SpeedLabs

MATHS

# CBSE 12th

# TEEVRA EDUTECH PVT. LTD.

# Three-Dimensional Geometry
# Exercise- 11.1



**1.** If a line makes angles 90°, 135°, 45° with x, y and z-axes respectively, find its direction cosines.

**Ans.** Let direction cosines of the line be l, m, and n.

$l = \cos 90° = 0$

$m = \cos 135° = -\frac{1}{2}$

$n = \cos 45° = \frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}, \text{and} \frac{1}{\sqrt{2}}$

**2.** Find the direction cosines of a line which makes equal angles with the coordinate axes.

**Ans.** Let the direction cosines of the line make an angle α with each of the coordinate axes.

$\therefore l = \cos a, m = \cos a, n = \cos a$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$

$\Rightarrow 3\cos^2 \alpha = 1$

$\Rightarrow \cos^2 \alpha = \frac{1}{3}$

$\Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes, are

$\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \text{and} \pm \frac{1}{\sqrt{3}}$

**3.** If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

**Ans.** If a line has direction ratios of $-18, 12,$ and $-4$, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}$$

i.e, $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Thus, the direction cosines are $\frac{-9}{11}, \frac{6}{11}, \text{and} \frac{-2}{11}$

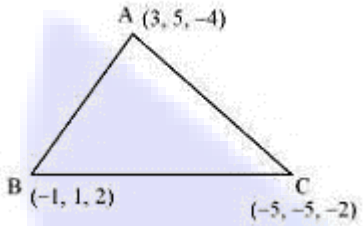
**4.** Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

Ans. The given points are A (2, 3, 4), B (− 1, − 2, 1), and C (5, 8, 7). It is known that the direction ratios of line joining the points, $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$, are given by, $x_2 - x_1$, $y_2 - y_1$, and $z_2 - z_1$. The direction ratios of AB are (−1 − 2), (−2 − 3), and (1 − 4) i.e., −3, −5, and −3. The direction ratios of BC are (5 − (− 1)), (8 − (− 2)), and (7 − 1) i.e., 6, 10, and 6. It can be seen that the direction ratios of BC are −2 times that of AB i.e., they are proportional.

Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B, and C are collinear.

**5.** Find the direction cosines of the sides of the triangle whose vertices are (3, 5, − 4), (− 1, 1, 2) and (− 5, − 5, − 2)

**Ans.** The vertices of ΔABC are A (3, 5, −4), B (−1, 1, 2), and C (−5, −5, −2)




The direction ratios of side AB are (−1 − 3), (1 − 5), and (2 − (−4)) i.e., −4, −4, and 6.

Then, $\sqrt{(-4)^2 + (-4)^2 + (6)^2} = \sqrt{16 + 16 + 36}$

$= \sqrt{68}$

$= 2\sqrt{17}$

Therefore, the direction cosines of AB are

$$\frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}$$

$$\frac{-4}{2\sqrt{17}}, -\frac{4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$$

$$\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of BC are (−5 − (−1)), (−5 − 1), and (−2 − 2) i.e., −4, −6, and −4. Therefore, the direction cosines of BC are

$$\frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-6}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}$$

The direction ratios of CA are (−5 − 3), (−5 − 5), and (−2 − (−4)) i.e., −8, −10, and 2.

Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{-5}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{2}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}$$

$$\frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}$$