Standard derivation $=\sqrt{\text{Variance(X)}}$

$=\sqrt{4.78}$

= 2.186 ≈ 2.19

Q.15 In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var(X).

Sol: It is given that $P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$$P\left(X=1\right)=70\%=\frac{70}{100}=0.7$$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, $E(X) = \Sigma X_i P(X_i)$

$= 0 \times 0.3 + 1 \times 0.7$

$= 0.7$

$E(X^2) = \Sigma X_i^2 . P(X_i)$

$= 0^2 \times 0.3 + (1)^2 \times 0.7$

$= 0.7$

It is known that, $Var(X) = E(X^2)-[E(X]^2$

$= 0.7 - (0.7)2$

$= 0.7 - 0.49$

$= 0.21$

Q.16 The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2

(C) 5 (D) $\frac{8}{3}$

Sol: Let X be the random variable representing a number on the die. The total number of observations is six

$\therefore P(X=1) = \frac{3}{6} = \frac{1}{2}$

$P(X=2) = \frac{2}{6} = \frac{1}{3}$

$P(X=5) = \frac{1}{6}$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean $= E(X) = \Sigma p_i x_i$

$= \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} . 5$

$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$

$= \frac{3+4+5}{6}$

$= \frac{12}{6}$

$= 2$

The correct answer is B.

Q.17 Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$

(C) $\frac{1}{13}$ (D) $\frac{2}{13}$

Sol: Let X denote the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

$\therefore$ P (X = 0) = P (0 ace and 2 non-ace cards) = $\frac{{}^{4}C_{0} \times {}^{48}C_{2}}{{}^{52}C_{2}} = \frac{1128}{1326}$

P (X = 1) = P (1 ace and 1 non-ace cards) = $\frac{{}^{4}C_{1} \times {}^{48}C_{1}}{{}^{52}C_{2}} = \frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non- ace cards) = $\frac{{}^{4}C_{1} \times {}^{48}C_{0}}{{}^{52}C_{2}} = \frac{6}{1326}$

Thus, the probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, E(X) = $\Sigma p_i x_i$

$= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326}$

$= \frac{204}{1326}$

$= \frac{2}{13}$

Therefore, the correct answer is D.