

Class – 10th

Topic – Ratio and Proportion

1. If x, y and z are in continued proportion, prove that: $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{y}$

[2010]

Solution:

If x, y and z are in continued proportion, then

$$\frac{x}{y} = \frac{y}{z} \Rightarrow x = \frac{y^2}{z}$$

Applying componendo and dividendo

$$\begin{aligned} \frac{x+y}{x-y} &= \frac{y+z}{y-z} \\ \Rightarrow \frac{x+y}{x-y} &= \frac{y+z}{y-z} \end{aligned}$$

Squaring both sides

$$\begin{aligned} \Rightarrow \frac{(x+y)^2}{(y+z)^2} &= \left(\frac{y+z}{y-z}\right)^2 \\ \Rightarrow \frac{(x+y)^2}{(y+z)^2} &= \left(\frac{y^2+z^2+2yz}{y^2+z^2-2yz}\right)^2 \end{aligned}$$

Substituting

$$\begin{aligned} \Rightarrow \frac{(x+y)^2}{(y+z)^2} &= \left(\frac{\frac{y^2}{z} + y}{y-z}\right)^2 \\ \Rightarrow \frac{(x+y)^2}{(y+z)^2} &= \left(\frac{y^2 + yz}{z(y-z)}\right)^2 \\ &= \frac{y^2}{z^2} = \frac{zx}{z^2} = \frac{x}{z} \end{aligned}$$

2. Given $x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$. Use componendo and dividendo to prove that: $b^2 = \frac{2a^2x}{x^2 + 1}$

Solution:

$$\text{Given } x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$$

Applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{(\sqrt{a^2+b^2} + \sqrt{a^2-b^2}) + (\sqrt{a^2+b^2} - \sqrt{a^2-b^2})}{(\sqrt{a^2+b^2} + \sqrt{a^2-b^2}) - (\sqrt{a^2+b^2} - \sqrt{a^2-b^2})}$$

Simplifying

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

Square both sides

$$\frac{x^2+1+2x}{x^2-2x+1} = \frac{a^2+b^2}{a^2-b^2}$$

Applying componendo and dividendo

$$\frac{x^2+1+2x+x^2-2x+1}{x^2+1+2x-x^2+2x-1} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

$$\frac{2(x^2+1)}{4x} = \frac{2a^2}{2b^2}$$

$$\frac{x^2+1}{2x} = \frac{a^2}{b^2}$$

Simplifying

$$b^2 = \frac{2a^2x}{x^2+1}$$

3. If $\frac{x^2+y^2}{x^2-y^2} = 2\frac{1}{8}$, find

:

[2014]

(i) $\frac{x}{y}$

(ii) $\frac{x^3+y^3}{x^3-y^3}$

Solution:

(i) Given $\frac{x^2+y^2}{x^2-y^2} = 2\frac{1}{8} = \frac{17}{8}$

Applying componendo and dividendo

$$\frac{x^2 + y^2 + x^2 - y^2}{x^2 - y^2 - x^2 + y^2} = \frac{17 + 8}{17 - 8}$$

$$\frac{2x^2}{2y^2} = \frac{25}{9}$$

Simplifying, we get

$$\frac{x}{y} = \frac{5}{3}$$

$$(ii) \frac{x^3 + y^3}{x^3 - y^3}$$

Applying componendo and dividendo

$$\frac{x^3 + y^3 + x^3 - y^3}{x^3 + y^3 - x^3 + y^3}$$

$$= \frac{x^3}{y^3} = \frac{125}{9}$$

$$\Rightarrow \frac{x^3 + y^3}{x^3 - y^3} = \frac{125 + 9}{125 - 9}$$

$$= \frac{134}{116}$$

4. Using componendo and dividendo, find the value of x :

$$\frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = 9$$

[2011]

Solution:

$$\text{Given } \frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = 9$$

Applying componendo and dividendo

$$\frac{(\sqrt{3x+4} + \sqrt{3x-5}) + (\sqrt{3x+4} - \sqrt{3x-5})}{(\sqrt{3x+4} + \sqrt{3x-5}) - (\sqrt{3x+4} - \sqrt{3x-5})} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

Simplifying

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Square both sides

$$\frac{3x+4}{3x-5} = \frac{25}{14}$$

$$42x + 56 = 75x - 125$$

Simplifying we get $x = 7$

5. If $x = \frac{\sqrt{a+1} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}}$, using properties of proportion show that:

$$\frac{x^2 - 2ax}{x + 1}$$

[2012]

Solution:

$$\text{Given} = \frac{\sqrt{a+1} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}}$$

Applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{(\sqrt{a+1} + \sqrt{a-1}) + (\sqrt{a+1} - \sqrt{a-1})}{(\sqrt{a+1} - \sqrt{a-1}) - (\sqrt{a+1} - \sqrt{a-1})}$$

Simplify

$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}}{\sqrt{a-1}}$$

Now square both sides

$$\frac{x^2 + 1 + 2x}{x^2 - 2x + 1} = \frac{a+1}{a-1}$$

$$\Rightarrow \text{Componendo and Dividendo} = \frac{(x^2 + 1 + 2x) + (x^2 + 1 - 2x)}{(x^2 + 1 + 2x) - (x^2 - 2x + 1)} = \frac{(a+1)(a-1)}{(a+1)(a-1)}$$

Simplifying

$$x^2 + 1 = 2ax$$

$$\text{or } x^2 - 2ax + 1 = 0$$

6. Given, $\frac{a}{b} = \frac{c}{d}$, prove that: $\frac{3a - 5b}{3a + 5b} = \frac{3c - 5d}{3c + 5d}$

Solution:

$$\text{Given } \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{3a}{5b} = \frac{3c}{5d}$$

By componendo and dividendo

$$\frac{3a + 5d}{3a - 5d} = \frac{3c + 5d}{3c - 5d}$$

By Alternendo

$$\frac{3a + 5d}{3a - 5d} = \frac{3c + 5d}{3c - 5d}$$

7. If $x = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$, prove that: $3bx^2 - 2ax + 3b = 0$. [2007]

Solution:

$$\text{Given } x = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$$

Applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{(\sqrt{a+3b} + \sqrt{a-3b}) + (\sqrt{a+3b} - \sqrt{a-3b})}{(\sqrt{a+3b} + \sqrt{a-3b}) - (\sqrt{a+3b} - \sqrt{a-3b})}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

Squaring both sides

$$\frac{x^2 + 2x + 1}{x^2 - 2x + 1} = \frac{a + 3b}{a - 3b}$$

Applying componendo and dividendo once again

$$\frac{(x^2 + 2x + 1) + (x^2 - 2x + 1)}{(x^2 + 2x + 1) - (x^2 - 2x + 1)} = \frac{(a + 3b) + (a - 3b)}{(a + 3b) - (a - 3b)}$$

Simplifying

$$\frac{x^2 + 1}{2x} = \frac{a}{3b}$$

$$3b(x^2 + 1) = 2ax$$

$$3bx^2 - 2ax + 3b = 0 \text{ Hence proved}$$

8. Using the properties of proportion, solve for x . Given: $\frac{(x^4 + 1)}{2x^2}$
 $= \frac{17}{8}$ [2013]

Solution:

$$\text{Given } \frac{(x^4 + 1)}{2x^2} = \frac{17}{8}$$

Applying componendo and dividendo

$$\frac{(x^4 + 1) + 2x^2}{(x^4 + 1) - 2x^2} = \frac{17 + 8}{17 - 8}$$

$$\frac{(x^2 + 1)^2}{(x^2 - 1)^2} = \frac{25}{9}$$

Taking the square root of both sides

$$\frac{x^2 + 1}{x^2 - 1} = \frac{5}{3}$$

$$3x^2 + 3 = 5x^2 - 5$$

$$x^2 = 4 \text{ or } x = \pm 2$$

9. What least number must be added to each of the numbers 6, 15, 20, and 43 to make them proportional.
[2005, 2013]

Solution:

Let the number added be x

$$\text{Therefore } (6 + x):(15 + x) = (20 + x):(43 + x)$$

$$\Rightarrow (6 + x) \times (43 + x) = (20 + x) \times (15 + x)$$

$$\Rightarrow x^2 + 49x + 258 = x^2 + 35x + 300$$

$$\Rightarrow x = 3$$

10. The monthly pocket money of Ravi and Sanjeev are in the ratio of 5: 7 Their expenditures are in the ratio of 3: 5. If each saves Rs. 80 per month, find their monthly pocket money.
[2012]

Solution:

Let monthly pocket of Ravi and Sanjeev be x and y respectively.

$$\frac{x}{y} = \frac{5}{7} \Rightarrow x = \frac{5}{7}y$$

$$\frac{x - 80}{y - 80} = \frac{3}{5}$$

$$\text{Substituting } \frac{5}{7}y - 80 = \frac{3}{5}(y - 80)$$

$$\frac{25}{7}y - 400 = 3y - 240 \Rightarrow y = 280$$

Substituting

$$x = \frac{5}{7} \times 280 = 200$$

11. If $\frac{x-9}{3x+6}$ is the duplicate ratio of 4:9, find x. [2014]

Solution:

$$\frac{x-9}{3x+6} = \frac{4^2}{9^2} = \frac{16}{81}$$

$$81x - 729 = 48x + 96$$

$$x = 25$$

12. If a: b = 5: 3, find (5a + 8b): (6a – 7b). [2002]

Solution:

$$\text{Given } a: b = 5: 3$$

$$\text{or } \frac{a}{b} = \frac{5}{3} \Rightarrow a = b \frac{5}{3}$$

Now substituting

$$\frac{5a + 8b}{6a - 7b} = \frac{5 \times b \frac{5}{3} + 8b}{6 \times b \frac{5}{3} - 7b} = \frac{25 + 24}{30 - 21} = \frac{49}{9}$$

$$\text{Hence } (5a + 8b): (6a - 7b) = \frac{49}{9}$$

13. The work done by (x – 3) men in (2x + 1) days and the work done by (2x + 1) men in (x + 4) days are in the ratio 3: 10. Find the value of x. [2003]

Solution:

$$\text{Amount of work done by } (x - 3) \text{ men in } (2x + 1) \text{ days} = (x - 3)(2x + 1)$$

$$\text{Similarly, amount of work done by } (2x + 1) \text{ men in } (x + 4) \text{ days} = (2x + 1)(x + 4)$$

Given $\frac{(x-3)(2x+1)}{(2x+1)(x+4)} = \frac{3}{10}$

$$10(2x^2 + x - 6x - 3) = 3(2x^2 + 8x + x + 4)$$

Simplifying $2x^2 - 11x - 6 = 0$

$$(x-6)(2x+1) = 0 \Rightarrow x = 6 \text{ or } x = -\frac{1}{2} \text{ (not possible)} \Rightarrow x = 6.$$

14. What number should be subtracted from each of the numbers 23, 30, 57 and 78; so that the ratios

are in proportion?

[2004]

Solution:

Let the number subtracted = x

$$\text{Therefore } (23 - x):(30 - x) = (57 - x):(78 - x)$$

$$\frac{23 - x}{30 - x} = \frac{57 - x}{78 - x} \Rightarrow 78 - x$$

Simplifying

$$x^2 - 101x + 1794 = x^2 - 87x + 1710$$

$$\Rightarrow x = 6$$

15. 6 is the mean proportion between two numbers x and y and 48 is the third proportion to x and y .

Find the numbers.

[2011]

Solution:

Given 6 is the mean proportion between two numbers x and y

$$\text{Therefore } \frac{x}{y} = \frac{6}{y}$$

$$\Rightarrow xy = 36$$

$$\Rightarrow x = \frac{36}{y} \dots \dots \dots \text{(i)}$$

Also given 48 is the third proportion to x and y

