| Board – | Class – | Topic – |
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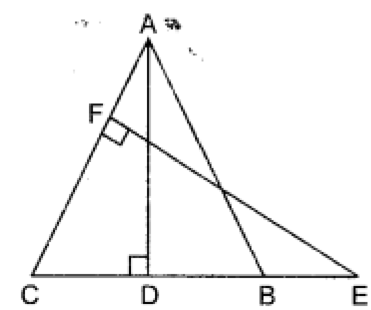
Short Answer Type Questions II [3 Marks]

**Question 1.**

In figure, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE ⊥ AC. If AD ⊥ CB, prove that AB X EF = AD X EC. [**2010**]

**Solution:**




In $\Delta ADB$ and $\Delta EFC$,

$\angle D = \angle F$ [Each 90°]

and $\angle B = \angle C$

[Angles opp. to equal sides of a triangle are equal]

$\Rightarrow \Delta ABD \sim \Delta ECF$ [AA similarity]

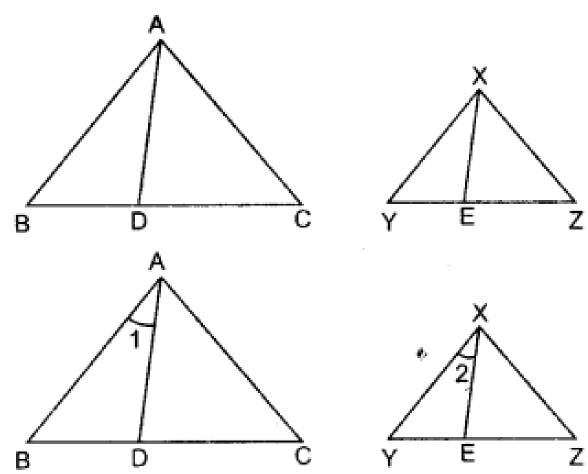
$\therefore \dfrac{AB}{EC} = \dfrac{AD}{EF}$ [Corresponding

$\therefore AB \times EF = AD \times EC$

**Question 2.**

In given figure ΔABC is similar to ΔXYZ and AD and XE are angle bisectors of ∠A and ∠X respectively such that AD and XE in centimetres are 4 and 3 respectively, find the ratio of area of ΔABD and area of ΔXYE. [**2011**]

**Solution:**




AD bisects $\angle A$ $\therefore$ $\angle 1 = \frac{1}{2}\angle A$

Similarly $\angle 2 = \frac{1}{2}\angle X$

$\because \Delta ABC \sim \Delta XYZ$

$\therefore \angle A = \angle X$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle X \Rightarrow \angle 1 = \angle 2$

Also $\angle B = \angle Y$

$\therefore \Delta ABD \sim \Delta XYE$

$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta XYE} = \frac{AD^2}{XE^2} = \frac{4^2}{3^2} = \frac{16}{9}$$

**Question 3.**

Right angled triangles BAC and BDC are right angled at A and D and they are on same side of BC. If AC and BD intersect at P, then prove that AP x PC = PB X DP. [**2014**]

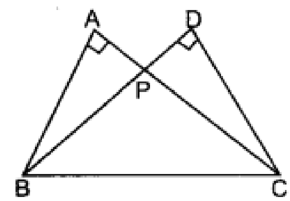
**Solution:**

In $\Delta APB$ and $\Delta DPC$,

$\angle BAP = \angle CDP = 90°$ [Given]

$\angle APB = \angle DPC$ [Vertically opposite angles]

$\therefore \Delta APB \sim \Delta DPC$ [by AA similarity of triangles.]




$$\therefore \frac{AP}{DP} = \frac{PB}{PC}$$

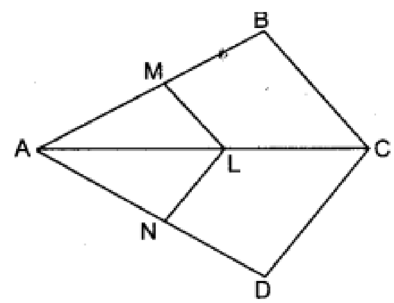
$$AP \times PC = PB \times DP$$

Hence proved.

**Question 4.**

In the given figure, if LM || CB and LN|| CD, prove that AM × AD = AB × AN. [**2014**]

**Solution:**




**Given:** LM || CB

LN || CD

**To prove:** AM × AD = AB × AN

**Proof:** In $\Delta BAC$,

LM || BC

$$\therefore \frac{AM}{MB} = \frac{AL}{LC} \quad \text{...(i) [By Thales Theorem]}$$

In $\Delta DAC$,

LN || CD

$$\therefore \frac{AN}{ND} = \frac{AL}{LC} \quad \text{...(ii) [By Thales Theorem]}$$

$\therefore$ From (*i*) and (*ii*)

$$\frac{AM}{MB} = \frac{AN}{ND}$$

or

$$\frac{MB}{AM} = \frac{ND}{AN}$$

$$\frac{MB}{AM}+1 = \frac{ND}{AN}+1 \quad \text{[Adding 1 on both sides]}$$

$$\frac{MB+AM}{AM} = \frac{ND+AN}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\therefore AM \times AD = AB \times AN$$

Hence proved.

**Question 5.**

Equiangular triangles are drawn on sides of right angled triangle in which perpendicular is double of its base. Show that area of triangle on the hypotenuse is the sum of areas of the other two triangles? [**2014**]

**Solution:**

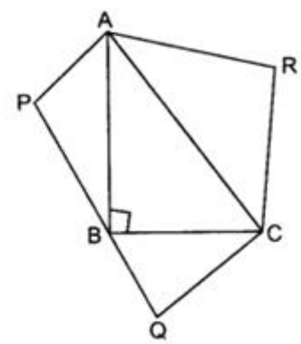
**Given:** A right angled triangle ABC with right angled at B.

Equiangular triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

Let BC = $x$ and AB = $2x$

**To prove:** ar($\Delta$PAB) + ar($\Delta$QBC) = ar($\Delta$RAC)

**Proof:** $\because$ Equiangular triangles are equilateral also,




$$\therefore \quad \text{Area of } \Delta PAB = \frac{\sqrt{3}}{4} \times (2x)^2 = \sqrt{3}x^2 \quad ...(i)$$

$$\text{Area of } \Delta QBC = \frac{\sqrt{3}}{4} \times (x)^2 = \frac{\sqrt{3}}{4}x^2 \quad ...(ii)$$

$$\text{Area of } \Delta RAC = \frac{\sqrt{3}}{4} \times (AC)^2 = \frac{\sqrt{3}}{4} \times 5x^2$$

[$\because$ In $\Delta$ABC, $AB^2 + BC^2 = AC^2$
$AC^2 = (2x)^2 + x^2 = 5x^2$]

$$= \frac{5\sqrt{3}x^2}{4} \quad ...(iii)$$

Adding ($i$) and ($ii$)

$$ar(\Delta PAB) + ar(\Delta QBC) = \sqrt{3}x^2 + \frac{\sqrt{3}x^2}{4} = \frac{4\sqrt{3}x^2 + \sqrt{3}x^2}{4} = \frac{5\sqrt{3}x^2}{4}$$

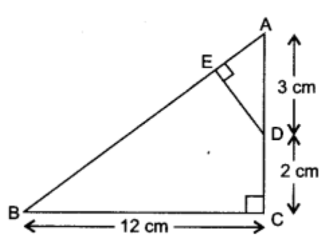
$$= ar(\Delta RAC) \quad \text{[From } (iii)]$$

$$\therefore \quad ar(\Delta PAB) + ar(\Delta QBC) = ar(\Delta RAC)$$

Hence proved.

**Question 6.**

In figure, AABC is right angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE. [**2009**]



Solution:

**Given:** $\Delta ABC$ and $\Delta ADE$ right angled at C and E.

**Proof:** In $\Delta ABC$ and $\Delta ADE$

$$\angle C = \angle E \quad [90^\circ \text{ Each}]$$
$$\angle A = \angle A \quad [\text{Common angle}]$$
$$\Delta ABC \sim \Delta ADE \quad [\text{By AA similarity}]$$

Since, $\Delta ABC \sim \Delta ADE$

In $\Delta ABC$, $AB^2 = AC^2 + BC^2$

$$AB^2 = 25 + 144 = 169$$

$\Rightarrow$ $AB = 13$

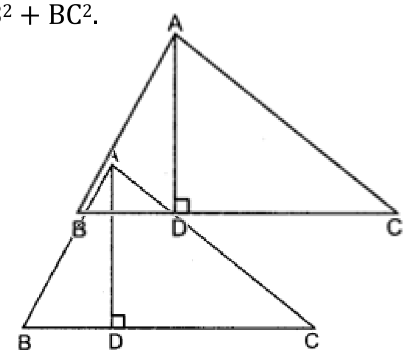
Now, $$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

Then, $$AE = \frac{15}{13}, DE = \frac{36}{13}$$

Question 7.

In figure, $AD \perp BC$ and $BD = \frac{1}{3}CD$. Prove that $2CA^2 = 2AB^2 + BC^2$. [**2009**]




Solution:

**In figure, $AD \perp BC$ and $BD = \frac{1}{3}CD$.**

**Prove that $2CA^2 = 2AB^2 + BC^2$.** [All India]

**Given:** In $\Delta ABC$, $AD \perp BC$ and $BD = \frac{1}{3}CD$

**To prove:** $2CA^2 = 2AB^2 + BC^2$

**Proof:** $\because$ $BC = BD + CD$ and $BD = \frac{1}{3}CD$ [Given]

$\therefore$ $$BC = \frac{1}{3}CD + CD = \frac{4}{3}CD$$

$\Rightarrow$ $$CD = \frac{3}{4}BC \quad ...(i)$$

In right angled $\Delta ADC$,

$$AC^2 = CD^2 + AD^2 \quad [\text{By Pythagoras theorem}] ...(ii)$$

In right angled $\Delta ADB$,

$$AB^2 = AD^2 + BD^2$$

$\Rightarrow$ $$AD^2 = AB^2 - BD^2$$

Substituting in (*ii*), we get

$$\Rightarrow \quad AC^2 = CD^2 + AB^2 - BD^2$$

$$\Rightarrow \quad AC^2 = CD^2 + AB^2 - \left(\frac{1}{3}CD\right)^2 \qquad \left[\text{Put } BD = \frac{1}{3}CD\right]$$

$$\Rightarrow \quad AC^2 = CD^2 - \frac{1}{9}CD^2 + AB^2$$

$$\Rightarrow \quad AC^2 = \frac{8}{9}CD^2 + AB^2$$

$$\Rightarrow \quad AC^2 = \frac{8}{9}\left(\frac{3}{4}BC\right)^2 + AB^2 \qquad [\text{Using } (i)]$$

$$\Rightarrow \quad AC^2 = \frac{8}{9} \times \frac{9}{16} BC^2 + AB^2$$

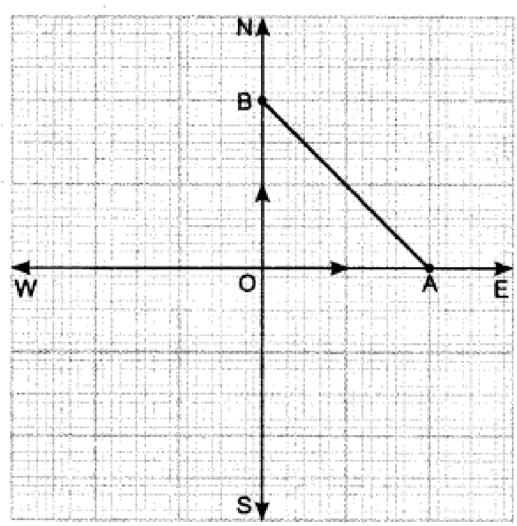
$$\Rightarrow \quad AC^2 = \frac{1}{2}BC^2 + AB^2$$

$$\Rightarrow \quad 2AC^2 = BC^2 + 2AB^2 \qquad \text{Hence proved.}$$

**Question 8.**

From airport two aero planes start at the same time. If the speed of first aero plane due North is 500 km/h and that of other due East is 650 km/h, then find the distance between two aeroplanes after 2 hours. [**2016**]

**Solution:**




Speed of aeroplane along north = 500 km/h

Speed of aeroplane along east = 650 km/h

Distance travelled by aeroplane in 2 hours in North direction.

$$= OB = 500 \times 2$$
$$= 1000 \text{ km.}$$

Distance travelled by aeroplane in 2 hours in East direction.

$$= OA = 650 \times 2$$
$$= 1300 \text{ km}$$

Distance between both ae roplanes after 2 hours = AB.

$$\therefore \quad AB^2 = OB^2 + ON^2$$

[By Pythagoras theorem in $\Delta AOB$]

$$= (1000)^2 + (1300)^2$$
$$= 1000000 + 1690000$$
$$= 2690000$$
$$AB = 100\sqrt{269} \text{ km}$$

Question 9.

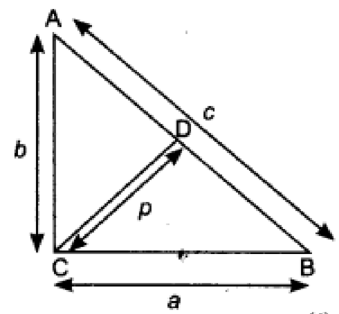
$\Delta$ABC, is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle$A,$\angle$B, $\angle$C respectively then prove that

$$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Solution:

**Given:** In $\Delta$ACB, $\angle C = 90°$, and CD $\perp$ AB.
Also, AB = $c$, BC = $a$, CA = $b$ and CD = $p$

**To prove:** $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$




**Proof:** In $\Delta$ABC, $\angle C = 90°$
Apply Pythagoras theorem

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow \quad c^2 = a^2 + b^2 \quad ...(i)$$

Now, area of $\Delta ABC = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times p \times c$ ...(ii)

Also area of $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times b \times a$ ...(iii)

Equating ($i$ ıd ($ii$), we have

$$\frac{1}{2}pc = \frac{1}{2}ab$$

$$pc = ab$$

$$c = \frac{ab}{p} \text{ and } c^2 = \frac{a^2b^2}{p^2}$$

Put value of $c^2$ in equation ($i$)

$$\frac{a^2b^2}{p^2} = a^2 + b^2$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} \quad \text{(dividing both sides by } a^2b^2\text{)}$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

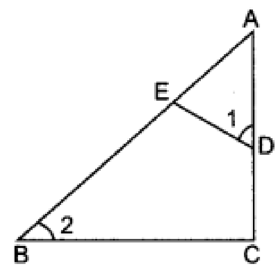
Long Answer Type Questions [4 Marks]

Question 10.

In ΔABC, if ∠ADE = ∠B, then prove that ΔADE ~ ΔABC. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.

[2015]

Solution:




**Given:** ∠ADE = ∠B, i.e. ∠1 = ∠2

**To prove:** ΔADE ~ ΔABC

**Proof:** In ΔADE and ΔABC

$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\Rightarrow \quad \dfrac{AD}{AB} = \dfrac{DE}{BC}$

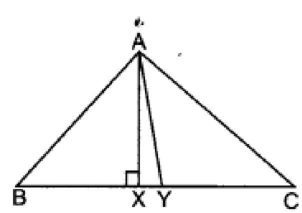
$\Rightarrow \quad \dfrac{7.6}{7.2+4.2} = \dfrac{DE}{8.4}$ $\{\because\ AB = AE + BE = 7.2 + 4.2\}$

$\Rightarrow \quad \dfrac{7.6}{11.4} = \dfrac{DE}{8.4} \Rightarrow DE = \dfrac{7.6 \times 8.4}{11.4} = 5.6$

Hence, DE = 5.6 cm.

Question 11.

**In ΔABC, AX ⊥ BC and Y is middle point of BC.**
**Then prove that,**




(*i*) $AB^2 = AY^2 + \dfrac{BC^2}{4} - BC.XY$

(*ii*) $AC^2 = AY^2 + \dfrac{BC^2}{4} + BC.XY$

**Given:** A ΔABC in which AX ⊥ BC and Y is mid-point of BC.

**Proof:** (*i*) In $\Delta ABX$.

$$AB^2 = AX^2 + BX^2 \quad \text{[By Pythagoras Theorem]}$$
$$AB^2 = AX^2 + (BY - XY)^2$$
$$AB^2 = AX^2 + \left(\frac{BC}{2} - XY\right)^2 \quad [\because \text{ Y is mid point of BC}]$$
$$AB^2 = AX^2 + \frac{BC^2}{4} + XY^2 - 2\left(\frac{BC}{2}\right)(XY)$$
$$AB^2 = (AX^2 + XY^2) + \frac{BC^2}{4} - \frac{2BC}{2}.XY$$
$$AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY \quad [\because \text{ In } \Delta AXY,\ AX^2 + XY^2 = AY^2]$$

Hence proved.

(*ii*) In $\Delta AXC$,

$$AC^2 = AX^2 + XC^2 \quad \text{[By Pythagoras Theorem]}$$
$$AC^2 = AX^2 + (XY + YC)^2$$
$$AC^2 = AX^2 + \left(XY + \frac{BC}{2}\right)^2 \quad [\because \text{ In Y is mid-point of BC}]$$
$$AC^2 = (AX^2 + XY^2) + \frac{BC^2}{4} + 2(XY).\left(\frac{BC}{2}\right)$$
$$AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY \quad [\because \text{ In } \Delta AXY,\ AX^2 + XY^2 = AY^2]$$

Hence proved.

Question 12.

In $\Delta ABC$, $\angle B = 90°$, $BD \perp AC$, ar $(\Delta ABC) = A$ and $BC = a$, then prove that [**2014**]

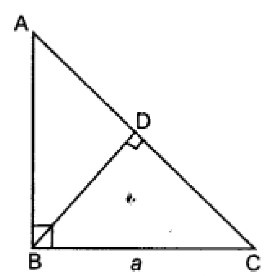
$$\mathbf{BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}}$$

Solution:

**Given:** Area of $\Delta ABC = A$

$BC = a$ and $BD \perp AC$

**To prove:** $BD = \dfrac{2Aa}{\sqrt{4A^2 + a^4}}$




**Proof:** $A$ = Area of $\Delta ABC$

$$A = \frac{1}{2}BC \times AB = \frac{1}{2} \times a \times AB$$
$$AB = \frac{2A}{a} \quad ...(i)$$

In $\Delta ADB$ and $\Delta ABC$

$$\angle ADB = \angle ABC = 90°$$
$$\angle BAD = \angle BAC \quad \text{[Common]}$$

$\therefore$ By AA similarity criterion, $\Delta ADB \sim \Delta ABC$

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{BC} \qquad \text{...}(ii)$$

In $\Delta ABC$, $AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]

$$\frac{4A^2}{a^2} + a^2 = AC^2$$

$$AC = \sqrt{\frac{4A^2 + a^4}{a^2}} = \frac{\sqrt{4A^2 + a^4}}{a}$$

From ($i$) and ($ii$), we get

$$\frac{2A}{a \times AC} = \frac{BD}{a}$$

$$BD = \frac{2A}{AC} = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Hence proved.

**Question 13.**

In the figure, ABC is a right triangle, right angled at B. AD and CF are two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$cm .Find the length of CE. [**2013**]

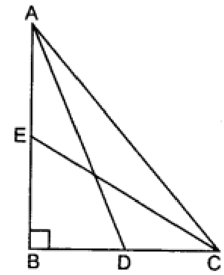
**Solution:**

In right-triangle ABD, $\angle B = 90°$

$\therefore \quad AD^2 = AB^2 + BD^2$ [By Pythagoras Theorem]

$\Rightarrow \quad AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ BD = DC]

$\Rightarrow \quad AD^2 = AB^2 + \frac{BC^2}{4}$ ...(*i*)




Now, in right-triangle EBC, $\angle B = 90°$,

$\therefore \quad CE^2 = BC^2 + BE^2$ [By Pythagoras Theorem]

$\Rightarrow \quad CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$ [$\because$ BE = AE]

$\Rightarrow \quad CE^2 = BC^2 + \frac{AB^2}{4}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$AD^2 + CE^2 = AB^2 + \frac{BC^2}{4} + BC^2 + \frac{AB^2}{4}$$

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

$$AD^2 + CE^2 = \frac{5}{4}AC^2 \quad [\because \text{By Pythagoras Theorem in } \Delta ABC\ AC^2 = AB^2 + BC^2]$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4} \times 25$$

$$CE^2 = \frac{125}{4} - \frac{45}{4}$$

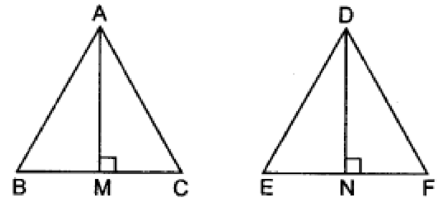
$$CE^2 = 20$$

$$\therefore \quad CE = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

**Question 14.**

The area of two similar triangles is 49 $cm^2$ and 64 $cm^2$ respectively. If the difference of the corresponding altitudes is 10 cm, then find the lengths of altitudes (in centimeters). [**2011**]

**Solution:**




$\Delta ABC \sim \Delta DEF$ (Given)

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{49}{64} = \frac{BC^2}{EF^2} \Rightarrow \frac{BC}{EF} = \frac{7}{8}$$

$$\text{Also} \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN} \Rightarrow \frac{49}{64} = \frac{BC}{EF} \times \frac{AM}{DN}$$

$$\Rightarrow \quad \frac{49}{64} = \frac{7}{8} \times \frac{AM}{DN} \quad \Rightarrow \frac{7}{8} = \frac{AM}{DN} \quad \Rightarrow \quad DN = \frac{8}{7}AM$$

Also $DN - AM = 10$ (Given)

$$\Rightarrow \quad \frac{8}{7}AM - AM = 10 \quad \Rightarrow \frac{1}{7}AM = 10$$

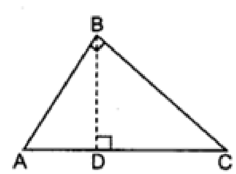
$$AM = 70 \text{ cm}$$

$$\therefore \quad DN = 80 \text{ cm}$$

**Question 15.**

Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Point D is the mid-point of the side BC of a right triangle ABC, right angled at C. Prove that, $4AD^2 = 4AC^2 + BC^2$. [**2010**]

**Solution:**




**Given:** In $\Delta ABC$, $\angle B = 90°$

**To prove:** $AC^2 = AB^2 + BC^2$

**Construction:** Draw $BD \perp AC$

**Proof:** Since, in $\Delta ABC$, $\angle B = 90°$ and $BD \perp AC$

so, $\Delta ADB \sim \Delta ABC$ [If a $\perp$ is drawn from the vertex of the rt. angle of rt. $\Delta$ to the hypotenuse then $\Delta$'s on both sides of the $\perp$ are similar to the whole $\Delta$ and to each other]

and $\Delta BDC \sim \Delta ABC$

Now, $\Delta ADB \sim \Delta ABC$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB^2 = AC.AD \quad ...(i)$$

Again, $\Delta BDC \sim \Delta ABC$

$$\Rightarrow \quad \frac{CD}{BC} = \frac{BC}{AC} \Rightarrow BC^2 = AC.CD \quad ...(ii)$$

Adding equation (*i*) and (*ii*), we get

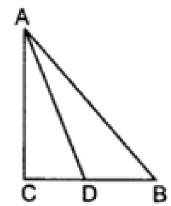
$$AB^2 + BC^2 = AC \cdot AD + AC \cdot CD$$
$$= AC(AD + CD)$$
$$= AC \cdot AC$$
$$= AC^2$$

$\therefore \quad AB^2 + BC^2 = AC^2$ Hence proved

**Other part:**

In $\Delta ADC$, $\angle C = 90°$

$\therefore \quad AD^2 = AC^2 + CD^2 \quad ...(i)$




D is mid point of BC

$\therefore \quad CD = \frac{1}{2}BC$

Putting this value in equation (*i*), we get

$$AD^2 = AC^2 + \left(\frac{BC}{2}\right)^2$$
$$AD^2 = AC^2 + \frac{BC^2}{4}$$
$$4AD^2 = 4AC^2 + BC^2$$

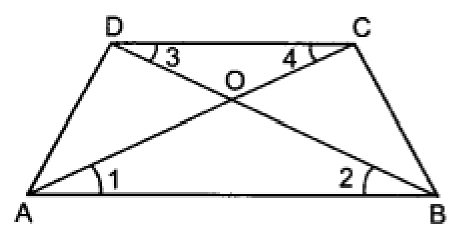
Question 16.

**Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.** [2009]

Using the above, do the following:

In a trapezium ABCD, AC and BD intersecting at OAB|| DC and AB = 2CD, if area of ΔAOB = 84 cm2, find the area of ΔCOD

Solution:




**Given:** ABCD is a trapezium

$AB \parallel CD$

Also $AB = 2CD$

**To find:** Area of $\Delta COD$

Now, in $\Delta AOB$ and $\Delta COD$

$$\angle 1 = \angle 4$$
$$\angle 2 = \angle 3$$

$\therefore \quad \Delta AOB \sim \Delta COD$

$$\frac{ar\Delta AOB}{ar\Delta COD} = \frac{(AB)^2}{(CD)^2}$$
$$\frac{84}{ar\Delta COD} = \frac{(2CD)^2}{(CD)^2}$$
$$\frac{84}{ar\Delta COD} = \frac{4(CD)^2}{(CD)^2}$$
$$\frac{84}{ar\Delta COD} = \frac{4}{1}$$
$$ar\ \Delta COD = 21\ cm^2$$