SpeedLabs

MATHS

CBSE 11th

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1. Find the 20th and nth terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Ans The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Here, a = First term $= \frac{5}{2}$,

$$r = \text{Common ratio} = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$$

$$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Ans Common ratio, r = 2

Let a be the first term of the G.P.

$\therefore a_8 = ar^{8-1} = ar7$

$\Rightarrow ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6(3)$

$$\Rightarrow a = \frac{(2)^6 \times 3}{(\ )^7} = \frac{3}{2}$$

$$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

3. The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

Ans Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_5 = a\,r^{5-1} = a\,r^4 = p$ … (1)

$a_8 = a\,r^{8-1} = a\,r^7 = q$ … (2)

$a_{11} = a\,r^{11-1} = a\,r^{10} = s$ … (3)

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \ldots\ldots\ldots (4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$

$$r^3 = \frac{s}{q} \ldots\ldots\ldots (5)$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

4. The 4^{th} term of a G.P. is square of its second term, and the first term is –3. Determine its 7^{th} term. Class XI

Ans Let a be the first term and r be the common ratio of the G.P.

$\therefore a = -3$

It is known that, $a_n = ar^{n-1}$

$\therefore a_4 = ar^3 = (-3)\ r^3$

$a_2 = a\ r_1 = (-3)\ r$

According to the given condition,

$(-3)\ r^3 = [(-3)\ r]^2$

$\Rightarrow -3r^3 = 9\ r^2$

$\Rightarrow r = -3$

$a7 = a\ r^{7-1} = a\ r^6 = (-3)\ (-3)^6 = -\ (3)^7 = -2187$

Thus, the seventh term of the G.P. is –2187

5. Which term of the following sequences:

$(1)\, 2, 2\sqrt{2}, 4 \ldots\ldots$

(b) $\sqrt{3}, 3, 3\sqrt{3} \ldots\ldots$ is 729?

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots$ is $\frac{1}{19683}$?

Ans. (a) The given sequence is $(1)\, 2, 2\sqrt{2}, 4 \ldots\ldots$

Here, a = 2 and r = $\frac{2\sqrt{2}}{2} = \sqrt{2}$

Let the nth term of the given sequence be 128

$a_n = ar^{n-1}$

$\Rightarrow (2)\left(\sqrt{2}\right)^{n-1} = 128$

$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$

$\Rightarrow (2)^{\frac{n-1}{2}} = (2)^7$

$\Rightarrow \therefore \frac{n-1}{2} = 6$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

Thus, the 13th term of the given sequence is 128.

(b) The given sequence is $\sqrt{3}, 3, 3\sqrt{3}, \ldots..$

Here, $a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let the nth term of the given sequence be 729.

$a_n = ar^{n-1}$

$\therefore \left(\sqrt{3}\right)\left(\sqrt{3}\right)^{n-1} = 729$

$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$

$\Rightarrow (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$

$\therefore \frac{1}{2} + \frac{n-1}{2} = 6$

$\Rightarrow \frac{1+n-1}{2} = 6$

$\Rightarrow n = 12$

Thus, the 12th term of the given sequence is 729.

(c) The given sequence is $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots..$

Here, $a, \frac{1}{3}$ and $r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3} \ldots..$

Let the nth term of the given sequence be $\frac{1}{19683}$.

$a_n = ar^{n-1}$

$\therefore ar^{n-1} = \frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$

$\Rightarrow \left(\frac{1}{3}\right)^{n} = \left(\frac{1}{3}\right)^{9}$

$\Rightarrow n = 9$

Thus, the 9th term of the given sequence is $\frac{1}{19683}$.

6. For what values of x, the numbers are in G.P?

Ans The given numbers are $\frac{2}{7}, x, \frac{-7}{2}$.

Common ratio = $\frac{\frac{x}{-2}}{7} = \frac{-7x}{2}$

Also, common ratio = $\frac{\frac{-7}{2}}{x} = \frac{-7}{2x}$

$\therefore \frac{-7x}{2} = \frac{-7}{2x}$

$\Rightarrow x^2 = \frac{-2\times 7}{-2\times 7} = 1$

$\Rightarrow x = \sqrt{1}$

$\Rightarrow x = \pm 1$

Thus, for x = ± 1, the given numbers will be in G.P.

7. Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 …

Ans The given G.P. is 0.15, 0.015, 0.00015, …

Here, a = 0.15 and $r = \frac{0.015}{0.15} = 0.1$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_n = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$

$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

8. Find the sum to n terms in the geometric progression $\sqrt{7}, \sqrt{21}, \sqrt[3]{7}.....$

Ans. The given G.P. is $\sqrt{7}, \sqrt{21}, \sqrt[3]{7}.....$

Here, $a = \sqrt{7}$

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$\therefore S_n = \frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}}$$

$$\frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}} \times \frac{1\times\sqrt{3}}{1+\sqrt{3}} \quad \text{(By rationalizing)}$$

$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)\left[1-\left(\sqrt{3}\right)^n\right]}{1-3}$$

$$= \frac{-\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[1-(3)^{\frac{n}{2}}\right]$$

$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[(3)^{\frac{n}{2-1}}\right]$$

9. Find the sum to n terms in the geometric progression

Ans The given G.P. is $1, -a, a^2, -a^3, ...$

Here, first term = a_1 = 1

Common ratio = r = – a

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$\therefore S_n = \frac{1\left[1-(-a)^n\right]}{1-(-a)} = \frac{\left[1-(-a)^n\right]}{1+a}$$

10. Find the sum to n terms in the geometric progression $x^3, x^5, x^7 \ldots (\text{if } x \neq \pm 1)$

Ans. The given G.P. is $x^3, x^5, x^7 \ldots$

Here, $a = x^3$ and $r = x^2$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3\left[1-(x^2)^n\right]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

11. Evaluate $\sum_{k=1}^{11}(2+3^k)$

Ans. $\sum_{k=1}^{11}(2+3^k) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11}3^k = 2(11) + \sum_{k=1}^{11}3^k = 22 + \sum_{k=1}^{11}3^k \ldots (1)$

$$\sum_{k=1}^{11}3^k = 3^1 + 3^2 + 3^3 + \ldots 3^{11}$$

The terms of this sequence 3, 3^2, 3^3, … forms a G.P.

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$\Rightarrow S_n = \frac{3\left[(3)^{11}-1\right]}{3-1}$$

$$\Rightarrow S_n = \frac{3}{2}(3^{11}-1)$$

$$\therefore \sum_{k=1}^{11}3^k = \frac{3}{2}(3^{11}-1)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11}(2+3^k) = 22 + \frac{3}{2}(3^{11}-1)$$

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Ans. Let $\frac{a}{r}, a, ar$

be the first three terms of the G.P.

$$\frac{a}{r}+a+ar=\frac{39}{10}.....(1)$$

$$\left(\frac{a}{r}\right)(a)(ar)=1......(2)$$

From (2), we obtain

$a^3 = 1$

$\Rightarrow a = 1$ (Considering real roots only)

Substituting $a = 1$ in equation (1), we obtain

$$\frac{1}{r}+1+r=\frac{39}{10}$$

$$\Rightarrow 1+r+r^2=\frac{39}{10}r$$

$$\Rightarrow 10+10r+10r^2-39r=0$$

$$\Rightarrow 10r^2-29r+10=0$$

$$\Rightarrow 10r^2-25r-4r+10=0$$

$$\Rightarrow 5r(2r-5)-2(2r-5)=0$$

$$\Rightarrow (5r-22)(2r-5)=0$$

$$\Rightarrow r=\frac{2}{5}\text{ or }\frac{5}{2}$$

Thus, the three terms of G.P. are $\frac{5}{2}, 1, \text{and } \frac{2}{5}$

13. How many terms of G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

Ans The given G.P. is $3, 3^2, 3^3, \ldots$

Let n terms of this G.P. be required to obtain the sum as 120.

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

Here, $a = 3$ and $r = 3$

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3-1}$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$$\therefore n = 4$$

Thus, four terms of the given G.P. are required to obtain the sum as 120.

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Ans Let the G.P. be a, ar, ar^2, ar^3, …

According to the given condition,

$a + ar + ar^2 = 16$ and $ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow a(1 + r + r^2) = 16$ … (1)

$ar^3(1 + r + r^2) = 128$ … (2)

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\therefore r = 2$$

Substituting r = 2 in (1), we obtain

$a(1 + 2 + 4) = 16$

$\Rightarrow a(7) = 16$

$$\Rightarrow a = \frac{16}{7}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{16}{7}\frac{(2^n - 1)}{2 - 1} = \frac{16}{7}(2^n - 1)$$

15. Given a G.P. with a = 729 and 7^{th} term 64, determine S7.

Ans a = 729

$a_7 = 64$

Let r be the common ratio of the G.P.

It is known that, $a_n = a\, r^{n-1}$

$a_7 = ar^{7-1} = (729)\, r^6$

$\Rightarrow 64 = 729\, r^6$

$$\Rightarrow r^6 = \frac{64}{729}$$

$$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow r = \frac{2}{3}$$

Also, it is known that $S_n \frac{a\left(1-r^n\right)}{1-r}$

$$\therefore S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 3\times 729\left[1-\left(\frac{2}{3}\right)^7\right]$$

$$= (3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

16. Find a G.P. for which sum of the first two terms is –4 and the fifth term is 4 times the third term.

Ans Let a be the first term and r be the common ratio of the G.P.

According to the given conditions,

$$S_2 = -4 = \frac{a\left(1-r^2\right)}{1-r}.......(1)$$

$a_5 = 4 \times a_3$

$ar_4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\therefore r = \pm 2$

From (1), we obtain

$$-4=\frac{a\left[1-(2)^{2}\right]}{1-2} \text{ for } r=2$$

$$\Rightarrow -4=\frac{a(1-4)}{-1}$$

$$\Rightarrow -4=a(3)$$

$$\Rightarrow a=\frac{-4}{3}$$

$$\text{Also}, -4=\frac{a\left[1-(-2)^{2}\right]}{1-(-2)} \text{ for } r=-2$$

$$\Rightarrow -4=\frac{a(1-4)}{1+2}$$

$$\Rightarrow -4=\frac{a(-3)}{3}$$

$$\Rightarrow a=4$$

Thus, the required G.P. is

$$\frac{-4}{3},\frac{-8}{3},\frac{-16}{3},\ldots\ldots \text{or } 4,-8,16,-32,\ldots\ldots$$

17. If the 4th, 10th and 16th terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P

Ans. Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_4 = a r^3 = x$ ... (1)

$a_{10} = a r^9 = y$ ... (2)

$a_{16} = a r^{15} = z$ ... (3)

Dividing (2) by (1), we obtain

$$\frac{y}{x}=\frac{ar^{9}}{ar^{3}}\Rightarrow\frac{y}{x}=r^{6}$$

Dividing (3) by (2), we obtain

$$\frac{z}{y}=\frac{ar^{15}}{ar^{9}}\Rightarrow\frac{z}{y}=r^{6}$$

$$\therefore\frac{y}{x}=\frac{z}{y}$$

Thus, x, y, z are in G. P.

18. Find the sum to n terms of the sequence, 8, 88, 888, 8888…

Ans The given sequence is 8, 88, 888, 8888…

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

Sn = 8 + 88 + 888 + 8888 + …………… to n terms

$$= \frac{8}{9}[9 + 99 + 999 + 9999 + ...\text{to n terms}]$$

$$= \frac{8}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1)(10^4 - 1) + ...\text{to n terms}\right]$$

$$= \frac{8}{9}\left[(10 + 10^2 + ...\text{n terms}) - (1 + 1 + 1 + ...\text{n terms})\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

$$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$$

19. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Ans. Required sum = $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

$$= 64\left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}\right]$$

Here, 4, 2, 1, $\frac{1}{2}, \frac{2}{2^2}$ is a G.P.

First term, a = 4

Common ratio, r = $\frac{1}{2}$

It is known that, $S_n = \frac{a(1 - r^n)}{1 - r}$

$$\therefore S_5 = \frac{4\left[1 - \left(\frac{1}{2}\right)^5\right]}{1 - \frac{1}{2}} = \frac{4\left[1 - \frac{1}{32}\right]}{\frac{1}{2}} = 8\left(\frac{32 - 1}{32}\right) = \frac{31}{4}$$

$$\therefore \text{Required sum} = 64\left(\frac{31}{4}\right) = (16)(31) = 496$$

20. Show that the products of the corresponding terms of the sequences form $a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ a G.P, and find the common ratio.

Ans It has to be proved that the sequence, aA, arAR, ar^2AR^2, …$ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}} = \frac{arAR}{aA} = rR$$

$$\frac{\text{Second term}}{\text{First term}} = \frac{ar^2AR^2}{arAR} = rR$$

Thus, the above sequence forms a G.P. and the common ratio is rR.

21. Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Ans Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9$

$\Rightarrow ar^2 = a + 9$ … (1)

$a_2 = a_4 + 18$

$\Rightarrow ar = ar^3 + 18$ … (2)

From (1) and (2), we obtain

$a(r^2 - 1) = 9$ … (3)

$ar(1 - r^2) = 18$ … (4)

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$\Rightarrow -r = 2$

$\Rightarrow r = -2$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$\Rightarrow 3a = 9$

$\therefore a = 3$

Thus, the first four numbers of the G.P. are 3, $3(-2)$, $3(-2)^2$, and $3(-2)^3$ i.e., 3, –6, 12, and –24.

22. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q} = 1$

Ans Let A be the first term and R be the common ratio of the G.P.

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$a^{q-r} b^{r-p} c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

23. If the first and the n^{th} term of a G.P. are a ad b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Ans. The first term of the G.P is a and the last term is b.

Therefore, the G.P. is a, ar, ar^2, ar^3, … ar^{n-1}, where r is the common ratio.

$b = ar^{n-1}$ … (1)

P = Product of n terms

$= (a)(ar)(ar^2)\ldots(ar^{n-1})$

$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$

$= a^n r^{1+2+\ldots(n-1)}$ … (2)

Here, 1, 2, …(n – 1) is an A.P.

$$\therefore 1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}\left[2+(n-1-1)\times 1\right] = \frac{n-1}{2}[2+n-2] = \frac{n(n-1)}{2}$$

$$P = a^n r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$= \left[a^2 r^{(n-1)}\right]^n$$

$$= (ab)^n \qquad [\text{Using } (1)]$$

Thus, the given result is proved.

24. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from

$(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$

Ans Let a be the first term and r be the common ratio of the G.P.

$$\text{Sum of first n terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are n terms from $(n+1)^{th}$ to $(2n)^{th}$ term, Sum of terms from (n + 1) th to (2n) th term $\frac{a_{n+1}\left(1-r^{n}\right)}{\left(1-r\right)}$

$a^{n+1} = ar^{n+1-1} = ar^{n}$

Thus, required ratio = $\frac{a\left(1-r^{n}\right)}{\left(1-r\right)} \times \frac{\left(1-r\right)}{ar^{n}\left(1-r^{n}\right)} = \frac{1}{r^{n}}$

Thus, the ratio of the sum of first n terms of a G.P. to the sum of terms from (n + 1) th to (2n) th term is $\frac{1}{r^{n}}$.

25. If a, b, c and d are in G.P. show that $\left(a^{2}+b^{2}+c^{2}\right)\left(b^{2}+c^{2}+d^{2}\right)=\left(ab+bc+cd\right)^{2}$

Ans a, b, c, d are in G.P.

Therefore,

bc = ad … (1)

b^2 = ac … (2)

c^2 = bd … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc - cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d(a + c)]^2$

$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c_2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

= L.H.S.

∴ L.H.S. = R.H.S.

$\left(a^{2}+b^{2}+c^{2}\right)\left(b^{2}+c^{2}+d^{2}\right)=\left(ab+bc+cd\right)^{2}$

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Ans Let G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1, G_2, 81, forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

$\therefore 81 = (3)\,(r)^3$

$\Rightarrow r^3 = 27$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3)\,(3) = 9$

$G_2 = ar^2 = (3)\,(3)^2 = 27$

Thus, the required two numbers are 9 and 27

27. Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Ans. G. M. of a and b is $\sqrt{ab}$. By the given condition $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$,

Squaring both sides, we obtain

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2}=ab$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=(ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$$

$$\Rightarrow a^{2n+2}-a^{2n+1}b=ab^{2n+1}(a-b)$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$$

$$\Rightarrow 2n+1=0$$

$$\Rightarrow n=\frac{-1}{2}$$

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Ans Let the two numbers be a and b.

G.M. $= \sqrt{ab}$

According to the given condition,

$$a+b=6\sqrt{ab}\,.......(1)$$

$$\Rightarrow (a+b)^2=36(ab)$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$

$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$

$$=4\sqrt{2}\sqrt{ab}........(2)$$

Adding (1) and (2), we obtain

$$2a=\left(6+4\sqrt{2}\right)\sqrt{ab}$$

$$\Rightarrow a=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substituting the value of a in (1), we obtain

$$b=6\sqrt{ab}-\left(3+2\sqrt{2}\right)\sqrt{ab}$$

$$\Rightarrow b=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

$$\frac{a}{b}=\frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $3+2\sqrt{2}:3-2\sqrt{2}$

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are .

Ans It is given that A and G are A.M. and G.M. between two positive numbers. Let these two positive numbers be a and b.

$$\therefore AM=A=\frac{a+b}{2}......(1)$$

$$GM=G=\sqrt{ab}......(2)$$

From (1) and (2), we obtain

a + b = 2A ... (3)

ab = G^2 ... (4)

Substituting the value of a and b from (3) and (4) in the identity $(a-b)^2=(a+b)^2-4ab$, we obtain

$(a-b)^2=4A^2-4G^2=4\,(A^2-G^2)$

$(a-b)2=4\,(A+G)\,(A-G)$

$$(a-b)=2\sqrt{(A+G)(A-G)}........(5)$$

From (3) and (5), we obtain

$$2a=2A+2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a=a+\sqrt{(A+G)(A-G)}$$

Substituting the value of a in (3), we obtain

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

30 The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4th hour and nth hour?

Ans It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, a = 30 and r = 2

$\therefore$ a3 = $ar^2 = (30)(2)^2 = 120$

Therefore, the number of bacteria at the end of 2^{nd} hour will be 120.

$a^5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4^{th} hour will be 480.

$a^{n+1} = ar^n = (30)\, 2^n$

Thus, number of bacteria at the end of n^{th} hour will be $30(2)^{n.}$

31. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Ans The amount deposited in the bank is Rs 500.

At the end of first year, amount = $\text{Rs } 500\left(1+\frac{1}{10}\right)$ = Rs 500 (1.1)

At the end of 2^{nd} year, amount = Rs 500 (1.1) (1.1)

At the end of 3^{rd} year, amount = Rs 500 (1.1) (1.1) (1.1) and so on

$\therefore$ Amount at the end of 10 years = Rs 500 (1.1) (1.1) … (10 times) = Rs $500(1.1)^{10}$

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Ans Let the root of the quadratic equation be a and b.

According to the given condition,

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \ldots\ldots\ldots(1)$$

$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \ldots\ldots\ldots(2)$$

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a+b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$