



**SpeedLabs**

**MATHS**

**CBSE 12<sup>th</sup>**

**TEEVRA EDUTECH PVT. LTD.**

# Three-Dimensional Geometry

## Exercise- 11.3

1. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

(a)  $z = 2$                       (b)  $x + y + z = 1$

(c)  $2x + 3y - z = 5$       (d)  $5y + 8 = 0$

Ans. (a) The equation of the plane is  $z = 2$  or  $0x + 0y + z = 2$  ... (1)

The direction ratios of normal are 0, 0, and 1.

$$\therefore \sqrt{0^2 + 0^2 + 1^2} = 1$$

Dividing both sides of equation (1) by 1, we obtain

$$0x + 0y + 1z = 2$$

This is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of the perpendicular drawn from the origin. Therefore, the direction cosines are 0, 0, and 1 and the distance of the plane from the origin is 2 units.

(b)  $x + y + z = 1$  ... (1)

The direction ratios of normal are 1, 1, and 1.

$$\therefore \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

Dividing both sides of equation (1) by  $\sqrt{3}$ , we obtain

$$\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = \frac{1}{\sqrt{3}}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin.

Therefore, the direction cosines of the normal are  $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$  and  $\frac{1}{\sqrt{3}}$  and the distance of normal from the

origin is  $\frac{1}{\sqrt{3}}$  units.

(c)  $2x + 3y - z = 5$  ... (1)

The direction ratios of normal are 2, 3, and  $-1$ .

$$\therefore \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$$

Dividing both sides of equation (1) by  $\sqrt{14}$ , we obtain

$$\frac{2}{\sqrt{14}}x + \frac{3}{\sqrt{14}}y - \frac{1}{\sqrt{14}}z = \frac{5}{\sqrt{14}}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin. Therefore, the direction cosines of the normal to the plane

$$\text{are } \frac{2}{\sqrt{14}}x + \frac{3}{\sqrt{14}} \text{ and } \frac{-1}{\sqrt{14}} \text{ and}$$

the distance of normal from the origin is  $= \frac{5}{\sqrt{14}}$  units.

$$(d) 5y + 8 = 0$$

$$\Rightarrow 0x - 5y + 0z = 8 \dots (1)$$

The direction ratios of normal are 0, -5, and 0.

$$\therefore \sqrt{0 + (-5)^2 + 0} = 5$$

Dividing both sides of equation (1) by 5, we obtain

$$-y = \frac{8}{5}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin. Therefore, the direction cosines of the normal to the plane

are 0, -1, and 0 and the distance of normal from the origin is  $\frac{8}{5}$  units.

2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector  $3\hat{i} + 5\hat{j} - 6\hat{k}$

Ans. The normal vector is,  $\vec{n} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

$$\therefore \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{(3)^2 + (5)^2 + (6)^2}} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}$$

It is known that the equation of the plane with position vector  $\vec{r}$  is given by,  $\vec{r} \cdot \hat{n} = d$

$$\Rightarrow \vec{r} \cdot \left( \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7$$

This is the vector equation of the required plane.

3. Find the Cartesian equation of the following planes:

$$(a) \vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2 \quad (b) \vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

$$(c) \vec{r} \cdot [(s-2t)\hat{i} + (3+t)\hat{j} + (2s+t)\hat{k}] = 15$$

Ans. (a) It is given that equation of the plane is

$$\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2 \dots (1)$$

For any arbitrary point P (x, y, z) on the plane, position vector  $\vec{r}$  is given by,

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

Substituting the value of  $\vec{r}$  in equation (1), we obtain

$$(x\hat{i} + y\hat{j} - z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\Rightarrow x + y - z = 2$$

This is the Cartesian equation of the plane.

(b) For any arbitrary point P (x, y, z) on the plane, position vector  $\vec{r}$  is given by,

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

Substituting the value  $\vec{r}$  of in equation (1), we obtain

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$$

$$\Rightarrow 2x + 3y - 4z = 1$$

This is the Cartesian equation of the plane.

$$(c) \vec{r} \cdot [(s-2t)\hat{i} + (3+t)\hat{j} + (2s+t)\hat{k}] = 15$$

For any arbitrary point P (x, y, z) on the plane, position vector  $\vec{r}$  is given by,

$$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$$

Substituting the value of in equation (1), we obtain

$$(x\hat{i} + y\hat{j} - z\hat{k}) \cdot [(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$$

$$\Rightarrow (s-2t)x + (3-t)y + (2s+t)z = 15$$

This is the Cartesian equation of the given plane.

4. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$$(a) 2x + 3y + 4z - 12 = 0 \quad (b) 3y + 4z - 6 = 0$$

$$(c) x + y + z = 1 \quad (d) 5y + 8 = 0$$

Ans. (a) Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$ .

$$2x + 3y + 4z - 12 = 0$$

$$\Rightarrow 2x + 3y + 4z = 12 \dots (1)$$

The direction ratios of normal are 2, 3, and 4.

$$\therefore \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{19}$$

Dividing both sides of equation (1) by  $\sqrt{19}$ , we obtain

$$\frac{2}{\sqrt{19}}x + \frac{3}{\sqrt{19}}y + \frac{4}{\sqrt{19}}z = \frac{12}{\sqrt{19}}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin. The coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$ . Therefore, the coordinates of the foot of the perpendicular are

$$\frac{2}{\sqrt{19}} \cdot \frac{12}{\sqrt{19}}, \frac{3}{\sqrt{19}} \cdot \frac{12}{\sqrt{19}}, \frac{4}{\sqrt{19}} \cdot \frac{12}{\sqrt{19}} \text{ i.e., } \left( \frac{24}{19}, \frac{36}{19}, \frac{48}{19} \right)$$

(b) Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$ .

$$3y + 4z - 6 = 0$$

$$\Rightarrow 0x + 3y + 4z = 6 \dots (1)$$

The direction ratios of the normal are 0, 3, and 4.

$$\therefore \sqrt{0^2 + 3^2 + 4^2} = 5$$

Dividing both sides of equation (1) by 5, we obtain

$$0x + \frac{3}{5}y + \frac{4}{5}z = \frac{6}{5}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin. The coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$ . Therefore, the coordinates of the foot of the perpendicular are

$$\left( 0, \frac{3}{5} \cdot \frac{6}{5}, \frac{4}{5} \cdot \frac{6}{5} \right) \text{ i.e., } \left( 0, \frac{18}{25}, \frac{24}{25} \right)$$

(c) Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$ .

$$x + y + z = 1 \dots (1)$$

The direction ratios of the normal are 1, 1, and 1.

$$\therefore \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Dividing both sides of equation (1) by  $\sqrt{3}$  we obtain

$$\left( \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \right) \text{ i.e., } \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$$

(d) Let the coordinates of the foot of perpendicular P from the origin to the plane be  $(x_1, y_1, z_1)$ .

$$5y + 8 = 0$$

$$\Rightarrow 0x - 5y + 0z = 8 \dots (1)$$

The direction ratios of the normal are 0, -5, and 0.

$$\therefore \sqrt{0 + (-5)^2 + 0} = 5$$

Dividing both sides of equation (1) by 5, we obtain

$$-y = \frac{8}{5}$$

This equation is of the form  $lx + my + nz = d$ , where  $l, m, n$  are the direction cosines of normal to the plane and  $d$  is the distance of normal from the origin. The coordinates of the foot of the perpendicular are given by  $(ld, md, nd)$ . Therefore, the coordinates of the foot of the perpendicular are

$$\left( 0, -2\left(\frac{8}{5}\right), 0 \right) \text{ i.e., } \left( 0, -\frac{8}{5}, 0 \right).$$

5. Find the vector and Cartesian equation of the planes

(a) that passes through the point  $(1, 0, -2)$  and the normal to the plane is  $\hat{i} + \hat{j} - \hat{k}$

(b) that passes through the point  $(1, 4, 6)$  and the normal vector to the plane is

$$\hat{i} + 2\hat{j} + \hat{k}$$

Ans. (a) The position vector of point  $(1, 0, -2)$  is  $\vec{a} = \hat{i} - 2\hat{k}$

The normal vector  $\vec{N}$  perpendicular to the plane is  $\vec{N} = \hat{i} + \hat{j} - \hat{k}$

The vector equation of the plane is given by,  $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0 \dots (1)$$

$\vec{r}$  is the position vector of any point P  $(x, y, z)$  in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, equation (1) becomes

$$[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\begin{aligned} &\Rightarrow [(x-1)\hat{i} + y\hat{j} + (z+2)\hat{k}] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0 \\ &\Rightarrow (x-1) + y - (z+2) = 0 \\ &\Rightarrow x + y - z - 3 = 0 \\ &\Rightarrow x + y - z = 3 \end{aligned}$$

This is the Cartesian equation of the required plane.

(b) The position vector of the point (1, 4, 6) is  $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

The normal vector  $\vec{N}$  perpendicular to the plane is  $\vec{N} = \hat{i} - 2\hat{j} + \hat{k}$

The vector equation of the plane is given by,  $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \dots (1)$$

$\vec{r}$  is the position vector of any point P (x, y, z) in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, equation (1) becomes

$$\begin{aligned} &[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \\ &\Rightarrow [(x-1)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \\ &\Rightarrow (x-1) - 2(y-4) + (z-6) = 0 \\ &\Rightarrow x - 2y + z + 1 = 0 \end{aligned}$$

This is the Cartesian equation of the required plane.

6. Find the equations of the planes that passes through three points.

(a) (1, 1, -1), (6, 4, -5), (-4, -2, 3)

(b) (1, 1, 0), (1, 2, 1), (-2, 2, -1)

Ans. (a) The given points are A (1, 1, -1), B (6, 4, -5), and C (-4, -2, 3)

$$\begin{vmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{vmatrix} = (12 - 10)(18 - 20) - (-12 + 16)$$

$$= 2 + 2 - 4$$

$$= 0$$

Since A, B, C are collinear points, there will be infinite number of planes passing through the given points.

(b) The given points are A (1, 1, 0), B (1, 2, 1), and C (-2, 2, -1).

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = (-2-2) - (2+2) = -8 \neq 0$$

Therefore, a plane will pass through the points A, B, and C

It is known that the equation of the plane through the points,  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$ , and  $(x_3, y_3, z_3)$ , is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (-2)(x-1) - 3(y-1) + 3z = 0$$

$$\Rightarrow -2x - 3y + 3z + 2 = -5$$

$$\Rightarrow -2x - 3y + 3z = -5$$

$$\Rightarrow 2x + 3y - 3z = 5$$

This is the Cartesian equation of the required plane.

7. Find the intercepts cut off by the plane  $2x + y - z = 5$

Ans.  $2x + y - z = 5 \dots (1)$

Dividing both sides of equation (1) by 5, we obtain

$$\frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \frac{x}{5} + \frac{y}{5} + \frac{z}{-5} = 1 \dots (2)$$

It is known that the equation of a plane in intercept form is  $\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$



where a, b, c are the intercepts cut off by the plane at x, y, and z axes respectively. Therefore, for the given equation,

$$a = \frac{5}{2}, b = 5, \text{ and } c = -5$$

Thus, the intercepts cut off by the plane are  $\frac{5}{2}, b = 5, \text{ and } -5$

8. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOY plane.

Ans. The equation of the plane ZOY is

$$y = 0$$

Any plane parallel to it is of the form,  $y = a$

Since the y-intercept of the plane is 3,

$$\therefore a = 3$$

Thus, the equation of the required plane is  $y = 3$

9. Find the equation of the plane through the intersection of the planes

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0 \text{ and the point } (2, 2, 1)$$

Ans. The equation of any plane through the intersection of the planes,

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0, \text{ is}$$

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0, \text{ where } \alpha \in \mathbb{R} \dots (1)$$

The plane passes through the point (2, 2, 1). Therefore, this point will satisfy equation (1).

$$\therefore (3 \times 2 - 2 + 2 \times 1 - 4) + \alpha(2 + 2 + 1 - 2) = 0$$

$$\Rightarrow 2 + 3\alpha = 0$$

$$\Rightarrow \alpha = -\frac{2}{3}$$

Substituting  $\alpha = -\frac{2}{3}$  in equation (1), we obtain

$$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$$

$$\Rightarrow 3(3x - y + 2z - 4) - 2(x + y + z - 2) = 0$$

$$\Rightarrow (9x - 3y + 6z - 12) - 2(x + y + z - 2) = 0$$

$$\Rightarrow 7x - 5y + 4z - 8 = 0$$

This is the required equation of the plane.

10. Find the vector equation of the plane passing through the intersection of the planes.

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 = 0 \dots (1)$$

$$\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 = 0 \dots (2)$$

The equation of any plane through the intersection of the planes given in equations (1) and (2) is given by,

$$\left[ \vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 \right] + \lambda \left[ \vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 \right] = 0 \text{ where } \lambda \in \mathbb{R}$$

$$\vec{r} \cdot \left[ (2\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k}) \right] = 9\lambda + 7$$

$$\vec{r} \cdot \left[ (2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3\lambda - 3)\hat{k} \right] = 9\lambda + 7 \dots (3)$$

The plane passes through the point (2, 1, 3). Therefore, its position vector is given by,

$$\vec{r} \cdot 2\hat{i} + 2\hat{j} + 3\hat{k}$$

Substituting in equation (3), we obtain

$$(2\hat{i} + \hat{j} - 3\hat{k}) \cdot \left[ (2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3\lambda - 3)\hat{k} \right] = 9\lambda + 7$$

$$\Rightarrow (2 + 2\lambda) + (2 + 5\lambda) + (3\lambda - 3) = 9\lambda + 7$$

$$\Rightarrow 18\lambda - 3 = 9\lambda + 7$$

$$\Rightarrow 9\lambda = 10$$

$$\Rightarrow \lambda = \frac{10}{9}$$

Substituting  $\lambda = \frac{10}{9}$  in equation (3), we obtain

$$\vec{r} \cdot \left( \frac{38}{9}\hat{i} + \frac{68}{9}\hat{j} + \frac{3}{9}\hat{k} \right) = 17$$

$$\Rightarrow \vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$$

This is the vector equation of the required plane.

11. Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  which is perpendicular to the plane  $x - y + z = 0$

Ans. The equation of the plane through the intersection of the planes,  $x - y + z = 1$  and  $2x + 3y + 4z = 5$ , is

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\Rightarrow (2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z - (5\lambda + 1) = 0 \dots (1)$$

The direction ratios,  $a_1, b_1, c_1$ , of this plane are  $(2\lambda + 1), (3\lambda + 1)$ , and  $(4\lambda + 1)$ .

The plane in equation (1) is perpendicular to  $x - y - z = 0$

Its direction ratios,  $a_2, b_2, c_2$ , are 1, -1, and 1.

Since the planes are perpendicular,

$$\begin{aligned}
 a_1 a_2 + b_1 b_2 + c_1 c_2 &= 0 \\
 \Rightarrow (2\lambda + 1) - (3\lambda + 1) + (4\lambda + 1) &= 0 \\
 \Rightarrow 3\lambda + 1 &= 0 \\
 \Rightarrow \lambda &= -\frac{1}{3}
 \end{aligned}$$

Substituting  $\lambda = -\frac{1}{3}$  in equation (1), we obtain

$$\begin{aligned}
 \frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} &= 0 \\
 \Rightarrow x - z + 2 &= 0
 \end{aligned}$$

This is the required equation of the plane.

12. Find the angle between the planes whose vector equations are

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

- Ans. The equations of the given planes are  $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$

It is known that if  $\vec{n}_1$  and  $\vec{n}_2$  are normal to the planes,  $\vec{r} \cdot \vec{n}_1 = d_1$  and  $\vec{r} \cdot \vec{n}_2 = d_2$  then the angle between them, Q, is given by,

$$\cos Q = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|} \dots \dots (1)$$

Here,  $\vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k}$  and  $\vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$

$$\therefore \vec{n}_1 \cdot \vec{n}_2 = (2\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 2 \cdot 3 + 2 \cdot (-3) + (-3) \cdot 5 = -15$$

$$|\vec{n}_1| = \sqrt{(2)^2 + (2)^2 + (-3)^2} = \sqrt{17}$$

$$|\vec{n}_2| = \sqrt{(3)^2 + (-3)^2 + (5)^2} = \sqrt{43}$$

Substituting the value of  $\vec{n}_1 \cdot \vec{n}_2$ ,  $|\vec{n}_1|$  and  $|\vec{n}_2|$  in equation (1), we obtain

$$\cos Q = \frac{|-15|}{\sqrt{17} \cdot \sqrt{43}}$$

$$\Rightarrow \cos Q = \left( \frac{15}{\sqrt{731}} \right)$$

$$\Rightarrow \cos Q^{-1} = \left( \frac{15}{\sqrt{731}} \right)$$

13. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

(a)  $7x + 5y + 6z + 30 = 0$  and  $3x - y - 10z + 4 = 0$

(b)  $2x + y + 3z - 2 = 0$  and  $x - 2y + 5 = 0$

(c)  $2x - 2y + 4z + 5 = 0$  and  $3x - 3y + 6z - 1 = 0$

(d)  $2x - y + 3z - 1 = 0$  and  $2x - y + 3z + 3 = 0$

(e)  $4x - 8y + z - 8 = 0$  and  $y + z - 4 = 0$

Ans. The direction ratios of normal to the plane,  $L_1 : a_1x + b_1y + c_1z = 0$ , are  $a_1, b_1, c_1$  and

$$L_2 : a_2x + b_2y + c_2z = 0 \text{ are } a_2, b_2, c_2$$

$$L_1 \parallel L_2, \text{ if } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$L_1 \perp L_2, \text{ if } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

The angle between  $L_1$  and  $L_2$  is given by,

$$Q = \cos^{-1} \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

(a) The equations of the planes are  $7x + 5y + 6z + 30 = 0$  and  $3x - y - 10z + 4 = 0$

Here,  $a_1 = 7, b_1 = 5, c_1 = 6$

$$a_2 = 3, b_2 = -1, c_2 = -10$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 7 \times 3 + 5 \times (-1) + 6 \times (-10) = -44 \neq 0$$

Therefore, the given planes are not perpendicular.

$$\frac{a_1}{a_2} = \frac{7}{3}, \frac{b_1}{b_2} = \frac{5}{-1} = -5, \frac{c_1}{c_2} = \frac{6}{-10} = \frac{-3}{5}$$

$$\text{It can be seen that, } \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given planes are not parallel.

The angle between them is given by,

$$Q = \cos^{-1} \left| \frac{7 \times 3 + 5 \times (-1) + 6 \times (-10)}{\sqrt{(7)^2 + (5)^2 + (6)^2} \times \sqrt{(3)^2 + (-1)^2 + (-10)^2}} \right|$$

$$= \cos^{-1} \left| \frac{21 - 5 - 60}{\sqrt{110} \times \sqrt{110}} \right|$$

$$= \cos^{-1} \frac{44}{110}$$

$$= \cos^{-1} \frac{2}{5}$$