CBSE 10^{th}

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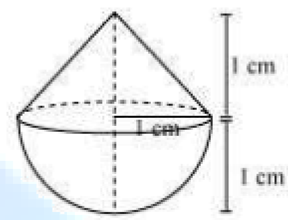
Q.1 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol: Radius (r1) of hemisphere = 4.2 cm



Radius (r2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the hemisphere will be the same in volume.

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3}\pi(4.2)^2 = \pi(6)^2 h$$

$$\frac{4}{3}\times\frac{4.2\times4.2\times.42}{36} = h$$

$$h = (1.4)^3 = 2.74 \text{ cm}$$

Hence, the height of the cylinder so formed will be 2.74 cm.

Q.2 Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol: Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.

Volume of 3 spheres = Volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

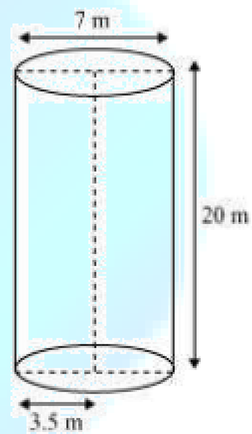
$$\frac{4}{3}\pi\left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$

$$r^3 = 216 + 512 + 1000$$

$$r^3 = 1728 \Rightarrow r = 12\,\text{cm}$$

Therefore, the radius of the sphere so formed will be 12 cm.

Q.3 A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol: The shape of the well will be cylindrical.

Depth (h) of well = 20 m

Radius (r) of circular end of well = 7/2 m

Area of platform = Length × Breadth = 22 × 14 m^2

Let height of the platform = H

Volume of soil dug from the well will be equal to the volume of soil scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

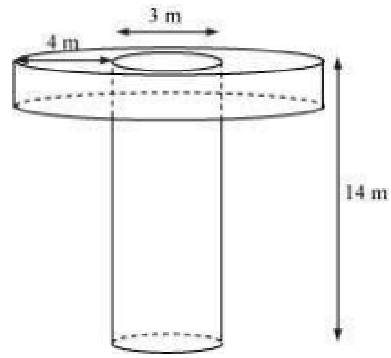
$$\pi \times r^2 \times h = \text{Area of platform} \times \text{Height of platform}$$

$$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$$

$$\therefore H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14} = \frac{5}{2}\,\text{m} = 2.5\text{m}$$

Therefore, the height of such platform will be 2.5 m.

Q.4 A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.



Sol: The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r_1) of the circular end of well = 3/2 m

Width of embankment = 4 m

From the figure, it can be observed that our embankment will be in a cylindrical

shape having outer radius (r_2) as and inner radius (r_1) as 3/2 m.

Let the height of embankment be h_2.

Volume of soil dug from well = Volume of earth used to form embankment

$$\pi \times r_1^2 \times h_1 = \pi\left(r_2^2 - r_1^2\right) \times h_2$$

$$= \pi\left(\frac{3}{2}\right)^2 \times 14 = \pi\left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h$$

$$\frac{9}{4} \times 14 = \frac{112}{4} \times h$$

$$h = \frac{9}{8} = 1.125 \text{m}$$

Therefore, the height of the embankment will be 1.125 m.

Q.5 A container shaped like a right circular cylinder having diameter 12 cm and height 15cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice ream.

Sol: Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container =12/2 = 6 cm

Radius (r_2) of circular end of ice-cream cone = 6/2 = 3 cm

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder =

n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3} r_2^3\right)$$

$$n = \frac{6^2 \times 15}{\frac{1}{3}\times 9\times 12 + \frac{2}{3}\times (3)^2}$$

$$n = \frac{36\times 15\times 3}{108+54} \Rightarrow n = 10$$

Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

Q.6 How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm ? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol: Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2}$ = 0.875 cm

Let n coins be melted to form the required cuboids.

Volume of n coins = Volume of cuboids

$$n\times \pi \times r^2 \times h_1 = l\times b\times h$$

$$n\times \pi \times (0.875)^2 \times 0.2 = 5.5\times 10\times 3.5$$

$$n = \frac{5.5\times 10\times 3.5\times 7}{(0.875)^2 \times 0.2\times 22} = 400$$

Therefore, the number of coins melted to form such a cuboid is 400.

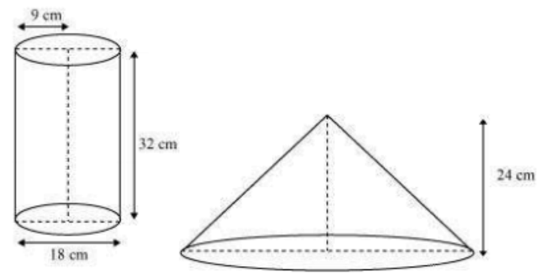
Q.7 A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Sol: Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.



The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$r_2^2 = \frac{3 \times 18^2 \times 32}{24} = 18^2 \times 4$$

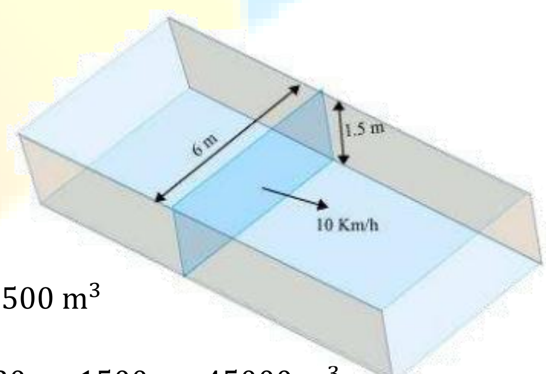
$$r_2 = 18 \times 2 = 36\,\text{cm}$$

$$\text{Slant height} = \sqrt{36^2 + 24^2} = \sqrt{12^2 \times \left(3^2 + 2^2\right)} = 12\sqrt{13}\,\text{cm}$$

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively

Q.8 Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. how much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol: Consider an area of cross − section of canal as ABCD.



Area of cross − section $= 6 \times 1.5 = 9\ \text{m}^2$

Speed of water $= 10\dfrac{\text{km}}{\text{h}} = \dfrac{10000}{60}$ metre/min

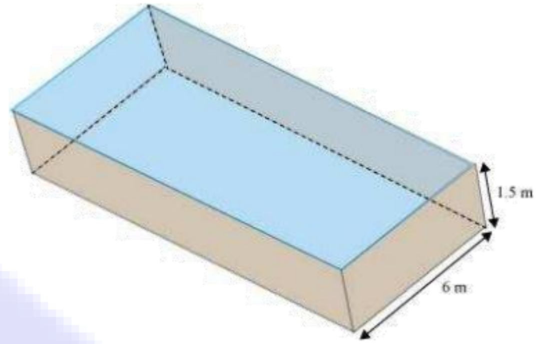
Volume of water that flows in 1 minute from canal $= = 1500\ \text{m}^3$

Volume of water that flows in 30 minutes from canal $= 30 \times 1500 = 45000\ \text{m}^3$

Volume of vessel = Volume of sphere + Volume of cylinder

Let the irrigated area be A. Volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Vol. of water flowing in 30 minutes from canal $=$ Vol. of water irrigating the reqd. area



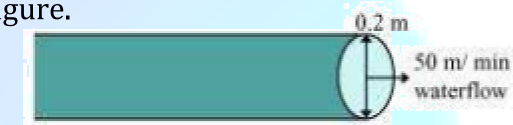
$$45000 = \frac{A \times 8}{100}$$

$$A = 562500 \text{ m}^2$$

Therefore, area irrigated in 30 minutes is 562500 m^2.

Q.9 A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol: Consider an area of cross-section of pipe as shown in the figure.

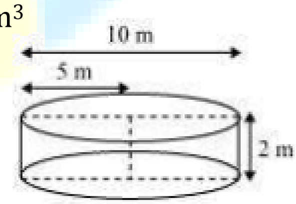


Radius (r_1) of circular end of pipe $= \frac{20}{200} = 0.1\text{m}$

Area of cross-section $= \pi r_1^2 = \pi(0.1)^2 = 0.01\pi \text{m}^2$

Speed of water = 3 km/h $= \frac{3000}{60} = 50 \text{ metre / min}$

Volume of water that flows in 1 minute from pipe $= 50 \times 0.01\pi = 0.5\pi \text{ m}^3$



Volume of water that flows in t minutes from pipe $= t \times 0.5\pi \text{ m}^3$

Radius (r_2) of circular end of cylindrical tank $= 10/2 = 5\text{m}$

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = 5^2 \times 2$

$t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.