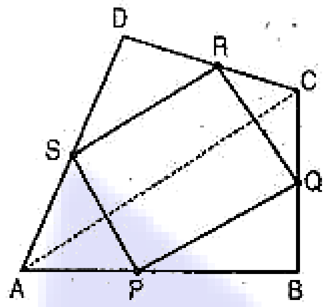
SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

Q.1 ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:



(i) SR ∥ AC and SR $= \frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Ans - In Δ ABC, P is the mid-point of AB and Q is the mid-point of BC.

Then PQ ∥ AC and PQ $= \frac{1}{2}$ AC

(i) In ΔACD, R is the mid-point of CD and S is the mid-point of AD.

Then SR ∥ AC and SR $= \frac{1}{2}$ AC

(ii) Since PQ $= \frac{1}{2}$ AC and SR $= \frac{1}{2}$ AC

Therefore, PQ = SR

(iii) Since PQ ∥ AC and SR ∥ AC

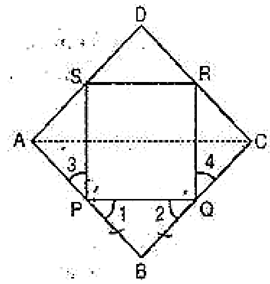
Therefore, PQ ∥ SR [two lines parallel to given line are parallel to each other]

Now PQ = SR and PQ ∥ SR

Therefore, PQRS is a parallelogram.

Q.2 ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.

Ans - Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.



To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ……….(i)

In ΔADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ ……….(ii)

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

$\therefore AB = BC$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC \;\; PB = BQ$

$\therefore \angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

$AP = CQ$ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, $AS = CR$ and $PS = QR$ [Opposite sides of a parallelogram]

$\therefore \Delta APS \cong \Delta CQR$ [By SSS congreuancy]

$\Rightarrow \angle 3 = \angle 4$ [By C.P.C.T.]

Now we have $\angle 1 + \angle SPQ + \angle 3 = 180^\circ$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$ [Linear pairs]

$\therefore \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore \angle SPQ = \angle PQR$ ……….(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \angle SPQ + \angle PQR = 180^\circ$……….(iv) [Interior angles]

Using eq. (iii) and (iv),

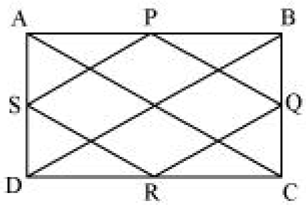
$\angle SPQ + \angle SPQ = 180^\circ \Rightarrow 2\angle SPQ = 180^\circ$

$\Rightarrow \angle SPQ = 90^\circ$

Hence PQRS is a rectangle.

Q.3 ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans - Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In ΔABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$ PQ ∥ AC and PQ = $\frac{1}{2}$ AC ……….(i)

In ΔADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$ SR ∥ AC and SR = $\frac{1}{2}$ AC ……….(ii)

From eq. (i) and (ii), PQ ∥ SR and PQ = SR ……….(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore$ AD = BC

$\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC AS = BQ ……….(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

∠PAS = ∠PBQ [Each 90°]

And AS = BQ [From eq. (iv)]

$\therefore$ ΔAPS ≅ ΔBPQ [By SAS congruency]

$\Rightarrow$ PS = PQ [By C.P.C.T.] ………(v)

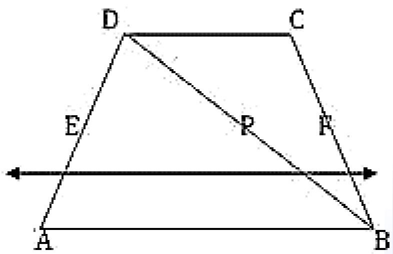
From eq. (iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Q.4 ABCD is a trapezium, in which AB ∥ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans - Let diagonal BD intersect line EF at point P.

In ΔDAB,

E is the mid-point of AD and EP ∥ AB [∵EF ∥ AB (given) P is the part of EF]

∴ P is the mid-point of other side, BD of ΔDAB.

[A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

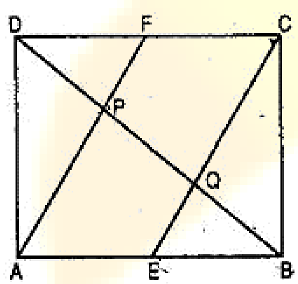
Now in ΔBCD,

P is the mid-point of BD and PF ∥ DC [∵ EF ∥ AB (given) and AB DC (given)]

∴ EF ∥ DC and PF is a part of EF.

∴ F is the mid-point of other side, BC of ΔBCD. [Converse of mid-point of theorem]

Q.5 In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans - Since E and F are the mid-points of AB and CD respectively.

∴ $AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$……….(i)

But ABCD is a parallelogram.

∴ AB = CD and AB ∥ DC

⇒ $\frac{1}{2}AB = \frac{1}{2}CD$ and AB ∥ DC

⇒ AE = FC and AE ∥ FC [From eq. (i)]

∴ AECF is a parallelogram.

⇒ FA ∥ CE ⇒FP ∥ CQ [FP is a part of FA and CQ is a part of CE] ………(ii)

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔDCQ, F is the mid-point of CD and $\Rightarrow FP \parallel CQ$

$\therefore$ P is the mid-point of DQ.

$\Rightarrow DP = PQ$ ……….(iii)

Similarly, In ΔABP, E is the mid-point of AB and$\Rightarrow EQ \parallel AP$

$\therefore$ Q is the mid-point of BP.

$\Rightarrow$ $BQ = PQ$ ……….(iv)

From eq. (iii) and (iv),

$DP = PQ = BQ$ ………(v)

Now $BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$

$\Rightarrow BQ = \frac{1}{3} BD$ ……….(vi)

From eq. (v) and (vi),

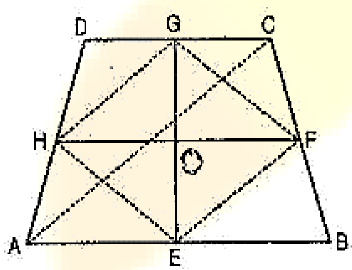
$DP = PQ = BQ = \frac{1}{3} BD$

$\Rightarrow$ Points P and Q trisects BD.

So AF and CE trisects BD.

Q.6 Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans - Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In ΔABC, E and F are the mid-points of respective sides AB and BC.

$\therefore EF \parallel AC$ and $EF \frac{1}{2} AC$ ……….(i)

Similarly, in ΔADC,

G and H are the mid-points of respective sides CD and AD.

$\therefore HG \parallel AC$ and $HG \frac{1}{2} AC$ ……….(ii)

From eq. (i) and (ii),

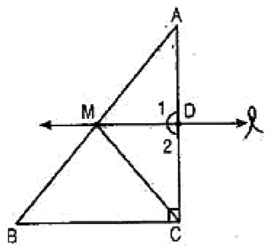
$EF \parallel HG$ and $EF = HG$

$\therefore$ EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other.

Q.7 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Ans - **(i)** In ΔABC, M is the mid-point of AB [Given]



$MD \parallel BC$

$\therefore AD = DC$ [Converse of mid-point theorem]

Thus D is the mid-point of AC.

(ii) $l \parallel BC$ (given) consider AC as a transversal.

$\therefore \angle 1 = \angle C$ [Corresponding angles]

$\Rightarrow \angle 1 = 90^\circ$ [$\angle C = 90^\circ$]

Thus $MD \perp AC$.

(iii) In ΔAMD and ΔCMD,

$AD = DC$ [proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

$MD = MD$ [common]

$\therefore \Delta AMD \cong \Delta CMD$ [By SAS congruency]

$\Rightarrow$ $AM = CM$ [By C.P.C.T.] ……….(i)

Given that M is the mid-point of AB.

$\therefore AM = \frac{1}{2} AB$ ……….(ii)

From eq. (i) and (ii),

$CM = AM = \frac{1}{2} AB$