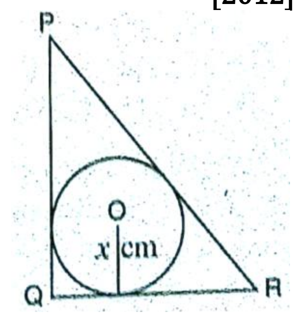
Example

Q.1 In triangle PQR, PQ = 24 cm, QR = 7 cm and $\angle PQR = 90°$.

Find the radius of the inscribed circle. **[2012]**



Sol: Since ΔPQR is a right-angled angle,

$$PR = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

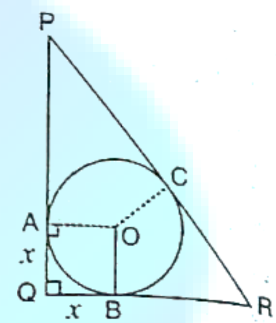
Let the given inscribed circle touches the sides of the given triangle at points A, B and C respectively.

Then, clearly, OAQB is a square.

$\Rightarrow$ AQ = BQ = x cm

PA = PQ – AQ = (24 – x) cm

RB = QR – BQ = (7 – x) cm



Since tangents from an exterior point to a circle are equal,

PC = PA = (24 – x) cm

And, RC = RB = (7 – x) cm

PR = PC + CR

$\Rightarrow$ 25 = (24 – x) + (7 – x)

$\Rightarrow$ 25 = 31 – 2x

$\Rightarrow$ 2x = 6

$\Rightarrow$ x = 3 cm

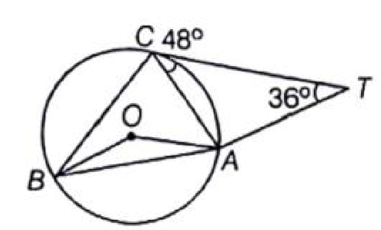
Hence, the radius of the inscribed circle is 3 cm.

Q.2 A, B, and C are three point on a circle. The tangent at C meet BA produced at T. that $\angle ATC = 36°$ and that $\angle ATC = 48°$, calculate the angle subtended by AB at the center of the circle **[2001]**

Sol: $\therefore \angle CAT = 180° - (48° + 36°)$

$= 180° - 84° = 96°$

$\therefore CAB = 180° - 96° = 84°$



Now, $\angle ABC = \angle ACT = 48°$

[angles made in alternate segments]

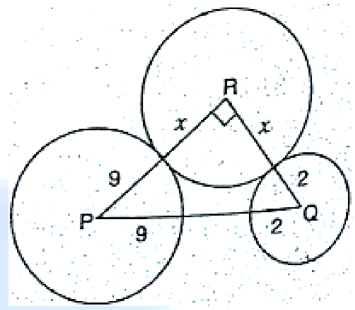
$\therefore \angle BCA = 180° - (\angle ABC + \angle CAB)$

$= 180° - (48° + 84°) = 48°$

$\therefore \angle BOA = 2 \times \angle BCA = 2 \times 48° = 96°$

Q.3 P and Q are centres of circles with radii 9 cm and 2 cm respectively. PQ = 17 cm. R is centre of a circle of radius x cm, which touches the above circles the above circles externally. Given that ∠PRO = 90°, write an equation in x and solve it. **[2004]**

Sol: According to the given statement the figure will be as shown alongside in which PQ =17 cm and RRQ = 90°. We know when two circles touch each other externally, the distance between their centres is equal to the sum of their radii.



∴ PR = (9 + x) cm and QR = (2 + x) cm.

Applying Pythagoras Theorem, we get :

$PR^2 + QR^2 + PQ^2 \Rightarrow (9+x)^2 + (2+x)^2 = 17^2$

i.e., $81 + 18x + x^2 + 4 + 4x + x^2 = 289$

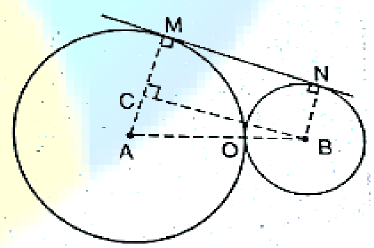
$\Rightarrow \quad 2x^2 + 22x - 204 = 289$ i.e., $x^2 + 11x - 102 = 0$

$\Rightarrow \quad x^2 + 17x - 6x - 102 = 0$ i.e., $(x+17)(x-6) = 0$

$\Rightarrow \quad x = -17$ or $x = 6$ i.e., $x = 6$

Q.4 Two circle with radii 25 cm and 9 cm touch each other externally. Find the length of the direct common tangent. **[1990]**

Sol: In the figure, A and B are the centres of the two circles touching each other externally at point O. Also MN is the direct common tangent.



Draw BC ⊥ AM.

Clearly, BCMN is a rectangle and so BC = MN.

Now, AB = OA + OB = 25 cm + 9 cm = 34 cm

AC = AM – BN = 25 cm – 9 cm = 16 cm

In right-angled triangle ABC

$AB^2 = AC^2 + BC^2$

$\Rightarrow \; 34^2 = 16^2 + BC^2$

$\Rightarrow \; BC^2 = 1156 - 256 = 900$

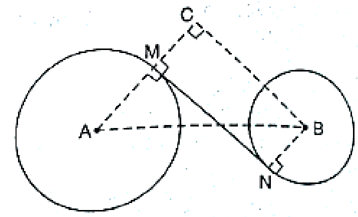
$\Rightarrow \; BC = 30$

∴ Length of direct common tangent = MN = BC

= 30cm

Q.5 The centre of two circles with radii 6 cm and 2 cm are 10 cm apart. Calculate the length of the transverse common tangent

Sol: According to the given statement the figure will be as shown alongside:



In the figure, A and B are centre of the two circles with radii 6 cm and 2 cm.

Also, MN is the transverse common tangent.

Hence, Draw AB = 10 cm, AM = 6 cm and BN = 2cm.

Draw BC perpendicular to AM Produced.

Clearly, BCMN is a rectangle, MN = BC and CM = BN = 2cm.

Now, AC = AM + CM = 6 cm + 2 cm = 8 cm

In right-angled triangle ABC

$AB^2 = AC^2 + BC^2$

$BC^2 = BC^2 - AC^2$

$\Rightarrow \; BC^2 = 10^2 - 8^2$

$\Rightarrow \; BC^2 = 100 – 64$

$\Rightarrow \; BC = 6$

∴ Length of transverse common tangent = MN = BC

= 6cm

Q.6 In the figure given alongside, PQ = QR, ∠RQP = 68° PC and QC are tangent to the circle with centre O. Calculate the values of (i) ∠QOP (ii) ∠QCP.

Sol: (i) In ΔPQR, PQ = QR

⇒ ∠QRP = ∠QPR = x (let)

∴ x + x + 68° = 180°

⇒ x = 56°

i.e., ∠QRP = 56°

Since, angle at centre is twice the angle at remaining circumference,

Therefore ∠QOP = 2∠QRP = 2 × 56° = 112°

(ii) In quadrilateral POQC,

∠QOP = 112° [Proved above]

∠OPC = 90° [Angle between radius and tangent]

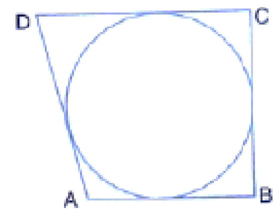
∠OQC = 90° [Angle between radius and tangent]

And ∠QOP + ∠OPC + ∠OQC + ∠QCP = 360°

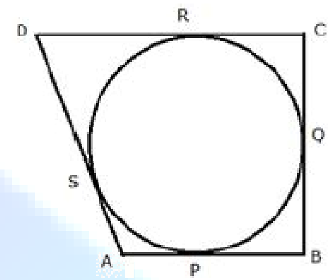
$\Rightarrow \quad 112° + 90° + 90° + \angle QCP = 360° \Rightarrow \angle QCP = 68°$

Q.7 If the sides of a quadrilateral ABCD touch a circle prove that:

$AB + CD = BC + AD$



Sol: Let the circle touch the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.



Since AP and AS are tangents to the circle from external point A

$AP = AS$ ....(i)

Similarly, we can prove that:

$BP = BQ$ ....(ii)

$CR = CQ$ ....(iii)

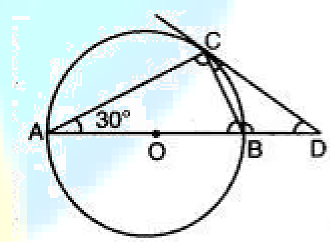
$DR = DS$ ....(iv)

Adding, $\Rightarrow AP + BP + CR + DR = AS + DS + BQ + CQ$

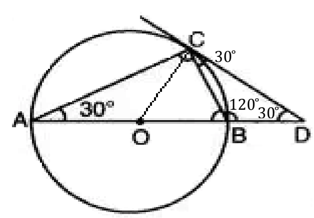
$AB + CD = AD + BC \quad \Rightarrow \quad$ Hence, $AB + CD = AD + BC$

Q.8 In the given figure, AB is the diameter and AC is the Chord of a circle such that $\angle BAC = 30°$ The tangent at C intersect AB produced at D. Prove that : BC = BD.

[2004]



Sol: Join OC.



$\angle ACB = 90°$ (Angle of the semicircle)

$\angle ABC = 60°$ (Angle sum property)

$\angle CBD = 120°$ (adj to angle CBA 30°)

$\angle OCD = 90°$ tangent

$\angle COB = 60°$ (Angle at the centre is equal to twice that of the circumference.)

$\angle OCB = 60°$ (Angle sum property)

$\angle BCD = \angle OCD - \angle OCB = 30°$

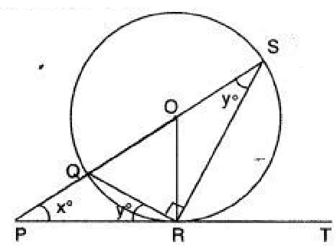
$\therefore \quad \angle BDC = \angle BDC = 30°$

BD = BC

Hence proved.

Q.9 In the figure, PT touches a circles with centre O at R. Diameter SQ when Produced meets PT at P. If ∠SPR = x° and ∠QRP = y°

show that x° + 2y° = 90°



Sol: We have,

∠PRQ = y and ∠QPR = x

We know that, if a chord is drawn through the point of contact of a tangent to a circle then the angles which this chord makes with the given tangent are equal to the angles formed in the corresponding alternate segments.

Now, ∠QSR = ∠PRQ (Angles in the alternate segment)

⇒∠QSR = y

Since, QS is the diameter of the circle, then

∠QRS = 90° (Angle in a semicircle is right angle)

Now, in ΔQRS,

∠SQR = 180° − (∠QRS + ∠QSR) [Angle sum property]

⇒ ∠SQR = 180° − 90° − y = 90°−y

Now, ∠SQR is an exterior angle of ΔPQR.

∠SQR = ∠QPR + ∠PRQ (Exterior angle theorem)

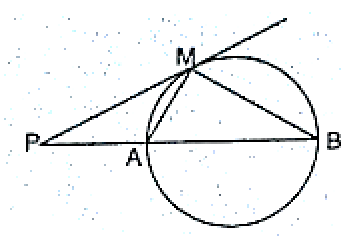
⇒ 90° − y = x + y

⇒ x + 2y = 90°

Q.10 In the given figure, PM is a tangent to the circle and PA = AM. Prove that:

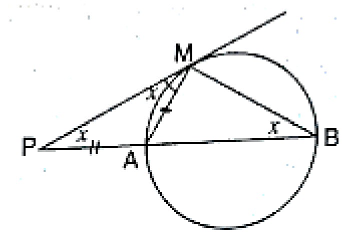
(i) ΔPMB is isosceles

(ii) PA × PB = MB^2 **[2005]**



Sol: (i) In ΔPAM, PA = PM

$\Rightarrow \quad \angle ABM = \angle AMP = x$ (suppose) [$\angle$s opp. to equal sides]



Also, $\angle ABM = \angle AMP = x$ [Angle of alternate segment]

$\Rightarrow \quad \angle APM = \angle ABM = x \Rightarrow MB = MP$

$\therefore$ ΔPMB is Isosceles. Hence proved

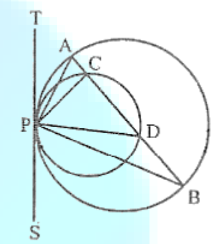
(ii) Since the chord AB produced and the tangent at point m intersect each other externally at point P.

$\therefore \quad PA \times PB = MP^2$

$\Rightarrow \; PA \times PB = MB^2$ [$\because$ MB = MP, proved above]

Hence Proved.

Q.11 Two circles touch each other internally at a point P. A chord AB of the bigger circle intersects the other circle in C and D. Prove that $\angle CPA = \angle DPB$



Sol: Given - Two circles touch each other internally at P.

A chord AB of bigger circle intersects the smaller circle at C and D.

AP, BP , CP and DP are joined.

To Prove - $\angle CPA = \angle DPB$

As, TPS is the tangent , PD is the chord.

$\therefore$ $\angle PAB = \angle BPS$ ....(i) [Angles in alt. segment]

Similarly, we can prove that

$\angle PCD = \angle DPS$ ....(ii)

subtract (i) from (ii),we get,

$\angle PCD - \angle PAB = \angle DPS - \angle BPS$

But in tr. PAC,

ext. $\angle PCD = \angle PAB + \angle CPA$

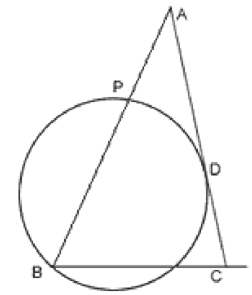
therefore,

$\angle PAB + \angle CPA - \angle PAB = \angle DPS - \angle BPS$

$\angle CPA = \angle DPB$

Hence the result.

Q.12 ABC is an isosceles triangle with AB =AC circle through B touches side AC at its middle point D and intersects AB in point P show AB = 4 × AP.



Sol: $AP \times AB = AD^2 = = \left(\frac{1}{2} A C\right)^2$

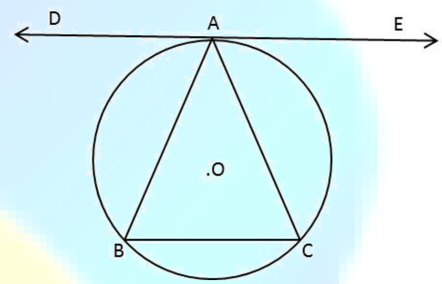
$AP \times AB = \frac{1}{4} AC^2$ [$AD = \frac{1}{2} AC$]

Or, $4\ AP.AB = AC^2$ [AC = AB]

Or, $4\ AP.AB = AB^2$

Or, 4 AP = AB

Q.13 The given figure shown an isosceles triangle ABC in a circle such that AB = AC. If DAE is a tangent to the circle at point A, Prove that DE is parallel to BC.



Sol: Let DAE be tangent at A to the circumcircle of ΔABC.

In ΔABC, AB = AC (Given)

∴ ∠ACB = ∠ABC --- (1) (Angles opposite to equal sides are equal)

According to alternate segment theorem, the angle between the tangent and chord at the point of contact is equal to the angles made by the chord in the corresponding alternative segment.

DAE is the tangent and AB is the chord.

∴ ∠DAB = ∠ACB ------ (2)

From (1) and (2), we have ∠ABC = ∠DAB

∴ DE is parallel to BC

(If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel)

Therefore, the tangent at A to the circumcircle of ΔABC is parallel BC.

Q.14 AB is the diameter od a circle with center O. A line PQ touches the given circle at point R and cuts the circle through A and B at points P and Q respectively. Prove that : ∠POQ = 90°

Sol: According to the given statement, the figure will be as shown alongside:

Join OR.

Now, show that Δ OBQ $\equiv$ ΔORQ

$\Rightarrow \quad \angle BOQ = \angle ROQ = x$ (let)

Then, show that Δ AOP $\equiv$ Δ ROP $= y$ (let)

Since, AOB is a straight line, therefore $\angle AOB = 180°$

$\Rightarrow \; x + x + y + y = 180°$

i.e., $\quad 2x + 2y = 180°$

$\Rightarrow \quad x + y = 90°$

$\Rightarrow \quad \angle POQ = 90°$