SpeedLabs

Physics

CBSE 11th

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Q.1 Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

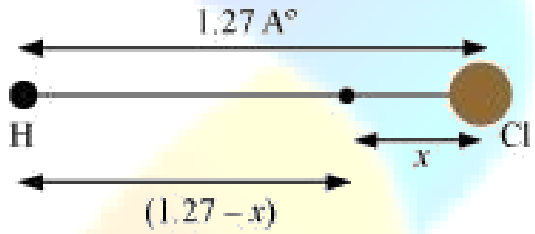
Ans Geometric centre; No

The centre of mass (C.M.) is a point where the mass of a body is supposed to be concentrated. For the given geometric shapes having a uniform mass density, the C.M. lies at their respective geometric centres.

The centre of mass of a body need not necessarily lie within it. For example, the C.M. of bodies such as a ring, a hollow sphere, etc., lies outside the body.

Q.2 In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å $(1\text{Å} = 10^{-10}\text{m})$ Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

Ans The given situation can be shown as:



Distance between H and Cl atoms = 1.27Å

Mass of H atom = m

Mass of Cl atom = 35.5m

Let the centre of mass of the system lie at a distance x from the Cl atom.

Distance of the centre of mass from the H atom = (1.27 – x)

Let us assume that the centre of mass of the given molecule lies at the origin. Therefore, we can have:

$$\frac{m(1.27-x)+35.5mx}{m+35.5m}=0$$

$$m(1.27-x)+35.5mx=0$$

$$1.27-x=-35.5x$$

$$\therefore x=\frac{-1.24}{(35.5-1)}=-0.037\ \text{Å}$$

$$x=-\frac{1.27}{35.5-1}$$

$= -0.037$ Å

Here, the negative sign indicates that the centre of mass lies at the left of the molecule. Hence, the centre of mass of the HCl molecule lies0.037Åfrom the Cl atom.

Q.3 A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

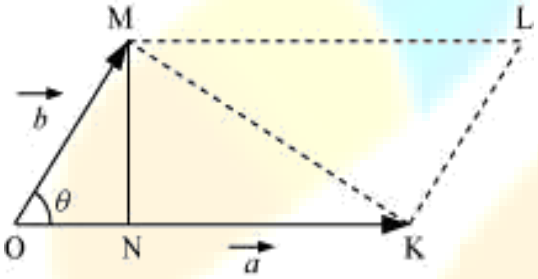
Ans No change

The child is running arbitrarily on a trolley moving with velocity v. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the child's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since no external force is involved in the (child+trolley) system, the child's motion will produce no change in the speed of the centre of mass of the trolley.

Q.4 Show that the area of the triangle contained between the vectors a and b is one half of the magnitude of $a \times b$.

Ans Consider two vectors $\overrightarrow{OK} = |\vec{a}|$ and $\overrightarrow{OM} = \vec{b}$, inclined at an angle θ, as shown in the following figure:

In ΔOMN, we can write the relation:



$$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\vec{b}|}$$

$$\therefore MN = |\vec{b}|\sin\theta$$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

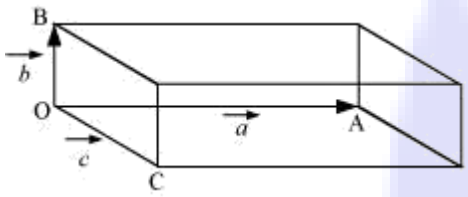
$$|\vec{a}\times\vec{b}| = OK.MN \times \frac{2}{2}$$

$$|\vec{a}\times\vec{b}| = 2 \times \text{Area of } \Delta OMK$$

$$\therefore \text{Area of } \Delta OMK = \frac{1}{2}|\vec{a}\times\vec{b}|$$

Q.5 Show that $a.(b \times c)$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors, a, b and c.

Ans A parallelepiped with origin O and sides a, b, and c is shown in the following figure.



Volume of the given parallelepiped = abc

$\overrightarrow{OC} = \vec{a}$

$\overrightarrow{OB} = \vec{b}$

$\overrightarrow{OC} = \vec{c}$

Let n be a unit vector perpendicular to both b and c. Hence, n and a have the same direction.

$\therefore \vec{b} \times \vec{c} = bc\sin\theta n$

$= bc\sin 90°n$

$= bcn$

$\vec{a}.(\vec{b} \times \vec{c}) = a.(bcn)$

$= abc\cos\theta\hat{n}$

$= abc\cos 0°$

$= abc$

= Volume of the parallelepiped

Q.6 Find the components along the x, y, z axes of the angular momentum 1 of a particle, whose position vector is r with components x, y, z and momentum is p with components , p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

Ans $l_x = yp_z - zp_y$

$l_y = zp_x - xp_z$

$l_z = xp_y - yp_x$

Linear momentum of the particle, $\vec{p} = p_x\hat{i} + p_y\, j + p_2 k$

Position vector of the particle, $\vec{r} = x\hat{i} + yj + zk$

Angular momentum, $\vec{l} = \vec{r} \times \vec{p}$

$$= \left(x\hat{i} + yj + zk\right)\left(p_x\hat{i} + p_y\, j + p_2 k\right)$$

$$= \begin{vmatrix} \hat{i} & j & k \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x\hat{i} + l_y j + l_2 k = \hat{i}\left(yp_z - zp_y\right) - j\left(xp_z - zp_x\right) + \hat{k}\left(xp_y - zp_x\right)$$

Comparing the coefficients of i j and k, we get:

$$\left.\begin{aligned} l_x &= yp_z - zp_y \\ l_y &= xp_z - zp_x \\ l_z &= xp_y - yp_x \end{aligned}\right\} \text{.....(i)}$$

The particle moves in the x-y plane. Hence, the z-component of the position vector and linear momentum vector becomes zero, i.e.,

$$z = p_z = 0$$

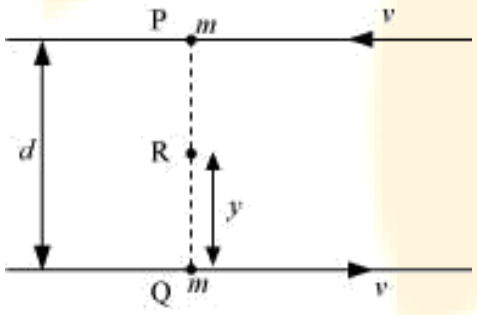
Thus, equation (i) reduces to:

$$\left.\begin{aligned} l_x &= 0 \\ l_y &= 0 \\ l_z &= xp_y - yp_x \end{aligned}\right\}.$$

Therefore, when the particle is confined to move in the x-y plane, the direction of angular momentum is along the z-direction.

Q.7 Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two-particle system is the same whatever be the point about which the angular momentum is taken.

Ans Let at a certain instant two p articles be at points P and Q, as shown in the following figure.



Angular momentum of the system about point P:

$$\overrightarrow{L_p} = mv \times 0 + mv \times d$$

$= mvd$ .....(i)

Angular momentum of the system about point Q:

$\overrightarrow{L_Q} = mv \times d + mv \times 0$

$= mvd$ .....(ii)

Consider a point R, which is at a distance y from point Q, i.e.,

$QR = y$

$\therefore PR = d - y$

Angular momentum of the system about point R:

$\overrightarrow{L_R} = mv \times (d - y) + mv \times y$

$= mvd - mvy + mvy$

$= mvd$ .....(iii)

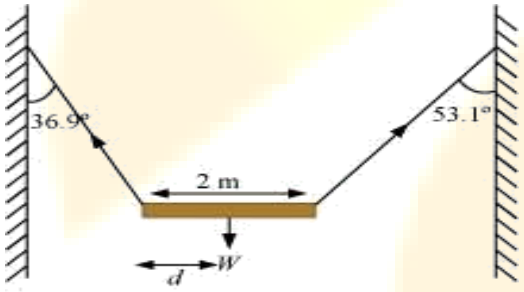
Comparing equations (i), (ii), and (iii), we get:

$\overrightarrow{L_p} = \overrightarrow{L_Q} = \overrightarrow{L_R}$ .....(iv)

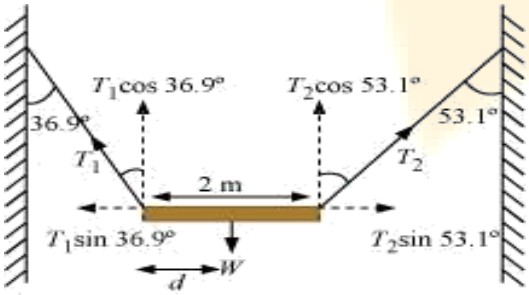
We infer from equation (iv)

that the angular momentum of a system does not depend on the point about which it is taken

Q.8 A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



Ans The free body diagram of the bar is shown in the following figure:



Length of the bar, l = 2 m

T_1 and T_2 are the tensions produced in the left and right strings respectively.

At translational equilibrium, we have:

$$T_1 \sin 36.9° = T_2 \sin 53.1$$

$$\frac{T_1}{T_2} = \frac{\sin 53.1°}{\sin 36.9}$$

$$\frac{T_1}{T_2} = \frac{0.800}{0.600} = \frac{4}{3}$$

$$\Rightarrow T_1 = \frac{4}{3} T_2$$

For rotational equilibrium, on taking the torque about the centre of gravity, we have:

$$T_1 \cos 36.9 \times d = T_2 \cos 53.1 (2 - d)$$

$$T_1 \times 0.800d = T_2 0.600(2 - d)$$

$$\frac{4}{3} \times T_2 \times 0.800d = T_2[0.600 \times 2 - 0.600d]$$

$$1.067d + 0.6d = 1.2$$

$$\therefore d = \frac{1.2}{1.67}$$

$$= 0.72m$$

Hence, the C.G. (centre of gravity) of the given bar lies 0.72 m from its left end.

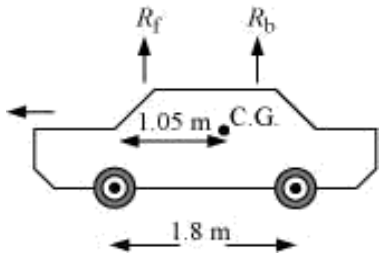
Q.9 A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Ans Mass of the car, m = 1800 kg

Distance between the front and back axles, d = 1.8 m

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure**.**



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$$R_f + R_b = mg$$

$$= 1800 \times 9.8$$

$$= 17640\text{N} \quad ....(i)$$

For rotational equilibrium, on taking the torque about the C.G., we have:

$$R_f(1.05) = R_b(1.8 - 1.05)$$

$$R_f \times (1.05) = R_b \times 0.75$$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$$R_b = 1.4R_f \quad .....(ii)$$

Solving equations (i) and (ii), we get:

$$1.4R_f + R_f = 17640$$

$$R_f = \frac{17640}{2.4} = 7350\text{N}$$

$$\therefore R_b = 17640 - 7350 = 10290\text{N}$$

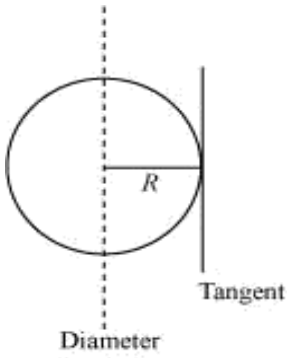
Therefore, the force exerted on each front wheel $= \frac{7350}{2} = 3675\text{N}$ and The force exerted on each back wheel $= \frac{10290}{2} = 514\text{N}$

Q.10 (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

Ans (a) $\frac{7}{5}MR^2$

The moment of inertia (M.I.) of a sphere about its diameter $= \frac{2}{5}MR^2$



According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere

$= \frac{2}{5}\mathrm{MR}^2 + \mathrm{MR}^2 = \frac{7}{5}\mathrm{MR}^2$

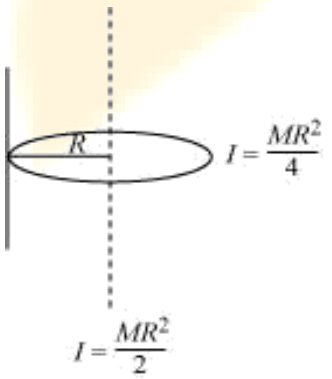
(b) $\frac{3}{2}\mathrm{MR}^2$

The moment of inertia of a disc about its diameter $= \frac{1}{4}\mathrm{MR}^2$

According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about its centre$= \frac{1}{4}\mathrm{MR}^2 + \frac{1}{4}\mathrm{MR}^2 = \frac{1}{2}\mathrm{MR}^2$

The situation is shown in the given figure.



Applying the theorem of parallel axes:

The moment of inertia about an axis normal to the disc and passing through a point on its edge

$= \frac{1}{2}\mathrm{MR}^2 + \mathrm{MR}^2 = \frac{3}{2}\mathrm{MR}^2$

Q.11 Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

Ans Let m and r be the respective mass and radius of the hollow cylinder and the solid sphere.

The moment of inertia of the hollow cylinder about its standard axis, $I_I = mr^2$

The moment of inertia of the solid sphere about an axis passing through its centre, $I_{II} = \frac{2}{5}mr^2$

We have the relation: $\tau = I\alpha$

Where,

$\propto$ = Angular acceleration

τ= Torque

I = Moment of inertia

For the hollow cylinder, $\tau_1 = I_1\alpha_1$

For the solid sphere, $\tau_{II} = I_{II}\alpha_{II}$

As an equal torque is applied to both the bodies, $\tau_1 = \tau_2$

$$\therefore \frac{\alpha_{II}}{\alpha_I} = \frac{I_I}{I_{II}} = \frac{mr^2}{\frac{2}{5}mr^2} = \frac{2}{5}$$

$\alpha_{II} > \alpha_I$ .....(i)

Now, using the relation:

$\omega = \omega_0 + \alpha t$

Where,

ω_0 = Initial angular velocity

t = Time of rotation

ω= Final angular velocity

For equal

and t, we have:

$\omega \propto \alpha$ .....(ii)

From equations (i) and (ii), we can write:

$\omega_{II} > \omega_I$

Hence, the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

Q.12 A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Ans Mass of the cylinder, m = 20 kg

Angular speed, ω = 100 rad s^{-1}

Radius of the cylinder, r = 0.25 m

The moment of inertia of the solid cylinder:

$$I = \frac{mr^2}{2}$$

$$= \frac{1}{2} \times 20 \times (0.25)^2$$

$$= 0.625 \text{ kgm}^2$$

$$\therefore \text{Kinetic energy} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} \times 6.25 \times (100)^2$$

$$= 3125 \text{J}$$

$$\therefore \text{Angular momentum, } L = I\omega$$

$$= 6.25 \times 100$$

$$= 62.5 \text{Js}$$

Q.13 (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

Ans (a) Initial angular velocity ω_1 = 40 rev/min

Final angular velocity = ω_2

The moment of inertia of the child with stretched hands = I_1

The moment of inertia of the child with folded hands = I_2

The two moments of inertia are related as:

$$I_2 = \frac{2}{5} I_1$$

Since no external force acts on the child, the angular momentum L is constant.

Hence, for the two situations, we can write:

$$I_2\omega_2 = I_1\omega_1$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

$$= \frac{I_1}{\frac{2}{5}I_1} \times 40 = \frac{5}{2} \times 40$$

$$= 100\,\text{rev/min}$$

(b) Initial Kinetic energy of rotation, $E_1 = \frac{1}{2}I_1{\omega_1}^2$

Final kinetic energy of rotation, $E_F = \frac{1}{2}I_2{\omega_2}^2$

$$\therefore \frac{E_F}{E_I} = \frac{\frac{1}{2}I_2{\omega_2}^2}{\frac{1}{2}I_1{\omega_1}^2}$$

$$\frac{E_F}{E_I} = \frac{\frac{1}{2} \times \frac{2}{5}I_1{\omega_2}^2}{\frac{1}{2}I_1{\omega_1}^2}$$

$$\frac{E_F}{E_I} = \frac{2}{5} \times \frac{{\omega_2}^2}{{\omega_1}^2}$$

$$\frac{E_F}{E_I} = \frac{2}{5} \times \frac{{\omega_2}^2}{{\omega_1}^2}$$

$$\frac{E_F}{E_I} = \frac{2}{5} \times \frac{(100)^2}{(40)^2}$$

$$\frac{E_F}{E_I} = 2.5$$

$$\therefore E_F = 2.5 \times E_1$$

The increase in the rotational kinetic energy is attributed to the internal energy of the boy.

Q.14 A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

Ans Mass of the hollow cylinder, m = 3 kg

Radius of the hollow cylinder, r = 40 cm = 0.4m

Applied force, F = 30N

The moment of inertia of the hollow cylinder about its geometric axis:

$I = mr^2$

$= 3 \times (0.4)^2$

$= 0.48\,kg\,m^2$

Torque,

$\tau = F \times r$

$\tau = 30 \times 0.4$

$= 12Nm$

For angular acceleration α, torque is also given by the relation:

$\tau = I\alpha$

$\therefore \alpha = \frac{\tau}{I} = \frac{12}{0.48}$

$= 25\,rads^{-2}$

Linear acceleration $= ra = 0.4 \times 25 = 10ms^{-2}$

Q.15 To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?

(Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100 % efficient.

Ans Angular speed of the rotor,ω= 200 rad/s

Torque required,= 180 Nm

The power of the rotor (P) is related to torque and angular speed by the relation:

$P = \tau\omega$

$P = 180 \times 200$

$P = 36 \times 10^3$

P= 36 kW

Hence, the power required by the engine is 36 kW.

Q.16 From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

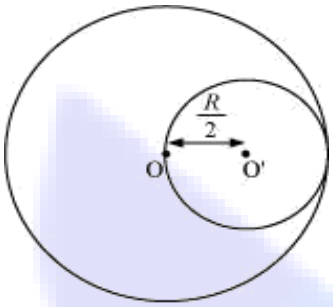
Ans From the original centre of the body and opposite to the centre of the cut portion.

Mass per unit area of the original disc = σ

Radius of the original disc = R

Mass of the original disc, M = $\pi R^2 \sigma$

The disc with the cut portion is shown in the following figure:



Radius of the smaller disc $= \frac{R}{2}$

Mass of the smaller disc, $M' = \pi\left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4}\pi R^2 \sigma = \frac{M}{4}$

Let O and O' b be the respective centers of the original disc and the disc cut off from the original. As per definition of the center of the center of mass, the center of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O'.

It is given that:

$OO' = \frac{R}{2}$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and $-M' = \left(\frac{M}{4}\right)$ concentrated at O'

(The negative sign indicates that this portion has been removed from the original disc.)

Let x be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centres of masses of two masses is given as:

$x = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$

For the given system, we can write:

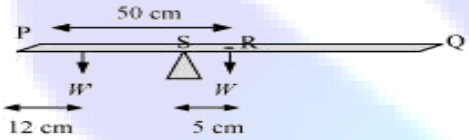
$$x = \frac{M \times 0 - M' \times \left(\frac{R}{2}\right)}{M + (-M')}$$

$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

(The negative sign indicates that the centre of mass gets shifted toward the left of point O.)

Q.17 A meter stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the meter stick?

Ans Let W and W′ be the respective weights of the meter stick and the coin.



The mass of the meter stick is concentrated at its mid-point, i.e., at the 50 cm mark.

Mass of the meter stick = m'

Mass of each coin, m = 5 g

When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from point R toward the end P. The centre of mass is located at a distance of 45 cm from point P.

The net torque will be conserved for rotational equilibrium about point R.

$$10 \times g(45 - 12) - m'g(50 - 45) = 0$$

$$\therefore m' = \frac{10 \times 33}{5} = 66g$$

Hence, the mass of the meter stick is 66 g.

Q.18 A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination.

(a) Will it reach the bottom with the same speed in each case?

(b) Will it take longer to roll down one plane than the other?

(c) If so, which one and why?

Ans (a) Yes (b) Yes (c) On the smaller inclination

(a)Mass of the sphere = m

Height of the plane = h

Velocity of the sphere at the bottom of the plane = v

At the top of the plane, the total energy of the sphere = Potential energy = mgh

At the bottom of the plane, the sphere has both translational and rotational kinetic energies.

Hence, total energy = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Using the law of conservation of energy, we can write:

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \quad ....(i)$$

For a solid sphere, the moment of inertia about its centre, $I = \frac{2}{5}mr^2$

Hence, equation (i) becomes:

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = mgh$$

$$\frac{1}{2}v^2 + \frac{1}{5}r^2\omega^2 = gh$$

But we have the relation, $v = r\omega$

$$\therefore \frac{1}{2}v^2 + \frac{1}{5}v^2 = gh$$

$$v^2\left(\frac{7}{10}\right) = gh$$

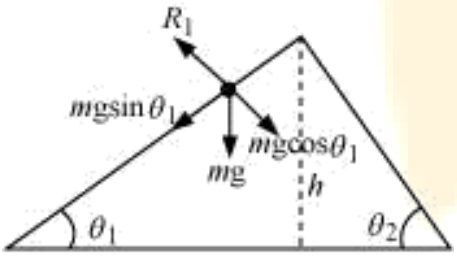
$$v = \sqrt{\frac{10}{7}gh}$$

Hence, the velocity of the sphere at the bottom depends only on height (h) and acceleration due to gravity (g). Both these values are constants. Therefore, the velocity at the bottom remains the same from whichever inclined plane the sphere is rolled.

(b), (c) Consider two inclined planes with inclinations θ_1 and θ_2, related as:

The acceleration produced in the sphere when it rolls down the plane inclined at θ_1 is: $g\sin\theta_1$

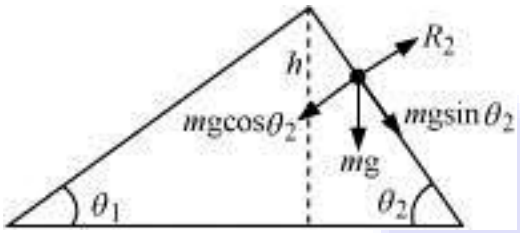
The various forces acting on the sphere are shown in the following figure.



R_1 is the normal reaction to the sphere.

Similarly, the acceleration produced in the sphere when it rolls down the plane inclined at θ_2 is: $g\sin\theta_2$

The various forces acting on the sphere are shown in the following figure.



R_2 is the normal reaction to the sphere.

$\theta_1 > \theta_2; \sin\theta_2 > \sin\theta_1$ .....(i)

$\therefore a_2 > a_1$ .....(ii)

Initial velocity, u= 0

Final velocity, v = Constant

Using the first equation of motion, we can obtain the time of roll as:

$v = u + at$

$\therefore t \propto \frac{1}{a}$

$$\left.\begin{array}{l} \text{For inclination } \theta_1 : t_1 \propto \frac{1}{a_1} \\ \text{For inclination } \theta_1 : t_2 \propto \frac{1}{a_2} \end{array}\right] \quad \text{.....(iii)}$$

From equations (ii) and (iii), we get: $t_2 < t_1$

Hence, the sphere will take a longer time to reach the bottom of the inclined plane having the smaller inclination.

Q.19 A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

Ans Radius of the hoop, r = 2 m

Mass of the hoop, m = 100 kg

Velocity of the hoop, $v = 20\,cm/s = 0.2\,m/s$

Total energy of the hoop = Translational KE + Rotational KE

$$E_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Moment of inertia of the hoop about its centre, $I = mr^2$

$$E_r = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$$

But we have the relation $v = r\omega$

$$\therefore E_1 = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

The work required to be done for stopping the hoop is equal to the total energy of the hoop.

∴Required work to be done,

$$W = mv^2 = 100 \times (0.2)^2 = 4J$$

Q.20 The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

Ans Mass of an oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Moment of inertia, $I = 1.94 \times 10^{-46}$ kg m^2

Velocity of the oxygen molecule, v = 500 m/s

The separation between the two atoms of the oxygen molecule = 2r

Mass of each oxygen atom = $\frac{m}{2}$

Hence, moment of inertia I, is calculated as:

$$\left(\frac{m}{2}\right)r^2 + \left(\frac{m}{2}\right)r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$\sqrt{\frac{1.94 \times 10^{-46}}{5.36 \times 10^{-26}}}$$

$$= 0.60 \times 10^{-10} m$$

It is given that:

$$KE_{rot} = \frac{2}{3}KE_{trans}$$

$$\frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$$

$$mr^2\omega^2 = \frac{2}{3}mv^2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{v}{r}$$

$$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6 \times 10^{-10}}$$

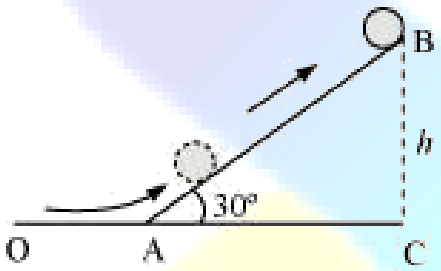
$$= 6.80 \times 10^{12}\,\text{rad/s}$$

Q.21 A solid cylinder rolls up an inclined plane of angle of inclination 30°.At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.

How far will the cylinder go up the plane?

How long will it take to return to the bottom?

Ans A solid cylinder rolling up an inclination is shown in the following figure.



Initial velocity of the solid cylinder, v = 5 m/s

Angle of inclination $\theta = 30°$

Height reached by the cylinder = h

Energy of the cylinder at point A:

$KE_{rot} = KE_{trans}$

$$\frac{1}{2}I\omega^{2=}\frac{1}{2}mv^2$$

Energy of the cylinder at point

$B = mgh$

Using the law of conservation of energy, we can write:

$$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 = mgh$$

Moment of inertia of the solid cylinder, $I = \frac{1}{2}mr^2$

$$\therefore \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{4}mr^2\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{4}r^2\omega^2 + \frac{1}{2}v^2gh$$

But we have the relation, $v = r\omega$

$$\therefore \frac{1}{4}v^2 + \frac{1}{2}v^2 = gh$$

$$\frac{3}{4}v^2 = gh$$

$$\therefore h = \frac{3}{4}\frac{v^2}{g}$$

$$= \frac{3}{4} \times \frac{5 \times 5}{9.8} = 1.9lm$$

In ΔABC:

$$\sin\theta = \frac{BC}{AB}$$

$$\sin 30° = \frac{h}{AB}$$

$$AB = \frac{1.91}{0.5} = 3.82m$$

Hence, the cylinder will travel 3.82 m up the inclined plane.

For radius of gyration K, the velocity of the cylinder at the instance when it rolls back to the bottom is given by the relation:

$$v = \left(\frac{2gh}{1 + \frac{k^2}{R^2}}\right)^{\frac{1}{2}}$$

$$\therefore v = \left(\frac{2gAB\sin\theta}{1 + \frac{k^2}{R^2}}\right)^{\frac{1}{2}}$$

For the solid cylinder $k^2 = \frac{R^2}{2}$

$$v = \left(\frac{2gAB\sin\theta}{1+\frac{1}{2}} \right)^{\frac{1}{2}}$$

$$v = \left(\frac{4}{3} gAB\sin\theta \right)^{\frac{1}{2}}$$

The time taken to return to the bottom is: $t = \frac{AB}{v}$

$$= \frac{AB}{\left(\frac{4}{3} gAB\sin\theta \right)^{\frac{1}{2}}} = \left(\frac{3AB}{4g\sin\theta} \right)^{\frac{1}{2}}$$

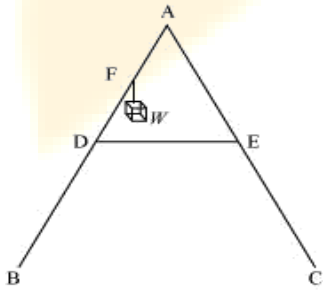
$$= \left(\frac{11.46}{19.6} \right)^{\frac{1}{2}}$$

$$= 0.764 \text{s}$$

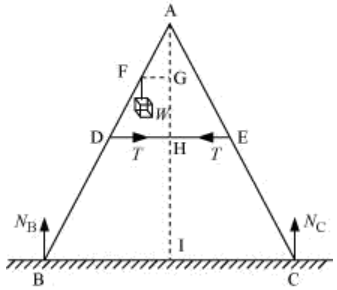
Therefore, the total time taken by the cylinder to return to the bottom is $(2 \times 0.764) = 1.53 \text{s}$.

Q.22 As shown in figure, the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. $(\text{Take } g = 9.8 \text{ m/s}^2)$

(Hint: Consider the equilibrium of each side of the ladder separately.)



Ans The given situation can be shown as:



N_B = Force exerted on the ladder by the floor point B

N_C = Force exerted on the ladder by the floor point C

T = Tension in the rope

BA = CA = 1.6 m

DE = 0. 5 m

BF = 1.2 m

Mass of the weight, m = 40 kg

Draw a perpendicular from A on the floor BC. This intersects DE at mid-point H.

ΔABI and ΔAIC are similar

∴BI = IC

Hence, I is the mid-point of BC.

DE || BC

BC = 2 × DE

= 1m

$AF = BA - BF$

$= 0.4\text{m}$ ..... (i)

D is the mid-point of AB.

Hence, we can write:

$$AD = \frac{1}{2} \times BA = 0.8\text{m} \quad ....(ii)$$

Using equations (i) and (ii), we get:

FE = 0.4 m

Hence, F is the mid-point of AD.

FG||DH and F is the mid-point of AD. Hence, G will also be the mid-point of AH.

ΔAFG and ΔADH are similar

$$\therefore \frac{FG}{DH} = \frac{AF}{AD}$$

$$\frac{FG}{DH} = \frac{0.4}{0.8} = \frac{1}{2}$$

$$FG = \frac{1}{2} DH$$

$$= \frac{1}{2} \times 0.25 = 0.125\text{m}$$

In ΔADH:

$$AH = \sqrt{\left(AD^2 - DH^2\right)}$$

$= \sqrt{(0.8)^2 - (0.25)^2}$

$= 0.76\text{m}$

For translational equilibrium of the ladder, the upward force should be equal to the downward force.

$N_C + N_B = mg = 392$ .....(iii)

For rotational equilibrium of the ladder, the net moment about A is:

$-N_B \times BI + mg \times FG + N_C \times CI + T \times AG - T \times AG = 0$

$-N_B \times 0.5 + 40 \times 9.8 \times 0.125 + N_c \times (0.5) = 0$

$(N_C - N_B) \times 0.5 = 49$

$N_C - N_B = 98$ .....(iv)

Solving equations (iii) and (iv), we get:

$N_C = 245\text{N}$

$N_B = 147\text{N}$

For rotational equilibrium of the side AB, consider the moment about A.

$-N_B \times BI + mg \times FG + T \times AG = 0$

$-245 \times 0.5 + 40 + 9.8 \times 0.125 + T \times 0.76 = 0$

$0.76T = 122.5 - 49$

$\therefore T = 96.7\text{N}$

Q.23 A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.

(a) What is his new angular speed? (Neglect friction.)

(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

Ans (a)Moment of inertia of the man-platform system = 7.6 kg m^2.

Moment of inertia when the man stretches his hands to a distance of 90 cm:

$2 \times m r^2$

$= 2 \times 5 \times (0.9)^2$

$= 8.1\text{kg m}^2$

Initial moment of inertia of the system,

$I_i = 7.6 + 8.1 = 15.7\text{kg m}^2$

Angular speed, $\omega_i = 300\,\text{rev/min}$

Angular momentum, $L_i = I_i\omega_i$

$= 15.7 \times 30$ ....(i)

Moment of inertia when the man folds his hands to a distance of 20 cm:

$2 \times mr^2$

$= 2 \times 5(0.2)^2$

$= 0.4\,\text{kg m}^2$

Final moment of inertia, $I_f = 7.6 + 0.4 = 8\,\text{kg m}^2$

Final angular speed = $= \omega_f$

Final angular momentum, $L_f = I_f\omega_f = 0.79\omega$ ....(ii)

From the conservation of angular momentum, we have:

$I_i\omega_i = I_f\omega_f$

$\therefore \omega_f = \dfrac{15.7 \times 30}{8}$

$= 58.88\,\text{rev/min}$

(b) Kinetic energy is not conserved in the given process. In fact, with the decrease in the moment of inertia, kinetic energy increases. The additional kinetic energy comes from the work done by the man to fold his hands toward himself.

Q.24 A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)

Ans Mass of the bullet, $m = 10\text{g} = 10 \times 10^{-3}\text{kg}$

Velocity of the bullet, v = 500 m/s

Width of the door, L = 1 m

Radius of the door, $r = \dfrac{1}{2}\text{m}$

Mass of the door, M = 12 kg

Angular momentum imparted by the bullet on the door:

$\alpha = mvr$

$= \left(10 \times 10^{-3}\right) \times \left(500\right) \times \frac{1}{2} = 2.5 \text{kg m}^2\text{s}^{-1}$ ....(i)

Moment of inertia of the door:

$I = \frac{1}{3} ML^2$

$= \frac{1}{3} \times 12 \times (1)^2 = 4 \text{kgm}^2$

But $\alpha = I\omega$

$\therefore \omega = \frac{\alpha}{I} = \frac{2.5}{4} = 0.625 \text{rads}^{-1}$

Q.25 Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

(a) What is the angular speed of the two-disc system?

(b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy?

Take $\omega_1 \neq \omega_2$

Ans (a) Moment of inertia of disc $I = I_1$

Angular speed of disc $I = \omega_1$

Moment of inertia of disc $II = I_2$

Angular speed of disc $II = \omega_2$

Angular momentum of disc $I, L_1 = I_1\omega_1$

Angular momentum of disc $II, L_2 = I_2\omega_2$

Total initial angular momentum $L_1 = I_1\omega_1 + I_2\omega_1$, $L_1 = I_1\omega_1 + I_2\omega_2$

When the two discs are joined together, their moments of inertia get added up. Moment of inertia of the system of two discs, $I = I_1 + I_2$

Let ω be the angular speed of the system.

Total final angular momentum, $L_f = \left(I_1 + I_2\right)\omega$

Using the law of conservation of angular momentum, we have:

$L_i = L_f$

$$I_1\omega_1 + I_2\omega_2 = (I_2 + I_2)\omega$$

$$\therefore \omega = \frac{I_1\omega_1 + I_2\omega}{I_1 + I_2}$$

(b) Kinetic energy of disc I, $E_1 = \frac{1}{2} I_1 {\omega_1}^2$

Kinetic energy of disc II, $E_2 = \frac{1}{2} I_1 {\omega_1}^2$ $E_2 = \frac{1}{2} I_2 {\omega_2}^2$

Total initial kinetic energy, $E_i = \frac{1}{2}\left(I_1 {\omega_1}^2 + I_2 {\omega_2}^2\right)$

When the discs are joined, their moments of inertia get added up.

Moment of inertia of the system, $I = I_1 + I_2$

Angular speed of the system = ω

Final kinetic energy: E_f

$$= \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$$

$$\therefore E_i = E_f$$

$$= \frac{1}{2}\left(I_1 {\omega_1}^2 + I_2 {\omega_2}^2\right) - \frac{\left(I_1\omega_1 + I_2\omega_2\right)^2}{2(I_1 + I_2)}$$

$$= \frac{1}{2} I_1 {\omega_1}^2 + \frac{1}{2} I_2 {\omega_2}^2 - \frac{1}{2}\frac{{I_1}^2 {\omega_1}^2}{(I_1 + I_2)} - \frac{1}{2}\frac{{I_2}^2 {\omega_2}^2}{(I_1 + I_2)} - \frac{1}{2}\frac{2I_1 I_2 \omega_1 \omega_2}{(I_1 + I_2)}$$

$$= \frac{1}{2}\left[{I_1}^2 {\omega_1}^2 + \frac{1}{2} I_1 I_2 {\omega_1}^2 + \frac{1}{2} I_1 I_2 {\omega_2}^2 + \frac{1}{2} {I_2}^2 \omega^2 - \frac{1}{2} {I_1}^2 {\omega_1}^2 - \frac{1}{2} {I_2}^2 {\omega_2}^2 - I_1 I_2 \omega_1 \omega_2 \right]$$

$$= \frac{I_1 I_2}{2(I_1 + I_2)}\left[{\omega_1}^2 + {\omega_2}^2 - 2\omega_1\omega_2 \right]$$

$$= \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

All the quantities on RHS are positive

$$\therefore E_i - E_f > 0$$

$E_i > E_f$

The loss of KE can be attributed to the frictional force that comes into play when the two discs come in contact with each other.

Q.26 Prove the theorem of perpendicular axes.

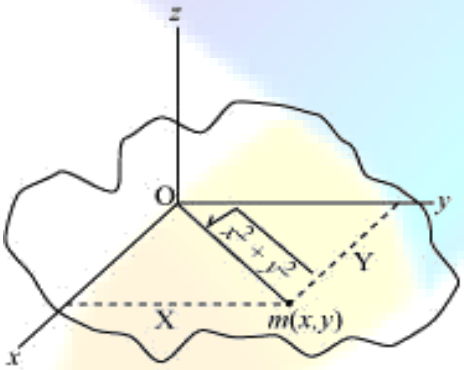
(Hint: Square of the distance of a point (x, y) in the x–y plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$)

Prove the theorem of parallel axes.

(Hint: If the centre of mass is chosen to be the origin $\sum m_i r_i = 0$)

Ans (a)The theorem of perpendicular axes states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

A physical body with centre O and a point mass m, in the x–y plane at (x, y) is shown in the following figure.



Moment of inertia about x-axis, $I_x = mx^2$

Moment of inertia about y-axis, $I_y = my^2$

Moment of inertia about z-axis, $I_z = m\left(\sqrt{x^2 + y^2}\right)^2$

$I_x + I_y = mx^2 + my^2$

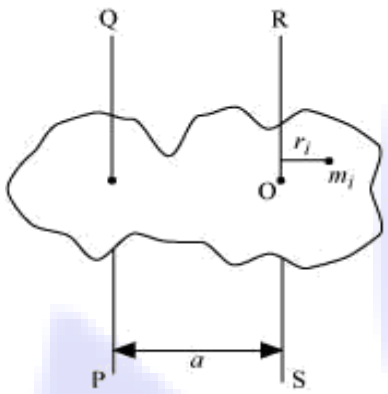
$= m\left(x^2 + y^2\right)$

$= m\left(\sqrt{x^2 + y^2}\right)^2$

$I_x + I_y = I_z$

Hence, the theorem is proved.

(b)The theorem of parallel axes states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.



Suppose a rigid body is made up of n particles, having masses $m_1, m_2, m_3, ..., m_n$, at perpendicular distances $r_1, r_2, r_3, ..., r_n$, respectively from the centre of mass O of the rigid body.

The moment of inertia about axis RS passing through the point O:

$$I_{RS} = \sum_{i=1}^{n} m_i r_i^2$$

The perpendicular distance of mass mi, from the axis $QP = a + r_i$

Hence, the moment of inertia about axis QP:

$$I_{QP} = \sum_{i=1}^{n} m_i \left(a + r_i\right)^2$$

$$= \sum_{i=1}^{n} m_i \left(a^2 + r_i^2 + 2ar_i\right)^2$$

$$= \sum_{i=1}^{n} m_i a^2 + \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i 2ar_i$$

$$= I_{RS} + \sum_{i=1}^{n} m_i a^2 + 2\sum_{i=1}^{n} m_i a r_i^2$$

Now, at the centre of mass, the moment of inertia of all the particles about the axis passing through the centre of mass is zero, that is,

$$2\sum_{i=1}^{n} m_i a r_i = 0$$

$$\because a \neq 0$$

$$\therefore \sum m_i r_i = 0$$

Also,

$\sum_{i=1}^{n} m_i = M; M$ Total mass of the rigid body

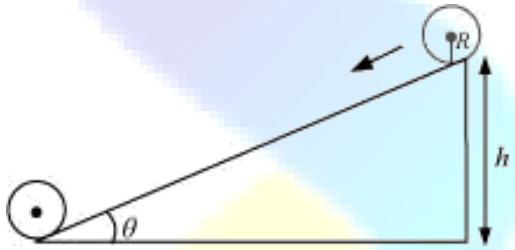
$\therefore I_{QP} = I_{RS} + Ma^2$

Hence, the theorem is proved.

Q.27 Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by $v^2 = \frac{2gh}{\left(1+k^2/R^2\right)}$ using dynamical consideration (i.e. by consideration of forces and torques).

Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Ans A body rolling on an inclined plane of height h, is shown in the following figure:



m = Mass of the body

R = Radius of the body

K = Radius of gyration of the body

v = Translational velocity of the body

h =Height of the inclined plane

g = Acceleration due to gravity

Total energy at the top of the plane, $E_1 = mgh$

Total energy at the bottom of the plane, $E_b = KE_{rot} + KE_{trans}$

$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

But $I = mk^2$ and $\omega = \frac{v}{R}$

$\therefore E_b = \frac{1}{2}\left(mk^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2} mv^2$

$$= \frac{1}{2}mv^2 \frac{k^2}{R^2} + \frac{1}{2}mv^2$$

$$= \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

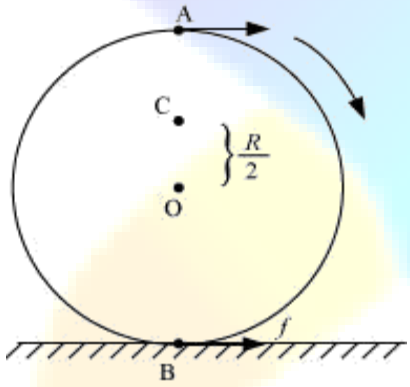
From the law of conservation of energy, we have: $E_T = E_b$

$$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

$$\therefore v = \frac{2gh}{\left(1+k^2/R^2\right)}$$

Hence, the given result is proved.

Q.28 A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in figure? Will the disc roll in the direction indicated?



Ans $v_A = R_0\omega;$

$v_B = R\omega_0;$

$v_C = \left(\frac{R}{2}\right)\omega_0;$

The disc will not roll

Angular speed of the disc = ω_0

Radius of the disc = R

Using the relation for linear velocity, $v = \omega_0 R$

For point A:

$v_A = R\omega_0$; in the direction tangential to the right

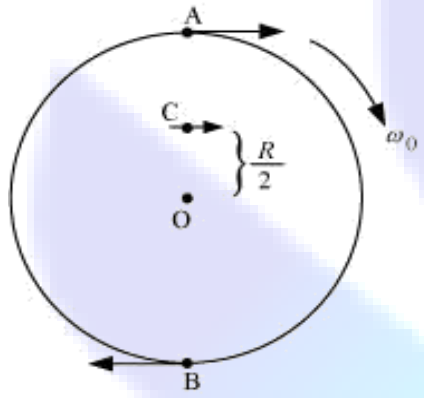
For point B:

$v_B = R\omega_0$; in the direction tangential to the left

For point C:

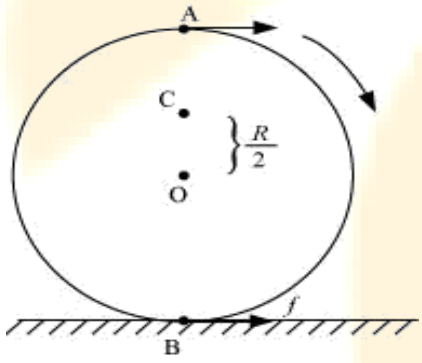
$v_c = \left(\frac{R}{2}\right)\omega_0$; in the direction same as that of

The directions of motion of points A, B, and C on the disc are shown in the following figure



Since the disc is placed on a frictionless table, it will not roll. This is because the presence of friction is essential for the rolling of a body.

Q.29 Explain why friction is necessary to make the disc in figure given roll in the direction indicated. Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins. What is the force of friction after perfect rolling begins?



Ans A torque is required to roll the given disc. As per the definition of torque, the rotating force should be tangential to the disc. Since the frictional force at point B is along the tangential force at point A, a frictional force is required for making the disc roll.

(a) Force of friction acts opposite to the direction of velocity at point B. The direction of linear velocity at point B is tangentially leftward. Hence, frictional force will act tangentially rightward. The sense of frictional torque before the start of perfect rolling is perpendicular to the plane of the disc in the outward direction.
(b) Since frictional force acts opposite to the direction of velocity at point B, perfect rolling will begin when the velocity at that point becomes equal to zero. This will make the frictional force acting on the disc zero.

Q.30 A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad s^{-1}. Which of the two will start to roll earlier? The co-efficient of kinetic friction is $\mu_k = 0.2$.

Ans Radii of the ring and the disc, r = 10 cm = 0.1 m

Initial angular speed, $\omega_0 = 10\pi \,\text{rad}\,\text{s}^{-1}$

Coefficient of kinetic friction, $\mu_k = 0.2$

Initial velocity of both the objects, u = 0

Motion of the two objects is caused by frictional force. As per Newton's second law of motion, we have frictional force, $f = ma$

$\mu_k mg = ma$

Where,

a = Acceleration produced in the objects

m = Mass

$\therefore a = \mu_k g$ .....(i)

As per the first equation of motion, the final velocity of the objects can be obtained as:

$V = u + at$

$= 0 + \mu_k gt$

$v = \mu_k gt$ .....(ii)

The torque applied by the frictional force will act in perpendicularly outward direction and cause reduction in the initial angular speed.

Torque $\tau = -Ia$

$\alpha =$ Angular acceleration

$\mu_x mgr = -I\alpha$

$\therefore \alpha = \dfrac{-\mu_k mgr}{I}$ ....(iii)

Using the first equation of rotational motion to obtain the final angular speed:

$\omega = \omega_0 + \alpha t$

$$= \omega_0 + \frac{-\mu_k gmrt}{I} \quad ....(iv)$$

Rolling starts when linear velocity, $v = r\omega$

$$\therefore v = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right) \quad ....(v)$$

Equating equations (ii) and (v), we get:

$$\mu_k gt = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right)$$

$$= r\omega_0 - \frac{\mu_k gm^2 rt}{I} \quad .....(vi)$$

$$I = mr^2$$

$$\therefore \mu_k gt = r\omega_0 - \frac{\mu_k gm^2 rt}{mr^2}$$

For the ring: $I = mr^2$

$$\therefore \mu_k gt = r\omega_0 - \frac{\mu_k gmr^2 t}{mr^2}$$

$$\mu_k gt = r\omega_0 - \mu_k gt$$

$$2\mu_k gt = r\omega_0$$

$$\therefore t_r = \frac{r\omega_0}{2\mu_k g}$$

$$= \frac{0.1 \times 10 \times 3.14}{2 \times 0.2 \times 9.8}$$

$$= 0.80s \quad ....(vii)$$

$$I = \frac{1}{2}mr^2$$

$$\therefore \mu_k gt_d = r\omega_0 - \frac{\mu_k gmr^2 t_d}{\frac{1}{2}mr^2}$$

$$\mu_k gt_d = r\omega_0 - \frac{\mu_k gt_d}{\frac{1}{2}}$$

$$\mu_k gt_d = r\omega_0 - 2\mu_k gt_d$$

$$3\mu_k gt_d = r\omega_0$$

$$\therefore t_d = \frac{r\omega_0}{3\mu_k g}$$

$$= \frac{0.1 \times 10 \times 3.14}{3 \times 0.2 \times 9.8}$$

$$= 0.53 s \quad .....(viii)$$

Since $t_d > t_{r,}$ the disc will start rolling before the ring.

Q.31 A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction μ_s 0.25

(a) How much is the force of friction acting on the cylinder?

(b) What is the work done against friction during rolling?

(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?

Ans Mass of the cylinder, m = 10 kg

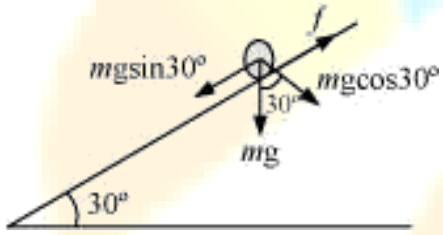
Radius of the cylinder, r = 1 cm = 0.15 m

Co-efficient of kinetic friction μ_s = 0.25

Angle of inclination $\theta = 30°$

Moment of inertia of a solid cylinder about its geometric axis, $I = \frac{1}{2}mr^2$

The various forces acting on the cylinder are shown in the following figure:



The acceleration of the cylinder is given as:

$$\alpha = \frac{mg\sin\theta}{m + \frac{1}{r^2}}$$

$$= \frac{mg\sin\theta}{m + \frac{1}{2}\frac{mr^2}{r^2}}$$

$$= \frac{2}{3}g\sin 30°$$

$= \frac{2}{3} \times 9.8 \times 0.5$

$= 3.27\,\mathrm{m/s^2}$

(a) Using Newton's second law of motion, we can write net force as:

$f_{net} = ma$

$mg\sin 30° - f = ma$

$f = mg\sin 30° - ma$

$= 10 \times 9.8 \times 0.5 - 10 \times 3.27$

$= 49 - 32.7$

$= 16.3\,\mathrm{N}$

(b) During rolling, the instantaneous point of contact with the plane comes to rest. Hence, the work done against frictional force is zero.

(c) For rolling without skid, we have the relation:

$\mu = \frac{1}{3}\tan\theta$

$\tan\theta = 3\mu$

$= 3 \times 0.25$

$= 0.75$

$\theta = \tan^{-1} 0.75$

$= 36.87°$

Q.32 Read each statement below carefully, and state, with reasons, if it is true or false;

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly frictionless inclined plane will undergo sliping (not rolling) motion.

Ans **(**a) **False**

Frictional force acts opposite to the direction of motion of the centre of mass of a body. In the case of rolling, the direct ion of motion of the centre of mass is backward. Hence, frictional force acts in the forward direction.

(b) **True**

Rolling can be considered as the rotation of a body about an axis passing through the point of contact of the body with the ground. Hence, its instantaneous speed is zero.

(c) **False**

When a body is rolling, its instantaneous acceleration is not equal to zero. It has some value.

(d) **True**

When perfect rolling begins, the frictional force acting at the lowermost point becomes zero. Hence, the work done against friction is also zero.

(e) **True**

The rolling of a body occurs when a frictional force acts between the body and the surface. This frictional force provides the torque necessary for rolling. In the absence of a frictional force, the body slips from the inclined plane under the effect of its own weight.

Q.33 Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass:

(a) Show $\mathrm{p_i} = \mathrm{p'_i} + \mathrm{m_i V}$

Where $\mathrm{p_i}$ is the momentum of the $\mathrm{i^{th}}$ particle (of mass m_i) and $\mathrm{p'_i} = \mathrm{m_i v'_i}$

Note $\mathrm{v'_i}$ is the velocity of the $\mathrm{i^{th}}$ particle relative to the centre of mass.

Also, prove using the definition of the centre of mass

$$\sum_{\mathrm{i}} \mathrm{p'_i} = 0$$

(b) Show $\mathrm{K} = \mathrm{K'} + \frac{1}{2}\mathrm{MV}^2$ Where K is the total kinetic energy of the system of particles, $\mathrm{K'}$ is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and $\frac{\mathrm{MV}^2}{2}$ is the kinetic energy of the translation of the system as a whole (i.e. of the centre of mass motion of the system). The result has been used in Sec. 7.14.

(c) Show $\mathrm{L} = \mathrm{L'} + \mathrm{R} \times \mathrm{MV}$

Where $\mathrm{L'} = \sum \mathrm{r'_i} \times \mathrm{p'_i}$ is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember $\mathrm{r'_i} = \mathrm{r_i} - \mathrm{R}$; rest of the notation is the standard notation used in the chapter.

Note $\mathrm{L'}$ and $\mathrm{MR} \times \mathrm{V}$ can be said to be angular momenta,

respectively, about and of the centre of mass of the system of particles.

(d) Show

$$\frac{dL'}{dt} = \sum_i r'_i \times \frac{d}{dt}\left(P'_i\right)$$

Further, show that

$$\frac{dL'}{dt} = \tau'_{ext}$$

where τ'_{ext} is the sum of all external torques acting on the system about the centre of mass.

(Hint: Use the definition of centre of mass and Newton's Third Law. Assume the internal forces between any two particles act along the line joining the particles.)

Ans (a)Take a system of i moving particles.

Mass of the i^{th} particle = m_i

Velocity of the i^{th} particle = v_i

Hence, momentum of the i^{th} particle, $p_i = m_i v_i$

Velocity of the centre of mass = V

The velocity of the i^{th} particle with respect to the centre of mass of the system is given as:

$v'_i = v_i - V$ ....(1)

Multiplying mi throughout equation (1),

we get:

$m_i v'_i = m_i v_i - m_i V$

$p'_i = p_i - m_i V$

Where,

$p'_i = m_i v'_i$ = Momentum of the i^{th}

particle with respect to the centre of mass of the system

$\therefore p_i = p'_i - m_i V$

We have the relation: $p'_i = m_i v_i'$

Taking the summation of momentum of all the particles with respect to the centre of mass of the system, we get:

$$\sum_i p'_r = \sum_i m_i v'_i = \sum_i m_i \frac{dr'_i}{dt}$$

Where,

r'_i = Position vector of i^{th} particle with respect to the centre of mass

$$v'_i = \frac{dr'}{dt}$$

As per the definition of the centre of mass, we have:

$$\sum_i m_i r'_i = 0$$

$$\therefore \sum_i m_i \frac{dr'_i}{dt} = 0$$

$$\sum_i p'_i = 0$$

(b) We have the relation for velocity of the i^{th} particle as:

$$v_i = v'_i + V$$

$$\sum_i m_i v_i = \sum_i m_i v'_i + \sum_i m_i V$$

Taking the dot product of equation (2) with itself, we get:

$$\sum_i m_i v_f \sum_i m_i v'_i + \sum_i m_i \left(v'_f + V\right) \sum_i m_i \left(v'_i + v\right)$$

$$M^2 \sum_i v^2_1 = M^2 \sum_i v^2_i + M^2 \sum_i v_i v'_j + M^2 \sum_i v'_i v_i = M^2 V^2$$

Here, for the centre of mass of the system of particles $\sum_i v_i v'_i = -\sum_i v'_i v_i$

$$M^2 \sum_i v^2_1 = M^2 \sum_i v^2_1 + M^2 V^2$$

$$\frac{1}{2} M \sum_i v^2_1 = \frac{1}{2} M \sum_i v'^2_1 + \frac{1}{2} M V^2$$

$$K = K' + \frac{1}{2} M V^2$$

Where,

$K = \frac{1}{2} M \sum_i v_1^2$ Total kinetic energy of the system of particles

$K' = \frac{1}{2} M \sum_i v'^2_1$ Total kinetic energy of the system of particles with respect to the centre of mass

$\frac{1}{2} M V^2$ = Kinetic energy of the translation of the system as a whole

(c) Position vector of the i^{th} particle with respect to origin = r_i Position vector of the i^{th} particle with respect to the centre of mass = r'_i

Position vector of the centre of mass with respect to the origin = R

It is given that:

$$r'_i = r_i - R$$

$$r_i = r'_i + R$$

We have from part (a),

$$p_i = p'_i + m_i V$$

Taking the cross product of this relation by r_i we get:

$$\sum_i r_i \times p_i = \sum_i r_i \times p'_i = \sum_i r_i \times m_i V$$

$$L = \sum_i (r_i \times R) \times p'_i + \sum_i (r'_i \times R) \times m_i V$$

$$= \sum_i r'_i \times p'_i + \sum_i R \times p'_i + \sum_i r'_i \times m_i V + \sum_i R \times m_i V$$

$$= L' + \sum_i R_i \times p'_i + \sum_i r'_i \times m_i V + \sum_i R \times m_i V$$

Where,

$$R \times \sum_i p'_i = 0 \text{ and}$$

$$\left(\sum_i r'_i \right) \times MV = 0$$

$$\sum_i m_i = M$$

$$\therefore L = L' + R \times MV$$

(d) We have the relation:

$$L' = \sum_i r'_i \times p'_i$$

$$\frac{dL'}{dt} = \frac{d}{dt}\left(\sum_i r'_i \times p'_i \right)$$

$$= \frac{d}{dt}\left(\sum_i r'_i \right) \times p'_i + \sum_i r'_i \times \frac{d}{dt}(p'_i)$$

$$= \frac{d}{dt}\left(\sum_i m_i r'_i \right) \times v'_i + \sum_i r'_i \times \frac{d}{dt}(p'_i)$$

Where, r'_i is the rate of change of velocity of the i^{th} particle with respect to the centre of mass of the system of particles.

$$\therefore \sum_i m_i r'_i = 0$$

$$\therefore \frac{dL'}{dt} = \sum_i r'_i \times \frac{d}{dt}(p'_i)$$

We have the relation:

$$\therefore \frac{dL'}{dt} = \sum_i r'_i \times \frac{d}{dt}(p'_i)$$

$$= \sum_i r'_1 \times m_i \frac{d}{dt}(v'_i)$$

Where, $\frac{d}{dt}(v'_i)$ is the rate of change of velocity of the i^{th} particle with respect to the centre of mass of the system

Therefore, according to Newton's third law of motion, we can write:

$m_i \frac{d}{dt}(v'_i) =$ External force acting on the i^{th} particle $\sum_i (\tau'_i)_{ext}$

i.e., $\sum_i r'_i \times m_i \frac{d}{dt}(v'_i) = \tau'_{ext}$ = External torque acting on the system as a whole

$$\therefore \frac{dL'}{dt} = \tau'_{ext}$$