SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Relations and Functions
Exercise- 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Ans. (i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

2. Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9 - x^2}$

Ans. (i) $f(x) = -|x|, x \in R$

We know that $|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$$

Since f(x) is defined for $x \in R$, the domain of f is R.

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

∴ The range of f is $(-\infty, 0]$

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to −3 and less than or equal to 3, the domain of f(x) is $\{x: -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of f(x) will lie between 0 and 3.

∴ The range of f(x) is {x: 0 ≤ x ≤ 3} or [0, 3].

3. A function f is defined by f (x) = 2x – 5. Write down the values of

(i) f (0), (ii) f (7), (iii) f (–3)

Ans. The given function is f(x) = 2x – 5.

Therefore,

(i) f (0) = 2 × 0 – 5 = 0 – 5 = –5

(ii) f (7) = 2 × 7 – 5 = 14 – 5 = 9

(iii) f (–3) = 2 × (–3) – 5 = – 6 – 5 = –11

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by. $t(C) = \frac{9C}{5} + 32$

Find (i) t (0) (ii) t (28) (iii) t (–10) (iv) The value of C, when t(C) = 212

Ans. The given function is. $t(C) = \frac{9C}{5} + 32$

Therefore,

(i)$t(0) = \frac{9\times}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9\times 28}{5} + 32 = \frac{252+160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9\times(-10)}{5} + 32 = 9\times(-2) + 32 = -18 + 32 = 14$

(iv) It is given that t(C) = 212

$\therefore 212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow 9C = 180\times 5$

$\Rightarrow C = \frac{180\times 5}{9} = 100$

Thus, the value of t, when t(C) = 212, is 100.

5. Find the range of each of the following functions.

(i) f (x) = 2 – 3x, x ∈ R, x > 0.

(ii) f (x) = x2 + 2, x, is a real number.

(iii) f (x) = x, x is a real number

Ans. (i) f (x) = 2 – 3x, x ∈ R, x > 0

The values of f(x) for various values of real numbers x > 0 can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
$F(x)$	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	...

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2. i.e., range of f = (– ∞, 2)

Alter:

Let x > 0

⇒ 3x > 0

⇒ 2 –3x < 2

⇒ f(x) < 2

∴Range of f = (–, 2)

(ii) f (x) = x^2 + 2, x, is a real number the values of f(x) for various values of real numbers x can be written in the tabular form as.

x	0	±0.3	±0.8	±1	±2	±3	...
$F(x)$	2	2.09	2.64	3	6	11	.....

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of f = [2, ∞)

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

⇒ $x^2 + 2 \geq 0 + 2$

⇒ $x^2 + 2 \geq 2$

⇒ f(x) ≥ 2

∴ Range of f = [2, ∞)

(iii) f (x) = x, x is a real number

It is clear that the range of f is the set of all real numbers.

∴ Range of f = R