



SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(i) 5 successes?

(ii) at least 5 successes?

(iii) at most 5 successes?

Sol: The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p = \frac{3}{6} = \frac{1}{2}$

$$\therefore q = 1 - p = \frac{1}{2}$$

X has a binomial distribution.

Therefore, $P(X = x) = {}^n C_{n-x} q^{n-x} p^x \times 48 C_0$, where $n = 0, 1, 2, \dots, n$

$$= {}^6 C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x$$

$$= {}^6 C_x \left(\frac{1}{2}\right)^6$$

(i) $P(5 \text{ successes}) = P(X = 5)$

$$= {}^6 C_5 \left(\frac{1}{2}\right)^6$$

$$= 6 \cdot \frac{1}{64}$$

$$= \frac{3}{32}$$

(ii) $P(\text{at least 5 successes}) = P(X \geq 5)$

$$= P(X = 5) + P(X = 6)$$

$$= {}^6 C_5 \left(\frac{1}{2}\right)^6 + {}^6 C_6 \left(\frac{1}{2}\right)^6$$

$$= 6 \cdot \frac{1}{64} + 1 \cdot \frac{1}{64}$$

$$= \frac{7}{64}$$

(iii) $P(\text{at most 5 successes}) = P(X \leq 5)$

$$= 1 - P(X > 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

Q.2 A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Sol: The repeated tosses of a pair of dice are Bernoulli trials. Let X denote the number of times of getting doublets in an experiment of throwing two dice simultaneously four times.

Probability of getting doublets in a single throw of the pair of dice is

$$p = \frac{6}{36} = \frac{1}{6}$$

$$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Clearly, X has the binomial distribution with $n = 4$, $p = \frac{1}{6}$ and $q = \frac{5}{6}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x$, where $x = 0, 1, 2, 3, \dots, n$

$$= {}^4C_x \left(\frac{5}{6}\right)^{4-x} \cdot \left(\frac{1}{6}\right)^x$$

$$= {}^4C_x \cdot \frac{5^{4-x}}{6^4}$$

$$\therefore P(2 \text{ successes}) = P(X = 2)$$

$$= {}^4 C_2 \cdot \frac{5^{4-2}}{6^4}$$

$$= 6 \cdot \frac{25}{1296}$$

$$= \frac{25}{216}$$

Q.3 There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Sol: Let X denote the number of defective items in a sample of 10 items drawn successively. Since the drawing is done with replacement, the trials are Bernoulli trials.

$$\Rightarrow p = \frac{5}{100} = \frac{1}{20}$$

$$\therefore q = 1 - \frac{1}{20} = \frac{19}{20}$$

X has a binomial distribution with $n = 10$ and $p = \frac{1}{20}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x, \text{ where } x = 0, 1, 2, \dots, n$$

$$= {}^{10} C_x \left(\frac{19}{20}\right)^{10-x} \cdot \left(\frac{1}{20}\right)^x$$

$$P(\text{not more than 1 defective item}) = P(X \leq 1)$$

$$= P(X = 0) + P(X = 1)$$

$$= {}^{10} C_0 \left(\frac{19}{20}\right)^{10-0} \cdot \left(\frac{1}{20}\right)^0 + {}^{10} C_1 \left(\frac{19}{20}\right)^{19} \cdot \left(\frac{1}{20}\right)^1$$

$$= \left(\frac{19}{20}\right)^{10} + 10 \left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left(\frac{29}{20}\right)$$

$$= \left(\frac{29}{20}\right) \cdot \left(\frac{19}{20}\right)^9$$

Q.4 Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards.

What is the probability that

(i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is a spade?

Sol: Let X represent the number of spade cards among the five cards drawn. Since the drawing of card is with replacement, the trials are Bernoulli trials. In a well shuffled deck of 52 cards, there are 13 spade cards.

$$\Rightarrow p = \frac{13}{52} = \frac{1}{4}$$

$$\therefore q = 1 - \frac{1}{4} = \frac{3}{4}$$

X has a binomial distribution with $n = 5$ and $p = \frac{1}{4}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x, \text{ where } x = 0, 1, \dots, n$$

$$= {}^5 C_x \left(\frac{3}{4}\right)^{5-x} \cdot \left(\frac{1}{4}\right)^x$$

(i) $P(\text{all five cards are spades}) = P(X = 5)$

$$= {}^5 C_5 \left(\frac{3}{4}\right)^0 \cdot \left(\frac{1}{4}\right)^5$$

$$= 1 \cdot \frac{1}{1024} = \frac{1}{1024}$$

(ii) $P(\text{only 3 cards are spades}) = P(X = 3)$

$$= {}^5 C_3 \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^3$$

$$= 10 \cdot \frac{9}{16} \cdot \frac{1}{64}$$

$$= \frac{45}{512}$$

(iii) $P(\text{none is a spade}) = P(X = 0)$

$$= {}^5 C_0 \left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^0$$

$$= 1 \cdot \frac{243}{1024} = \frac{243}{1024}$$

Q.5 The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. What is the probability that out of 5 such bulbs

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one

will fuse after 150 days of use.

Sol: Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that, $p = 0.05$

$$\therefore q = 1 - p = 1 - 0.05 = 0.95$$

X has a binomial distribution with $n = 5$ and $p = 0.05$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x, \text{ where } x = 0, 1, 2, \dots, n$$

$$= {}^5 C_x (0.95)^{5-x} \cdot (0.05)^x$$

(i) $P(\text{none}) = P(X = 0)$

$$= {}^5 C_0 (0.95)^5 \cdot (0.05)^0$$

$$= 1 \times (0.95)^5$$

$$= (0.95)^5$$

(ii) $P(\text{not more than one}) = P(X \leq 1)$

$$= P(X = 0) + P(X = 1)$$

$$= {}^5 C_0 (0.95)^5 \cdot (0.05)^0 + {}^5 C_1 (0.95)^4 \cdot (0.05)^1$$

$$= 1 \times (0.95)^5 + 5 \times (0.95)^4 \times (0.05)$$

$$= (0.95)^5 + (0.25)(0.95)^4$$

$$= (0.95)^4 + [0.95 + 0.25]$$

$$= (0.95)^4 \times 1.2$$

$$\text{(iii) } P(\text{more than 1}) = P(X > 1)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - P(\text{not more than 1})$$

$$= 1 - (0.95)^4 \times 1.2$$

$$\text{(iv) } P(\text{at least one}) = P(X \geq 1)$$

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^5C_0 (0.95)^5 \cdot (0.05)^0$$

$$= 1 - 1 \times (0.95)^5$$

$$= 1 - (0.95)^5$$

Q.6 A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Sol: Let X denote the number of balls marked with the digit 0 among the 4 balls drawn. Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binomial distribution with $n = 4$ and $p = \frac{1}{4}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x, x = 0, 1, 2, \dots, n$$

$$= {}^4C_x \left(\frac{9}{10}\right)^{4-x} \cdot \left(\frac{1}{10}\right)^x$$

$$P(\text{none marked with 0}) = P(X = 0)$$

$$= {}^4C_0 \left(\frac{9}{10}\right)^4 \cdot \left(\frac{1}{10}\right)^0$$

$$= 1 \cdot \left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^4$$

Q.7 In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Sol: Let X represent the number of correctly answered questions out of 20 questions.

The repeated tosses of a coin are Bernoulli trials. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$$\therefore p = \frac{1}{2}$$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

X has a binomial distribution with $n = 20$ and $p = \frac{1}{2}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x, \text{ where } x = 0, 1, 2, \dots, n$$

$$= {}^{20} C_x \left(\frac{1}{2}\right)^{20-x} \cdot \left(\frac{1}{2}\right)^x$$

$$= {}^{20} C_x \left(\frac{1}{2}\right)^{20}$$

$P(\text{at least 12 questions answered correctly}) = P(X \geq 12)$

$$= P(X = 12) + P(X = 13) + \dots + P(X = 20)$$

$$= {}^{20} C_{12} \left(\frac{1}{2}\right)^{20} + {}^{20} C_{13} \left(\frac{1}{2}\right)^{20} + \dots + {}^{20} C_{20} \left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20} \left[{}^{20} C_{12} + {}^{20} C_{13} + \dots + {}^{20} C_{20} \right]$$

Q.8 Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome.

(Hint: $P(X = 3)$ is the maximum among all $P(x_i)$, $x_i = 0, 1, 2, 3, 4, 5, 6$)

Sol: X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Then, $P(X = x) = {}^n C_x q^{n-x} p^x$

$$= {}^6 C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x$$

$$= {}^6 C_x \left(\frac{1}{2}\right)^6$$

It can be seen that $P(X = x)$ will be maximum, if ${}^6 C_x$ will be maximum.

$$\text{Then, } {}^6 C_0 = {}^6 C_6 = \frac{6!}{0!.6!} = 1$$

$${}^6 C_1 = {}^6 C_5 = \frac{6!}{1!.5!} = 6$$

$${}^6 C_2 = {}^6 C_4 = \frac{6!}{2!.4!} = 15$$

$${}^6 C_3 = \frac{6!}{3!.3!} = 20$$

The value of ${}^6 C_3$ is maximum. Therefore, for $x = 3$, $P(X = x)$ is maximum. Thus, $X = 3$ is the most likely outcome.

Q.9 On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Sol: The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$\therefore P(X = x) = {}^n C_x q^{n-x} p^x$

$$= {}^5 C_x \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^x$$

$$\begin{aligned} P(\text{guessing more than 4 correct answers}) &= P(X \geq 4) \\ &= P(X = 4) + P(X = 5) \end{aligned}$$

$$\begin{aligned}
&= {}^5C_4 \left(\frac{2}{3}\right) \cdot \left(\frac{1}{2}\right)^4 + {}^5C_5 \left(\frac{1}{3}\right)^5 \\
&= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243} \\
&= \frac{10}{243} + \frac{1}{243} \\
&= \frac{11}{243}
\end{aligned}$$

Q.10 A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- (a) at least once
- (b) exactly once
- (c) at least twice?

Sol: Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$$

(a) $P(\text{winning at least once}) = P(X \geq 1)$

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50}$$

$$= 1 - 1 \cdot \left(\frac{99}{100}\right)^{50}$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

$$(b) P(\text{winning exactly once}) = P(X = 1)$$

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^1$$

$$= 50 \left(\frac{1}{100}\right) \left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2} \left(\frac{99}{100}\right)^{49}$$

$$(c) P(\text{at least twice}) = P(X \geq 2)$$

$$= 1 - P(X < 2)$$

$$= 1 - P(X \leq 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= [1 - P(X = 0) - P(X = 1)]$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$$

$$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49}$$

Q.11 Find the probability of getting 5 exactly twice in 7 throws of a die.

Sol: The repeated tossing of a die are Bernoulli trials. Let X represent the number of times of getting 5 in 7 throws of the die.

Probability of getting 5 in a single throw of the die, $p = \frac{1}{6}$

$$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Clearly, X has the probability distribution with $n = 7$ and $p = \frac{1}{6}$

$$\therefore P(X = x) = {}^nC_x q^{n-x} p^x = {}^7C_x \left(\frac{5}{6}\right)^{7-x} \cdot \left(\frac{1}{6}\right)^x$$

$$P(\text{getting 5 exactly twice}) = P(X = 2)$$

$$= {}^7 C_2 \left(\frac{5}{6}\right)^5 \cdot \left(\frac{1}{6}\right)^2$$

$$= 21 \cdot \left(\frac{5}{6}\right)^5 \cdot \frac{1}{36}$$

$$= \left(\frac{7}{12}\right) \left(\frac{5}{6}\right)^5$$

Q.12 Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Sol: The repeated tossing of the die are Bernoulli trials. Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p = \frac{1}{6}$

$$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x = {}^6 C_x \left(\frac{5}{6}\right)^{6-x} \cdot \left(\frac{1}{6}\right)^x$$

$$P(\text{at most 2 sixes}) = P(X \leq 2)$$

$$= P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6 C_0 \left(\frac{5}{6}\right)^6 + {}^6 C_1 \cdot \left(\frac{5}{6}\right)^5 \cdot \left(\frac{1}{6}\right) + {}^6 C_2 \left(\frac{5}{6}\right)^4 \cdot \left(\frac{1}{6}\right)^2$$

$$= 1 \cdot \left(\frac{5}{6}\right)^6 + 6 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^5 + 15 \cdot \frac{1}{36} \cdot \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^4 \left[\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right) \right]$$

$$= \left(\frac{5}{6}\right)^4 \left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12} \right]$$

$$= \left(\frac{5}{6}\right)^4 \left[\frac{25 + 30 + 15}{36} \right]$$

$$= \frac{70}{36} \cdot \left(\frac{5}{6}\right)^4 = \frac{35}{18} \cdot \left(\frac{5}{6}\right)^4$$

Q.13 It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Sol: The repeated selections of articles in a random sample space are Bernoulli trials. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly, X has a binomial distribution with $n = 12$ and $p = 10\% = \frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x = {}^{12} C_x \left(\frac{9}{10}\right)^{12-x} \cdot \left(\frac{1}{10}\right)^x$$

$$P(\text{selecting 9 defective articles}) = {}^{12} C_9 \left(\frac{9}{10}\right)^3 \cdot \left(\frac{1}{10}\right)^9$$

$$= 220 \cdot \frac{9^3}{10^3} \cdot \frac{1}{10^9}$$

$$= \frac{22 \times 9^3}{10^3}$$

Q.14 In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

- (A) 10^{-1} (B) $\left(\frac{1}{2}\right)^5$
(C) $\left(\frac{9}{10}\right)^5$ (D) $\frac{9}{10}$

Sol: The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denote the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb, $p = \frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{10}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x = {}^5 C_x \left(\frac{9}{10}\right)^{5-x} \cdot \left(\frac{1}{10}\right)^x$$

$$P(\text{none of the bulbs is defective}) = P(X = 0)$$

$$= {}^5 C_0 \left(\frac{9}{10} \right)^5$$

$$= 1 \cdot \left(\frac{9}{10} \right)^5$$

$$= \left(\frac{9}{10} \right)^5$$

The correct answer is C.

Q.15 The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

(A) ${}^5 C_4 \left(\frac{4}{5} \right)^4 \frac{1}{5}$

(B) $\left(\frac{4}{5} \right)^4 \frac{1}{5}$

(C) ${}^5 C_1 \frac{1}{5} \left(\frac{4}{5} \right)^4$

(D) None of these

Sol: The repeated selection of students who are swimmers are Bernoulli trials. Let X denote the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q = \frac{1}{5}$

$$\therefore q = 1 - p = 1 - \frac{4}{5} = \frac{1}{5}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{4}{5}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x = {}^5 C_x \left(\frac{1}{5} \right)^{5-x} \cdot \left(\frac{4}{5} \right)^x$$

$$P(\text{four students are swimmers}) = P(X = 4) = {}^5 C_4 \left(\frac{1}{5} \right) \cdot \left(\frac{4}{5} \right)^4$$

Therefore, the correct answer is A.