CBSE 10th

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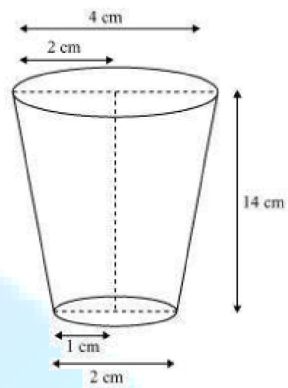
Q.1 A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Sol: Radius (r_1) of upper of glass $= \frac{4}{2} = 2$ cm

Radius (r_2) of lower base of glass $= \frac{2}{2} = 1$cm

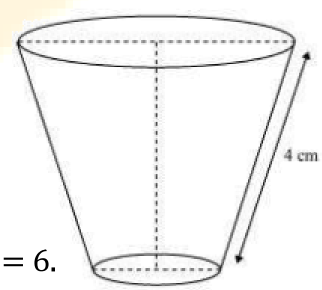
Capacity of glass = Volume of frustum of cone

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$
$$= \frac{1}{3}\pi h\left[(2)^2 + (1)^2 + (2)(1)\right]$$
$$= \frac{1}{3} \times \frac{22}{7} \times 14[4+1+2]$$
$$= \frac{308}{3} = 102\frac{2}{3}\text{cm}^2$$

Therefore, the capacity of the glass is $102\frac{2}{3}\text{cm}^2$

Q.2 The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. find the curved surface area of the frustum.

Sol: Perimeter of upper circular end of frustum = 18.



$2\pi r_1 = 18$

$r_1 = \frac{9}{\pi}$

Perimeter of lower end of frustum = 6 cm $\Rightarrow$ $2\pi r_2 = 6$.

$r_2 = \frac{3}{\pi}$

Slant height (l) of frustum = 4

CSA of frustum = π (r_1 + r_2) l

$$= \pi\left(\frac{9}{\pi}+\frac{3}{\pi}\right)4$$
$$= 12 \times 4 = 48 \text{ cm}^2$$

Therefore, the curved surface area of the frustum is 48 cm^2.

Q.3 A fez, the cap used by the Turks, is shaped like the frustum of a cone (see the figure given below). If its radius on the open side is 10 cm, radius at the upper base is 4 cm & its slant height is 15 cm, find the area of material use for making it. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol: Radius (r_2) at upper circular end = 4 cm

Radius (r_1) at lower circular end = 10 cm

Slant height (l) of frustum = 15 cm

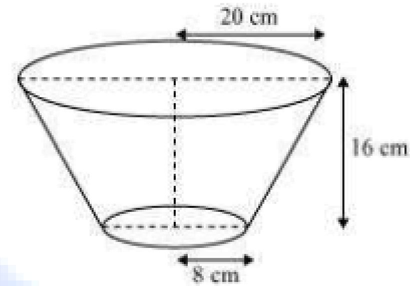
Area of material used for making the fez = CSA of frustum + Area of upper circular end

$$= \pi(r_1 + r_2)l + \pi r_2^2$$
$$= \pi\ (10\ +\ 4)\ 15\ +\ \pi\ (4)^2$$
$$= \pi\ (14)\ 15\ +\ 16\ \pi$$
$$= 210\pi + 16\pi = \frac{226 \times 22}{7} = 710\frac{2}{7}\text{cm}^2$$

Therefore, the area of material used for making it is $710\frac{2}{7}\text{cm}^2$

Q.4 A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs.20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs.8 per 100 cm^2 . [Take π = 3.14]

Sol: Radius (r_1) of upper end of container = 20 cm



Radius (r_2) of lower end of container = 8 cm

Height (h) of container = 16 cm

Slant height (l) of frustum $= \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + (16)^2} = \sqrt{(12)^2 + (16)^2}$

$= \sqrt{144 + 256} = 20\text{cm}$

Capacity of container = Volume of frustum

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 16\left[(20)^2 + (8)^2 + (20)(8)\right]$

$= \frac{1}{3} \times 3.14 \times 16(400 + 64 + 160)$

$= \frac{1}{3} \times 3.14 \times 16 \times 624$

$= 10449.92\,\text{cm}^3 = 10.45$ litres.

Cost of 1 litre milk = Rs 20

Cost of 10.45 litre milk = 10.45 × 20 = Rs 209

Area of metal sheet used to make the container

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi\ (20 + 8)\ 20 + \pi\ (8)^2$

$= 560\ \pi + 64\ \pi = 624\ \pi\ cm^2$

Cost of 100 cm^2 metal sheet = Rs 8

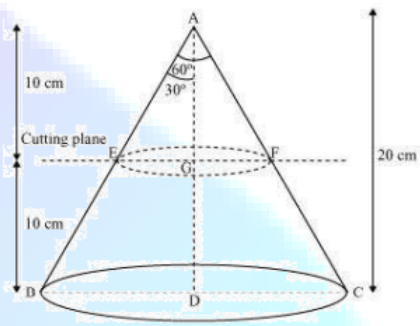
Cost of $624\pi\,\text{cm}^2$ metal sheet $= \frac{624 \times 3.14 \times 8}{100} = 156.75$

Therefore, the cost of the milk which can completely fill the container is

Rs 209 and the cost of metal sheet used to make the container is Rs 156.75.

Q.5 A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained is drawn into a wire of diameter 1/16 cm, find the length of the wire. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol:



In ΔAEG,

$$\frac{EG}{AG} = \tan 30^\circ$$

$$\frac{EG}{AG} = \frac{10}{\sqrt{3}}\text{cm} = \frac{10\sqrt{3}}{3}$$

In ΔABD,

$$\frac{BD}{AD} = \tan 30^\circ$$

$$BD = \frac{20}{\sqrt{3}}\text{cm} = \frac{20\sqrt{3}}{3}\text{cm}$$

Radius (r_1) of upper end of frustum $= \frac{10\sqrt{3}}{3}$ cm

Radius (r_2) of lower end of container $= \frac{20\sqrt{3}}{3}$ cm

Height (h) of container = 10 cm

Volume of frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3} \times \pi \times 10 \left[\left(\frac{10\sqrt{3}}{3} \right)^2 + \left(\frac{20\sqrt{3}}{3} \right)^2 + \frac{\left(10\sqrt{3}\right)\left(20\sqrt{3}\right)}{3 \times 3} \right]$$

$$= \frac{10}{3} \pi \left(\frac{100}{3} + \frac{400}{3} + \frac{200}{3} \right)$$

$$= \frac{10}{3} \times \frac{22}{7} \times \frac{700}{3} = \frac{22000}{3} \text{cm}^3$$

Radius (r) of wire $= \frac{1}{16} \times \frac{1}{2} = \frac{1}{32} \text{cm}$

Let the length of wire be l.

Volume of wire = Area of cross-section × Length

$= (\pi r^2)\ (l)$

$$= \pi \left(\frac{1}{32} \right)^2 \times l$$

Volume of frustum = Volume of wire

$$= \frac{22000}{9} = \frac{22}{7} \times \left(\frac{1}{32} \right)^2 \times l$$

$$= \frac{7000}{9} \times 1024 = l$$

$l = 796444.44 \text{cm}$

$= 7964.44 \text{ metres.}$