SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Relations and Functions
Exercise- 2.1

1. If, $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$ find the values of x and y.

Ans. It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\left(\frac{x}{3}+1, = \frac{5}{3}\right)$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-y-\frac{2}{3}=\frac{2}{3}$

$\Rightarrow x=2 \Rightarrow y=1$

$\therefore x = 2$ and $y = 1$

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Ans. It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A × B)

= (Number of elements in A) × (Number of elements in B) = 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G

Ans. G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as

P × Q = {(p, q): p∈ P, q ∈ Q}

∴ G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

Ans. (i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that x ∈ A and y ∈ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

Ans. (i) False

If P = {m, n} and Q = {n, m}, then

P × Q = {(m, m), (m, n), (n, m), (n, n)}

(ii) True

(iii) True

Q.5 If A = {–1, 1}, find A × A × A.

Ans. It is known that for any non-empty set A, A × A × A is defined as

A × A × A = {(a, b, c): a, b, c ∈ A}

It is given that A = {–1, 1}

∴ A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1),

(1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

6. If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Ans. It is given that A × B = {(a, x), (a, y), (b, x), (b, y)}

We know that the Cartesian product of two non-empty sets P and Q is defined as P × Q = {(p, q): p ∈ P, q ∈ Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {a, b} and B = {x, y}

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × C is a subset of B × D

Ans. (i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)

We have B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ

∴L.H.S. = A × (B ∩ C) = A × Φ = Φ

A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

∴ R.H.S. = (A × B) ∩ (A × C) = Φ

∴L.H.S. = R.H.S

Hence, A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) To verify: A × C is a subset of B × D

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A × C are the elements of set B × D.

Therefore, A × C is a subset of B × D.

8. Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Ans. A = {1, 2} and B = {3, 4}

∴ A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ n (A × B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2m.

Therefore, the set A × B has 24 = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},

{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)},

{(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

9. Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

Ans. It is given that n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.

We know that A = Set of first elements of the ordered pair elements of A × B

B = Set of second elements of the ordered pair elements of A × B.

∴ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since n(A) = 3 and n(B) = 2, it is clear that A = {x, y, z} and B = {1, 2}.

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

Ans. We know that if n(A) = p and n(B) = q, then n(A × B) = pq.

∴ n (A × A) = n(A) × n(A)

It is given that n (A × A) = 9

∴ n (A) × n(A) = 9

⇒ n(A) = 3

The ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.

We know that A × A = {(a, a): a ∈ A}. Therefore, –1, 0, and 1 are elements of A.

Since n(A) = 3, it is clear that A = {–1, 0, 1}.

The remaining elements of set A × A are (–1, –1), (–1, 1), (0, –1), (0, 0),

(1, –1), (1, 0), and (1, 1)