SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

1. Reduce the following equations into slope-intercept form and find their slopes and the y intercepts.

(i) $x + 7y = 0$ (ii) $6x + 3y - 5 = 0$ (iii) $y = 0$

Ans (i) The given equation is $x + 7y = 0$.

It can be written as

$$y = -\frac{1}{7}x = 0....(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$

$$y = -2x + \frac{5}{3}.....(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$

Therefore, equation (2) is in the slope-intercept form, where the slope and the y intercept are–2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$y = 0.x + 0$ ... (3)

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y intercept are 0 and 0 respectively.

2. Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$ (ii) $4x - 3y = 6$ (iii) $3y + 2 = 0$.

Ans (i) The given equation is $3x + 2y - 12 = 0$.

It can be written as

$$3x + 2y = 12$$

$$\frac{3x}{12} + \frac{2y}{12} = 1$$

$$\text{i.e, } \frac{x}{4} + \frac{y}{6} = 1 \ldots..(1)$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where a = 4 and b = 6.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is 4x – 3y = 6.

It can be written as

$$\frac{4x}{6} - \frac{3x}{6} = 1$$

$$\frac{2x}{3} - \frac{y}{2} = 1$$

$$\text{i.e, } \frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1 \ldots\ldots(2)$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{3}{2}$ and b = –2.

Therefore, equation (2) is in the intercept form, where the intercepts on the x and y axes are $\frac{3}{2}$ and –2 respectively.

(iii) The given equation is 3y + 2 = 0.

It can be written as

$$3y = -2$$

$$\text{i.e, } \frac{y}{\left(-\frac{2}{3}\right)} = 1 \ldots..(3)$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where a = 0 and $b = -\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

(i) $(i)\ x-\sqrt{3}y+8=0$ (ii) y – 2 = 0 (iii) x – y = 4

Ans (i) The given equation is $x-\sqrt{3}y+8=0$.

It can be reduced as:

$$x-\sqrt{3}y=8$$
$$\Rightarrow -x+\sqrt{3}y=8$$

On dividing both sides by $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{4}=2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\Rightarrow\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$\Rightarrow x\cos 120°+y\sin 120°=4.......(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x\cos\omega + y\sin\omega = p$, we obtain $\omega = 120°$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y - 2 = 0$.

It can be reduced as $0.x + 1.y = 2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain $0.x + 1.y = 2$

$\Rightarrow x\cos 90° + y\sin 90° = 2$ … (1)

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x\cos\omega + y\sin\omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90°.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)\,y = 4$

On dividing both sides by $\sqrt{0^2+(-1)^2}=\sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$$

$$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$$

$$\Rightarrow x\cos 315°+y\sin 315°=2\sqrt{2}......(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

x cos ω + y sin ω = p, we obtain ω = 315° and $p=2\sqrt{2}$

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315°

4. Find the distance of the point (–1, 1) from the line 12(x + 6) = 5(y – 2).

Ans The given equation of the line is 12(x + 6) = 5(y – 2).

⇒ 12x + 72 = 5y – 10

⇒12x – 5y + 82 = 0 … (1)

On comparing equation (1) with general equation of line Ax + By + C = 0, we obtain A = 12, B = –5, and C = 82.

It is known that the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

The given point is (x1, y1) = (–1, 1).

Therefore, the distance of point (–1, 1) from the given line

$$=\frac{|12(-1)+(-5)(1)+82|}{\sqrt{(12)^2+(-5)^2}}\text{units}=\frac{|-12-5+82|}{\sqrt{169}}\text{units}=\frac{|65|}{13}\text{units}=5\text{ units}$$

5. Find the points on the x-axis, whose distances from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

Ans The given equation of line is

$$\frac{x}{3}+\frac{y}{4}=1$$

$$\text{or },4x+3y-12=0.......(1)$$

On comparing equation (1) with general equation of line Ax + By + C = 0, we obtain A = 4, B = 3, and C = –12.

Let (a, 0) be the point on the x-axis whose distance from the given line is 4 units

It is known that the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x1, y1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

Therefore,

$$4=\frac{|4x_1+3\times 0-12|}{\sqrt{4^2+3^2}}$$

$$\Rightarrow 4=\frac{|4a-12|}{5}$$

$$\Rightarrow 4=|4a-12|=20$$

$$\Rightarrow \pm(4a-12)=20$$

$$\Rightarrow (4a-12)=20 \text{ or } -(4a-12)=20$$

$$\Rightarrow 4a=20+12 \text{ or } 4a=-20+12$$

$$\Rightarrow a=8 \text{ or } -2$$

Thus, the required points on the x-axis are (−2, 0) and (8, 0)

6. Find the distance between parallel lines

(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0

(ii) l (x + y) + p = 0 and l (x + y) – r = 0

Ans It is known that the distance (d) between parallel lines Ax + By + C1 = 0 and Ax + By + C2 = 0 is given by

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}.$$

(i) The given parallel lines are 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0.

Here, A = 15, B = 8, C1 = −34, and C2 = 31.

Therefore, the distance between the parallel lines is

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}=\frac{|-34-31|}{\sqrt{(15)^2+(8)^2}}\text{units}=\frac{|-65|}{17}\text{units}=\frac{65}{17}\text{units}$$

(ii) The given parallel lines are l (x + y) + p = 0 and l (x + y) – r = 0.

lx + ly + p = 0 and lx + ly – r = 0

Here, A = l, B = l, C_1 = p, and C_2 = −r.

Therefore, the distance between the parallel lines is

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}=\frac{|p+r|}{\sqrt{2l^2}}\text{units}=\frac{|p+r|}{l\sqrt{2}}\text{units}=\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|\text{units}$$

7. Find equation of the line parallel to the line 3x – 4y + 2 = 0 and passing through the point (−2, 3).

Ans The equation of the given line is

$$3x - 4y + 2 = 0$$

$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$$\text{or } y = \frac{3}{4}x + \frac{1}{2}$$

, which is of the form y = mx + c

∴ Slope of the given line $= \frac{3}{4}$

It is known that parallel lines have the same slope.

∴ Slope of the other line $= m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point (–2, 3) is

$$(y - 3) = \frac{3}{4}\{x - (-2)\}$$

$$4y - 12 = 3x + 6$$

$$\text{i.e, } 3x - 4y + 18 = 0$$

8. Find equation of the line perpendicular to the line x – 7y + 5 = 0 and having x intercept 3.

Ans The given equation of line is $x - 7y + 5 = 0$.

$$\text{or, } y = \frac{1}{7}x + \frac{5}{7}$$

which is of the form y = mx + c

∴ Slope of the given line $= \frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m = -\frac{1}{\left(\frac{1}{7}\right)} = -7$

The equation of the line with slope –7 and x-intercept 3 is given by

y = m (x – d)

⇒ y = –7 (x – 3)

⇒ y = –7x + 21

⇒ 7x + y = 21

9. Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Ans The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

$y = -\sqrt{3}x + 1 \ldots\ldots(1)$ and $y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}$

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\frac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$

$$\tan\theta \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or 180° – 30° = 150°

10. The line through the points (h, 3) and (4, 1) intersects the line 7x – 9y – 19 = 0. at right angle. Find the value of h.

Ans The slope of the line passing through points (h, 3) and (4, 1) is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line 7x – 9y – 19 = 0 or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$.

It is given that the two lines are perpendicular.

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$

$$\Rightarrow \frac{-14}{36-9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$

11. Prove that the line through the point (x1, y1) and parallel to the line Ax + By + C = 0 is A (x –x1) + B (y – y1) = 0.

Ans The slope of line Ax + By + C = 0 or $y=\left(\frac{-A}{B}\right)x+\left(\frac{-C}{B}\right)$ is $m=-\frac{A}{B}$

It is known that parallel lines have the same slope.

∴ Slope of the other line = $m=-\frac{A}{B}$

The equation of the line passing through point (x1, y1) and having a slope $m=-\frac{A}{B}$ is

$$y-y_1=-\frac{A}{B}(x_1-x_1)$$
$$B(y-y_1)=-A(x_1-x_1)$$
$$A(x_1-x_1)+B(y-y_1)=0$$

Hence, the line through point (x_1, y_1) and parallel to line Ax + By + C = 0 is A (x –x_1) + B (y – y_1) = 0

12. Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Ans. It is given that the slope of the first line, m_1 = 2.

Let the slope of the other line be m_2.

The angle between the two lines is 60°

$$\therefore m_1\times m_2=-1$$
$$\Rightarrow\left(\frac{-2}{4-h}\right)\times\left(\frac{7}{9}\right)=-1$$
$$\Rightarrow\frac{-14}{36-9h}=-1$$
$$\Rightarrow 14=36-9h$$
$$y-y_1=m(x_1-x_1)$$
$$\therefore \tan 60^\circ=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$\Rightarrow\sqrt{3}=\left|\frac{2-m_2}{1+2m_2}\right|$$
$$\Rightarrow\sqrt{3}=\pm\left(\frac{2-m_2}{1+2m_2}\right)$$

$$\Rightarrow \sqrt{3} = \frac{2-m_2}{1+2m_2} \text{ or } \sqrt{3} = -\left(\frac{2-m_2}{1+2m_2}\right)$$

$$\Rightarrow \sqrt{3}(1+2m_2) = 2-m_2 \text{ or } \sqrt{3}(1+2m_2) = -(1-2m_2)$$

$$\Rightarrow \sqrt{3}+2\sqrt{3}m_2+m_2 = 2 \text{ or } \sqrt{3}+2\sqrt{3}m_2-m_2 = -2$$

$$\Rightarrow \sqrt{3}+\left(2\sqrt{3}+1\right)m_2 = 2 \text{ or } \sqrt{3}+\left(2\sqrt{3}-1\right)m_2 = -2$$

$$\Rightarrow m_2 = \frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)} \text{ or } m_2 = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$$

CASE I: $m_2 = \left(\frac{2-\sqrt{3}}{2\sqrt{3}+1}\right)$

The equation of the line passing through point (2, 3) and having a slope of $\left(\frac{2-\sqrt{3}}{2\sqrt{3}+1}\right)$ is

$$(y-3) = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x+2)$$

$$\left(2\sqrt{3}+1\right)y-3\left(2\sqrt{3}+1\right) = \left(2-\sqrt{3}\right)x-2\left(2-\sqrt{3}\right)$$

$$\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y = -4+2\sqrt{3}+6\sqrt{3}+3$$

$$\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y = -1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y = -1+8\sqrt{3}$

CASE I: $m_2 = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}+1\right)}$

The equation of the line passing through point (2, 3) and having a slope of $\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}+1\right)}$ is

$$(y-3) = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}(x-2)$$

$$\left(2\sqrt{3}-1\right)y-3\left(2\sqrt{3}-1\right) = -\left(2+\sqrt{3}\right)x+2\left(2+\sqrt{3}\right)$$

$$\left(2\sqrt{3}-1\right)y+\left(2+\sqrt{3}\right)x = 4+2\sqrt{3}+6\sqrt{3}-3$$

$$\left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y = 1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y = 1+8\sqrt{3}$

Thus, the required equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$ or

$\left(2+\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$

13. Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Ans The right bisector of a line segment bisects the line segment at 90°.

The end-points of the line segment are given as A (3, 4) and B (–1, 2).

Accordingly, mid-point of AB $=\left(\frac{3-1}{2},\frac{4+2}{2}\right)=(1,3)$

Slope of AB $=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{-2}{-4}=\frac{1}{2}$

∴ Slope of the line perpendicular to AB $=-\frac{1}{\left(\frac{1}{2}\right)}=-2$

The equation of the line passing through (1, 3) and having a slope of –2 is

$(y-3)=-2(x-1)$

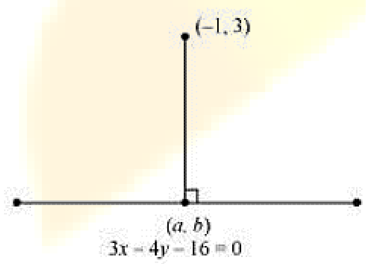
$y-3=-2x+2$

$2x+y=5$

Thus, the required equation of the line is $2x+y=5$.

14. Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line $3x-4y-16=0$.

Ans Let (a, b) be the coordinates of the foot of the perpendicular from the point (–1, 3) to the line $3x-4y-16=0$.



Slope of the line joining (–1, 3) and (a, b), $m_1=\frac{b-3}{a+2}$

Slope of the line $3x-4y-16=0$ or $y=\frac{3}{4}x-4, m_2=\frac{3}{4}$

Since these two lines are perpendicular, $m_1m_2=-1$

$\therefore \left(\frac{b-3}{a+1}\right)\times\left(\frac{3}{4}\right)=-1$

$\Rightarrow \frac{3b-9}{4a+4}=-1$

$\Rightarrow 3b-9=-4a-4$

$\Rightarrow 4a+3b=5$

Point (a, b) lies on line 3x – 4y = 16.

∴ 3a – 4b = 16 … (2)

On solving equations (1) and (2), we obtain

$a=\frac{68}{25}$ and $b=-\frac{49}{25}$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25},-\frac{49}{25}\right)$

15. The perpendicular from the origin to the line y = mx + c meets it at the point (−1, 2). Find the values of m and c.

Ans The given equation of line is y = mx + c.

It is given that the perpendicular from the origin meets the given line at (−1, 2).

Therefore, the line joining the points (0, 0) and (−1, 2) is perpendicular to the given line.

∴ Slope of the line joining (0, 0) and (−1, 2) $=\frac{2}{-1}=-2$

The slope of the given line is m.

$\therefore m\times-2=-1$ [The two lines are perpendicular]

$\Rightarrow m=\frac{1}{2}$

Since point (−1, 2) lies on the given line, it satisfies the equation y = mx + c.

$\therefore 2=m\left(-1\right)+c$

$\Rightarrow 2=\frac{1}{2}\left(-1\right)+c$

$\Rightarrow c=2+\frac{1}{2}=\frac{5}{2}$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$

16. If p and q are the lengths of perpendiculars from the origin to the lines x cos θ – y sin θ = k cos 2θ and x sec θ+ y cosec θ = k, respectively, prove that $p^2 + 4q^2 = k^2$

Ans The equations of given lines are

x cos θ – y sinθ = k cos 2θ … (1)

x secθ + y cosec θ= k ... (2)

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation (1) to the general equation of line i.e., Ax + By + C = 0,

we obtain A = cosθ, B = –sinθ, and C = –k cos 2θ.

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$\therefore p=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}=|-k\cos 2\theta|..........(3)$$

On comparing equation (2) to the general equation of line i.e., Ax + By + C = 0,

we obtain A = secθ, B = cosecθ, and C = –k.

It is given that q is the length of the perpendicular from (0, 0) to line (2).

$$\therefore q=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}..........(4)$$

From (3) and (4), we have

$$p^2+4q^2=\left(|-k\cos 2\theta|\right)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}\right)^2$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\sec^2\theta+\operatorname{cosec}^2\theta\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\sec^2\theta}+\frac{1}{\sin^2\theta}\right)}$$

$$= k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta$$

$$= k^2\cos^2 2\theta+k^2\left(2\sin\theta\cos\theta\right)^2$$

$$=k^2\cos^2 2\theta+k^2\sin^2 2\theta$$

$$=k^2\left(\cos^2 2\theta+k^2\sin^2 2\theta\right)$$

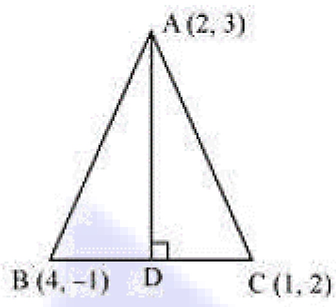
$$=k^2$$

Hence, we proved that $p^2 + 4q^2 = k^2$

17. In the triangle ABC with vertices A (2, 3), B (4, –1) and C (1, 2), find the equation and length of altitude from the vertex A.

Ans Let AD be the altitude of triangle ABC from vertex A.

Accordingly, AD⊥BC



The equation of the line passing through point (2, 3) and having a slope of 1 is

$(y - 3) = 1(x - 2)$

$\Rightarrow x - y + 1 = 0$

$\Rightarrow y - x = 1$

Therefore, equation of the altitude from vertex A = y – x = 1.

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$

$$\Rightarrow (y+1) = -1(x+4)$$

$$\Rightarrow y+1 = -x+4$$

$$\Rightarrow x+y-3 = 0 \ldots\ldots\ldots(1)$$

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line Ax + By + C = 0, we obtain A = 1, B = 1, and C = –3.

$$\therefore \text{Length of AD} = \frac{|1\times 2 + 1\times 3 - 3|}{\sqrt{1^2+1^2}}\text{units} = \frac{|2|}{\sqrt{2}}\text{units} = \frac{2}{\sqrt{2}}\text{units} = \sqrt{2}\text{units}$$

Thus, the equation and the length of the altitude from vertex A are y – x = 1 and $\sqrt{2}$ units respectively.

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} = \frac{1}{b^2}$

Ans. It is known that the equation of a line whose intercepts on the axes are a and b is

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\text{or } bx+ay=ab$$

$$\text{or } bx+ay-ab......(1)$$

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+c|}{\sqrt{A^2+B^2}}$$

On comparing equation (1) to the general equation of line Ax + By + C = 0, we obtain A = b, B = a, and C = –ab.

Therefore, if p is the length of the perpendicular from point (x1, y1) = (0, 0) to line (1),

we obtain

$$p=\frac{|A(0)+B(0)-ab|}{\sqrt{b^2+a^2}}$$

$$\Rightarrow p=\frac{|-ab|}{\sqrt{b^2+a^2}}$$

On squaring both sides, we obtain

$$p^2=\frac{(-ab)^2}{a^2b^2}$$

$$\Rightarrow p^2\left(a^2+b^2\right)=a^2b^2$$

$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, we showed that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$