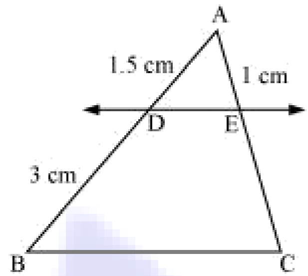
SpeedLabs

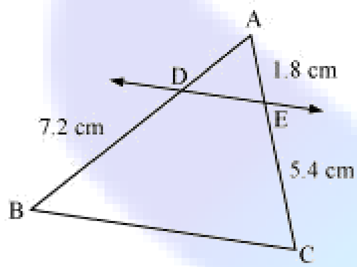
MATHS

CBSE 10th

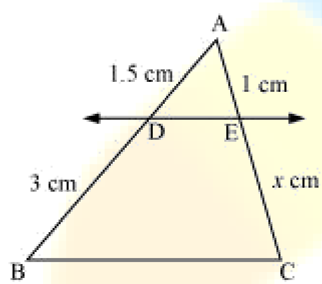
TEEVRA EDUTECH PVT. LTD.

Q.1: In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).





Sol: (i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

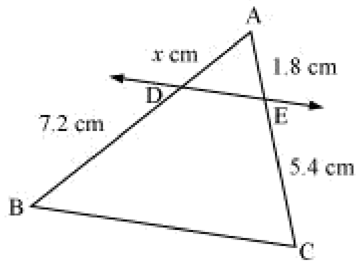
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$x = 2$

$\therefore$ EC = 2 cm

(ii)



Let AD = x cm It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$x = 2.4$

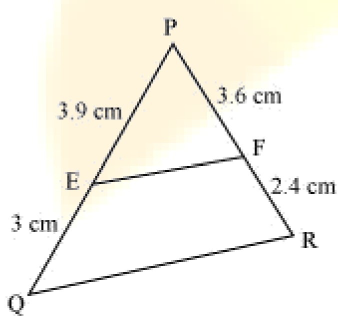
$\therefore$ AD = 2.4 cm

Q.2 E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm



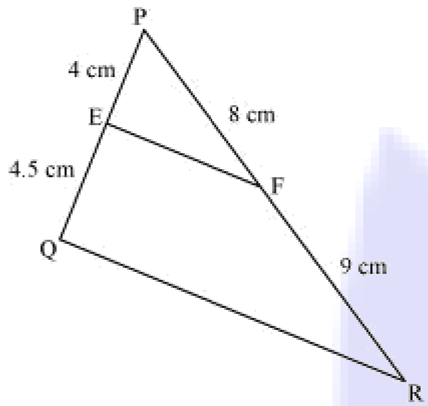
Sol: Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.



(II)

PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

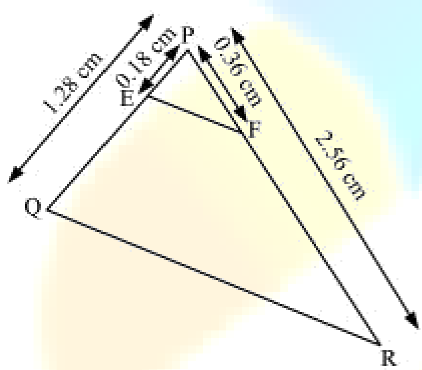
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(III)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

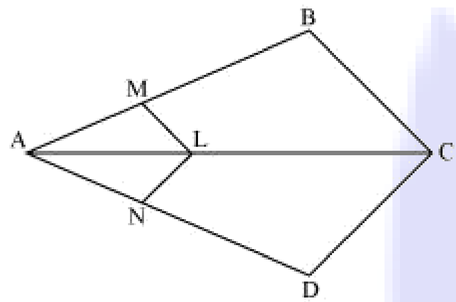
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

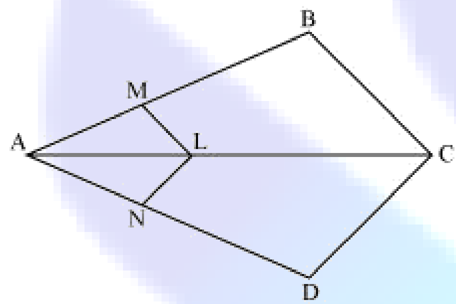
Therefore, EF is parallel to QR.

Q.3 In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Sol:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \ldots\ldots (i)$$

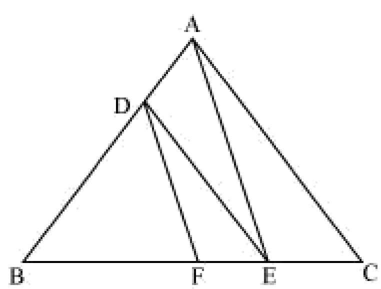
Similarly, LN||CD

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \ldots . (ii)$$

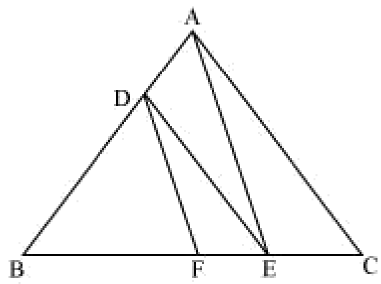
From (i) and (ii), we obtain $\frac{AM}{AB} = \frac{AN}{AD}$

Q.4: In the following figure, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$



Sol:



In ΔABC, DE || AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (Basic Proportionality Theorem) (i)

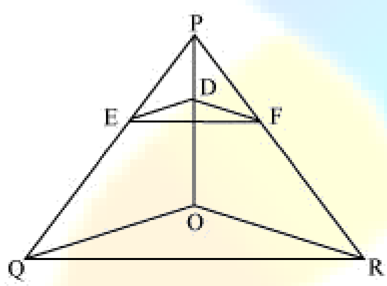
In ΔBAE, DF || AE

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ (Basic Proportionality Theorem) (ii)

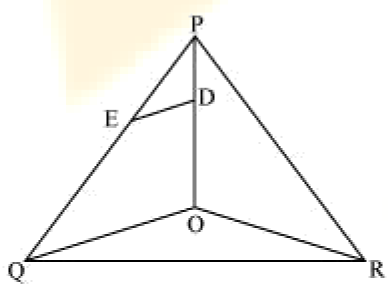
From (i) and (ii), we obtain

$\frac{BE}{EC} = \frac{BF}{FE}$

Q.5 In the following figure, DE || OQ and DF || OR, show that EF || QR.

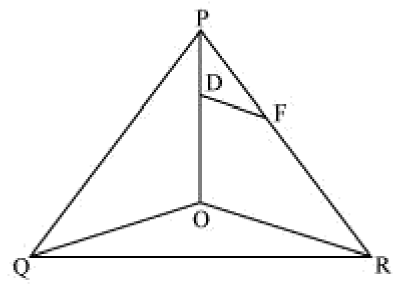


Sol:



In Δ POQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionaliy theorem) (i)



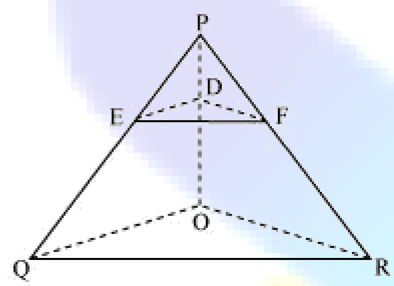
In Δ POR, DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

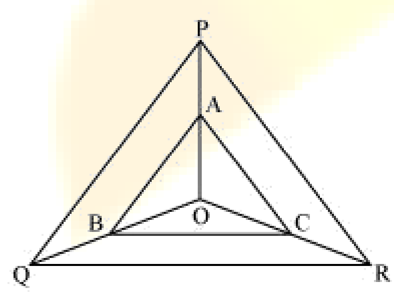
From (i) and (ii), we obtain

$\frac{PE}{EQ} = \frac{PF}{FR}$

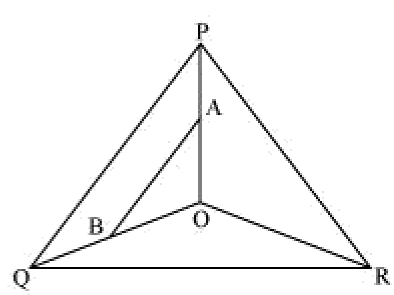
$\therefore$ EF||QR (Converse of basic propoprtionality theorem)



Q.6 In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

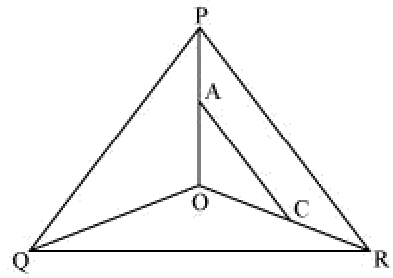


Sol:



In Δ POQ, AB || PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (Basic proportinality theorem) (i)



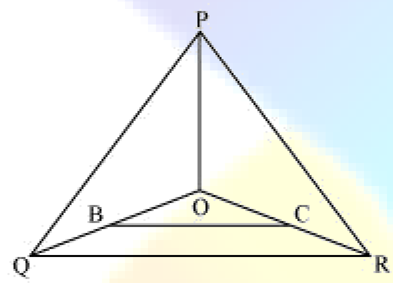
In Δ POR, AC || PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (Basic proportinality theorem) (ii)

From (i) and (ii), we obtain

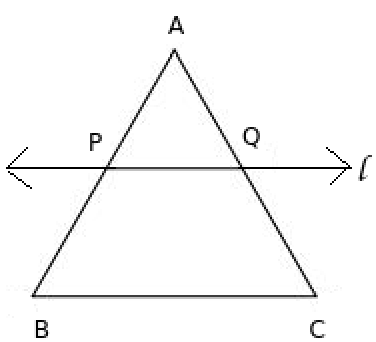
$\frac{OB}{BQ} = \frac{OC}{CR}$

$\therefore$ BC||QR (By the converse of basic proportionality theorem)



Q.7 Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side.

Sol:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that PQ||BC By using basic proportionality theorem, we obtain

$\frac{AQ}{QC} = \frac{AP}{PB}$

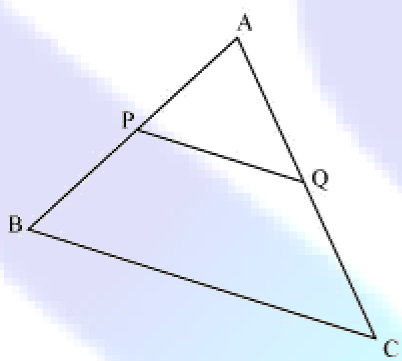
$\frac{AQ}{QC} = \frac{1}{1}$ (P is the mid point of AB $\therefore$ AP = PB)

AQ = QC

Or, Q is the mid-point of AC.

Q.8: Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side.

Sol:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$\frac{AP}{PB} = \frac{1}{1}$

and $\frac{AQ}{QC} = \frac{1}{1}$

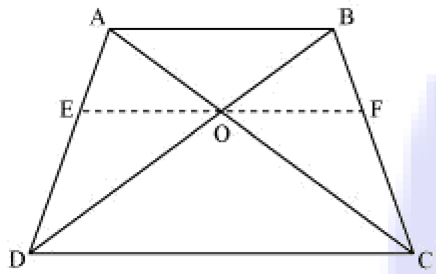
$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$

Hence, by using basic proportionality theorem, we obtain PQ || BC.

Q.9 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O.

Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Sol:



Draw a line EF through point O, such that EF||CD

In ΔADC, EO||CD

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, OE||AB

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

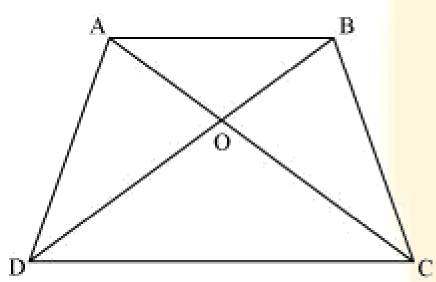
From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

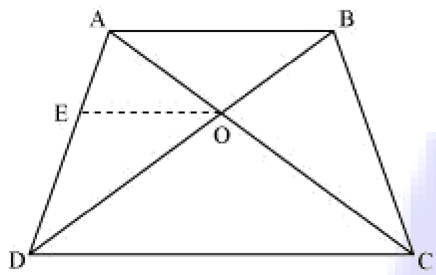
$$\frac{AO}{BO} = \frac{OC}{OD}$$

Q.10: The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{OD}$. Show that ABCD is a trapezium.

Sol: The diagonals of a quadrilateral ABCD intersect each other at the point O such that



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

∴ ABCD is a trapezium.