

Board – CBSE

Class – 12

Topic – Relations and functions

- Discuss whether each of the following relations are reflexive, symmetric or transitive:
 - Relation R_1 on the set R of all real numbers defined as $R_1 = \{(a, b) : a - b \geq 0\}$
 - $R = \{(a, b) : a = b\}$
- Let $A = \{1, 2, 3, 4\}$ and let $f = \{(1, 4), (2, 1), (3, 3), (4, 2)\}$ and $g = \{(1, 3), (2, 1), (3, 2), (4, 4)\}$, find $f \circ g$, $g \circ f$ and $g \circ g$.
- Find $f \circ g$ and $g \circ f$ if
 - $f(x) = |x|$ and $g(x) = |5x - 2|$
 - $f(x) = 8x^3$ and $g(x) = x^{1/3}$
- Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be defined as $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Find $f \circ g$ and $g \circ f$.
- Let $f(x) = \begin{cases} x^2 - 4x + 3 & x < 3 \\ x - 4 & x \geq 3 \end{cases}$
 $g(x) = \begin{cases} x - 3 & x < 4 \\ x^2 + 2x + 2 & x \geq 4 \end{cases}$
 Describe the function $\frac{f}{g}$.
- $f(x) = \begin{cases} x + 1 & x \leq 1 \\ 2x + 1 & 1 < x \leq 2 \end{cases}$
 $g(x) = \begin{cases} x^2 & -1 < x < 2 \\ x + 2 & 2 \leq x \leq 3 \end{cases}$
 Find $f \circ g$ and $g \circ f$.
- Are the following functions invertible in their respective domains? If so find the inverse of each.
 - $f(x) = \frac{x}{x+1}$
 - $f(x) = \sqrt{1-x^2}$
- If the function $f: R \rightarrow R$ be defined by $f(x) = x^2 + 5x + 9$. Find $f^{-1}(8)$.
- Let $S = \{0, 1, 2, 3, 4\}$ and $*$ be an operation on S defined by $a * b = a \oplus_3 b$. Prepare operation table for \otimes_3 and \oplus_3 .
- Let $*$ be a binary operation on A defined by $a * b = \text{LCM of } (a, b)$ for all $(a, b) \in N$.
 Find
 - $3 * 4$
 - $5 * 7$

- (iii) $20 * 16$
(iv) $6 * 9$
11. Find the number of binary operations on the set $\{3,4,5\}$ having identity element as 3.
12. Find the number of binary operations on the set $\{6,7,8\}$ having identity element as 6 and $7^{-1} = 7, 8^{-1} = 8$.
13. Check whether the following relations are functions or not. If yes, then find which functions are one one, onto or neither. $A = \{2,4,6,8\}$ and $B = \{a, b, c\}$
- (i) $f = \{(4, a), (6, c), (8, a)\}$
(ii) $g = \{(2, a), (4, b), (6, c), (8, a)\}$
(iii) $h = \{(2, a), (4, a), (6, b), (8, b)\}$
14. Check whether the following function $f: A \rightarrow B$ is invertible or not. Describe f^{-1} .
 $A = \{1,2,3,4\}$ $B = \{2,4,6,8\}$, $f(1)=2, f(2)=4, f(3)=6, f(4)=8$.
15. Determine whether the function $f: S \rightarrow S$ where $S = \{1,2,3\}$ and $f = \{(1,2), (2,1), (3,1)\}$ invertible or not.
16. Find the number of binary operations that can be defined on a set of 2 elements.
17. Find the number of binary operations on set $\{1,2,3\}$ having $e=3$ and $4^{-1} = 4, 5^{-1} = 4$
18. If R is the relation on $N \times N \rightarrow N$ defined by $(a, b) R (c, d)$ if $a + d = b + c$. Show that R is an equivalence relation.
19. Let a function $f: R \rightarrow R$ be defined by $f(x) = 3 - 4x$, prove that f is one one and onto.
20. Show that $f: N \rightarrow N$ given by $f(x) = x^2 + x + 1$ is not invertible
21. Let $A = \{-1,0,1\}$ and $f = \{(x, x^2), x \in A\}$. Show that $f: A \rightarrow A$ is neither one one nor onto.
22. Show that $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $\frac{x}{1+|x|}$, $x \in R$ is one one and onto.
23. If $f: [0, \infty) \rightarrow [0, \infty)$ and $f(x) = \frac{x}{1+x}$, show that f is one one but not onto.
24. Let $A = \{a, b, c\}$. State the reason for the following where R_1 and R_2 are relations on A .
- (i) $R_1 = \{(a, a), (b, b), (a, c)\}$ is not reflexive.
(ii) $R_2 = \{(a, b), (b, a), (a, c), (a, a), (b, b), (c, c)\}$ is not symmetric.
25. Let $A = N \times N$ and let $*$ be a binary operation on A defined by $\{(a, b) * (c, d) = (ad + bc)$ for all $(a, b), (c, d) \in N \times N\}$. Show that
- (i) $*$ is commutative on A
(ii) $*$ is associative on A
(iii) $*$ has no identity element.

Answer

1. (i) Reflexive, Transitive
(ii) Reflexive, Symmetric, Transitive
2. $f \circ g = \{(1,3), (2,4), (3,1), (4,2)\}$; $g \circ f = \{(1,4), (2,3), (3,2), (4,1)\}$;
 $f \circ f = \{(1,2), (2,4), (3,3), (4,1)\}$ $g \circ g = \{(1,2), (2,3), (3,1), (4,4)\}$
3. (i) $(f \circ g)(x) = ||5x - 2||$ $(g \circ f)(x) = |5|x| - 2|$
(ii) $(f \circ g)(x) = 8x$ $(g \circ f)(x) = 2x$
4. $(f \circ g)(x) = 4x^2 - 6x + 1$ $(g \circ f)(x) = 2x^2 + 6x - 1$
5. $\left(\frac{f}{g}\right)(x) = \begin{cases} x - 1, & x < 3 \\ \frac{x-4}{x-3}, & 3 \leq x < 4 \\ \frac{x^2+2x+2}{x-4}, & x \geq 4 \end{cases}$
7. (i) Yes, $f^{-1}(x) = \frac{x}{1-x}$ (ii) No
8. $\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}$
10. (i) 12 (ii) 35 (iii) 80 (iv) 18
11. 81
12. 9
13. (i) Not a function
(ii) Function but not one one (why)
(iii) Function but not onto (why)
14. Yes, $f^{-1}\{(2,1), (4,2), (6,3), (8,4)\}$
15. No (why)
16. 16
17. 3