Class – 10th	Topic – Factor And Remainder Theorems

1. 1 If $x^3 + ax^2 + bx + 6$ has $(x - 2)$ as a factor and leaves a remainder of 3 when divided by $(x - 3)$, find the value of a and b.
[2005]

Solution:

When x=2, Remainder = 0

$\Rightarrow\ (2)^3 + a(2)^2 + b(2) + 6 = 0$

$\Rightarrow\ 4a + 2b = -14 \ \ldots\ \ldots\ \ldots\ \ldots\ \ldots (i)$

When x = 3, Remainder = 3

$\Rightarrow\ (3)^3 + a(3)^2 + b(3) + 6 = 3$

$\Rightarrow\ 9a + 3b = -30 \ \ldots\ \ldots\ \ldots\ \ldots\ \ldots (ii)$

Solving i) and ii) a = -3 and b = -1

2. If (x-2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x-3), it leaves a remainder 52. Find the value of a and b.
[2013]

Solution:

When x=2, Remainder = 0

$\Rightarrow\ 2(2)^3 + a(2)^2 + b(2) - 14 = 0$

$\Rightarrow\ 4a + 2b = 2$

$\Rightarrow\ 2a + b = -1 \ \ldots\ \ldots\ \ldots\ \ldots\ \ldots (i)$

When x = 3, Remainder = 52

$\Rightarrow\ 2(3)^3 + a(3)^2 + b(3) - 14 = 52$

$\Rightarrow\ 9a + 3b = 12$

$\Rightarrow\ 3a + b = 4 \ \ldots\ \ldots\ \ldots\ \ldots\ \ldots (ii)$

Solving (i) and (ii), we get a = 5 and b = -11

3. **Find the value of a, if (x-a) is a factor of $\mathbf{x^3 - ax^2 + x + 2}$**

[2003]

Solution:

When $x = a$, Remainder $= 0$

Therefore $(a)^3 - a(a)^2 + (a) + 2 = 0$

$\Rightarrow\ a^3 - a^3 + a + 2 = 0$

$\Rightarrow a = 2$

4. **Using remainder theorem, factorize $\mathbf{x^3 + 10x^2 - 37x + 26}$ completely.**

[2014]

Solution:

For $x = 1$,

Remainder: $= (1)^3 + 10(1)^2 - 37(1) + 26 = 1 + 10 - 37 + 26 = 0$

Hence (x-1) is a factor of $x^3 + 10x^2 - 37x + 26$

$x - 1)\overline{x^3 + 10x^2 - 37x + 26}(x^2 + 11x - 26$

$(-)\ \underline{x^3 - x^2}$

$11x^2 - 37x + 26$

$(-)\ \underline{11x^2 - 11x}$

$-26x + 26$

$(-)\ \underline{-26x + 26}$

$\times$

$x^3 + 10x^2 - 37x + 26 = (x - 1)(x^2 + 11x - 26)$

$= (x - 1)(x^2 - 2x + 13x - 26)$

$= (x - 1)[x(x - 2) + 13(x - 2)]$

$= (x - 1)(x - 2)(x + 13)$

Hence $x^3 + 10x^2 - 37x + 26 = (x - 1)(x - 2)(x + 13)$

5. **When divided by (x-3) the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p+3)x - 6$ leave the same remainder. Find the value of p.**
[2010]

Solution:

When $x = 3$

$\text{Remainder1} = (3)^3 - p(3)^2 + (3) + 6$

$= 27 - 9p + 9$

$= 36 - 9p$

$\text{Remainder2} = 2(3)^3 - (3)^2 - (p+3)(3) - 6$

$= 54 - 9 - 3p - 9 - 6$

$= 30 - 3p$

Given Remainder 1 = Remainder 2

$36 - 9p = 30 - 3p$

$6 = 6p \Rightarrow p = 1$

6. **Use the remainder theorem to factorize the following expression: $2x^3 + x^2 - 13x + 6$.**
[2010]

Solution:

Let x =2

$\text{Remainder} = 2(2)^3 + (2)^2 - 13(2) + 6$

$= 16 + 4 - 26 + 6 = 0$

Hence (x-2) is a factor of $2x^3 + x^2 - 13x + 6$

$2x^3 + x^2 - 13x + 6 = (x-2)(2x^2 + 5x - 3)$

$= (x-2)(2x^2 + 6x - x - 3)$

$$x - 2)\overline{2x^3 + x^2 - 13x + 6}(2x^2 + 5x - 3$$

$$(-)\ \underline{2x^3 - 4x^2}$$

$$5x^2 - 13x + 6$$

$$(-)\ \underline{5x^2 - 10x}$$

$$-3x + 6$$

$$(-)\ \underline{-3x + 6}$$

$= (x-2)[2x(x+3)-(x+3)]$

$= (x-2)(x+3)(2x-1)$

Hence $2x^3 + x^2 - 13x + 6 = (x-2)(x+3)(2x-1)$

7. Find the value of k if (x-2) is a factor of $x^3 + 2x^2 - kx + 10$. Hence determine whether (x+5) is also a factor.

[2011]

Solution:

Let $f(x) = x^3 + 2x^2 - kx + 10$.

Since given that (x-2) is a factor $f(2) = 0$

Substituting the value of x =2 in the above function we get:

$f(2) = 0$

$f(2) = 8 + 8 - 2k + 10 = 0$

$\Rightarrow k = 13$

For $(x+5)$ to be a factor $f(-5) = 0$

Substituting the value of x =-5 in the above function we get:

$f(-5) = (-5)^3 + 2(-5)^2 - k(-5) + 10 = -125 + 50 + 65 + 10 = 0$

Hence (x+5) is a factor of $x^3 + 2x^2 - kx + 10$