(i) Direction Ratios and Direction Cosines

LEVEL-I

1. Write the direction-cosines of the line joining the points (1,0,0) and (0,1,1) [CBSE 2011]
2. Find the direction cosines of the line passing through the following points $(-2,4,-5), (1,2,3)$.
3. Write the direction cosines of a line equally inclined to the three coordinate axes

LEVEL-II

4. Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$
5. Write the direction ratios of a line parallel to the line $\frac{5-x}{3} = \frac{y+7}{-2} = \frac{z+2}{6}$
6. If the equation of a line AB $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+6}{3}$ Find the direction cosine.
7. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axis.

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL-I

8. Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-x}{2}$
9. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3).
10. Express the equation of the plane $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ in the Cartesian form.
11. Express the equation of the plane $\vec{r}.(2\hat{i} - 3\hat{j} + \hat{k}) + 4 = 0$ in the Cartesian form

(iii) Co-planer and skew lines

LEVEL-II

12. Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If intersecting. Find their point of intersection.
13. Show that the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ and $(-4,4,4,)$ are coplanar. Also, find the equation of the plane containing them.
14. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect Find their point of intersection.

LEVEL-III

15. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{-2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.
16. The point A(4, 5, 10), B (2,3,4) and C $(1, 2, -1)$ are three vertices of a parallelogram ABCD. Find the vector equation of the sides AB and BC and also find the coordinates
17. Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1, 1, 1).
18. Show that the four points $(0, -1, -1)$, $(4, 5, 1)$, $(3, 9, 4)$ and $(-4, 4, 4)$ are coplanar and find the equation of the common plane

(iv) Shortest distance between two lines

LEVEL-II

19. Find the shortest distance between the lines I_1 and I_2 given by the following:

    (a) $I_1: \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ $\quad$ $I_2: \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

    (b) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$

    $\vec{r} = (4\hat{i} + 2\mu)\hat{i} + (5 + 3\mu)\hat{j} + (6 + \mu)\hat{k}$.
20. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.
21. Find the shortest distance between the lines

    $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$
22. Find the shortest distance between the lines

    $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} + (2s + 1)\hat{k}$ [CBSE 2011]
23. Find the distance between the parallel planes $x + y - z = -4$ and $2x + 2y - 2z + 10 = 0$
24. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through $(3, 0, -4)$. Also find the distance between these two lines.

(v) Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

25. Find the equation of a plane passing through the origin and perpendicular to x-axis