SpeedLabs

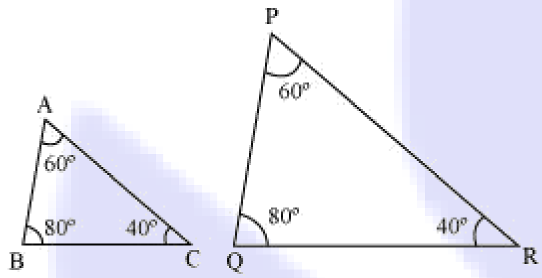
MATHS

CBSE 10th

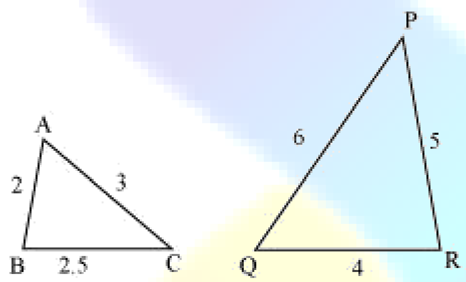
TEEVRA EDUTECH PVT. LTD.

Q.1 State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

(i)



(ii)



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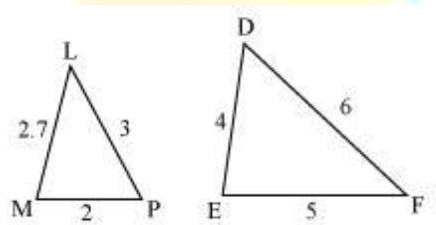

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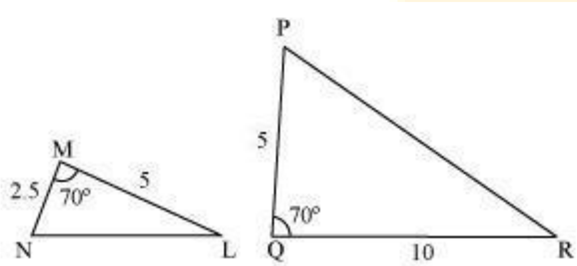

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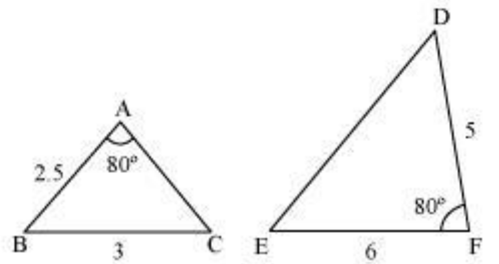
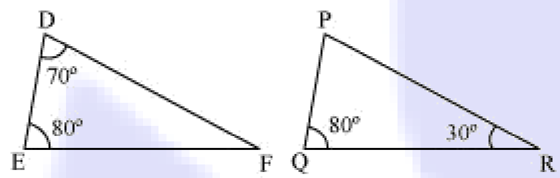


(vi)



Sol:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

(ii) $\therefore \Delta ABC \sim \Delta QRP$ [By SSS Similarity criterion]

(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) The given triangles are not similar as the corresponding sides are not proportional.

(v) The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180°$ (Sum of the measures of the angles of a triangle is 180º.)

$70° + 80° + \angle F = 180°$ $\angle F = 30°$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80° + 30° = 180°$

$\angle P = 70°$

In ΔDEF and ΔPQR,

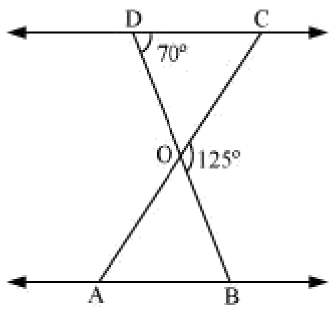
$\angle D = \angle P$ (Each 70°)

$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

$\therefore \Delta DEF \sim \Delta PQR$ [By AAA similarity criterion]

Q.2 In the following figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol: DOB is a straight line.

$\therefore \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$ (Sum of the measures of the angles of a triangle is 180º.)

$\Rightarrow \angle DCO + 70º + 55º = 180°$

$\Rightarrow \angle DCO = 55°$

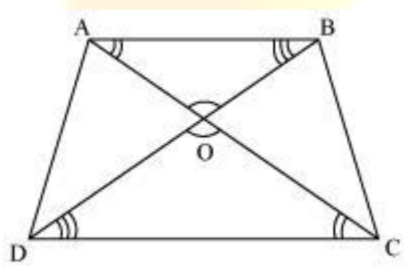
It is given that $\Delta ODC \sim \Delta OBA$.

$\therefore \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

$\Rightarrow \angle OAB = 55°$

Q.3 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Sol:



In ΔDOC and ΔBOA,

$\angle CDO = \angle ABO$ [Alternate interior angles as AB || CD]

$\angle DCO = \angle BAO$ [Alternate interior angles as AB || CD]

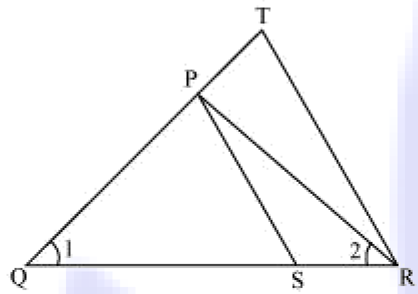
$\angle DOC = \angle BOA$ [Vertically opposite angles]

$\therefore \Delta DOC \sim \Delta BOA$ [AAA similarity criterion]

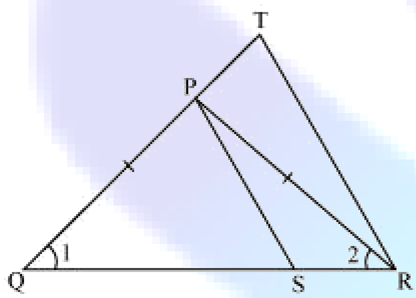
$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

$$\frac{OA}{OC} = \frac{OB}{OD}$$

Q.4 In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ Show that $\Delta PQS \sim \Delta TQR$.



Sol:



In ΔPQR,

$\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (I), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad \text{(ii)}$$

In ΔPQS and ΔTQR,

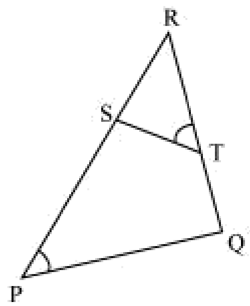
$$\frac{QR}{QS} = \frac{QT}{QP} \quad \text{[Using (ii)]}$$

$\angle Q = \angle Q$

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity criterion]

Q.5 S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Sol:



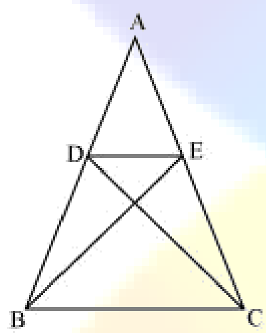
In ΔRPQ and ΔRST,

∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Q.6 In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Sol: It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

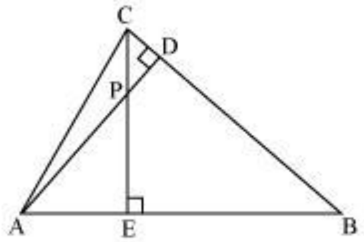
$$\frac{AD}{AB} = \frac{AE}{AC}$$

[Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Q.7 In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



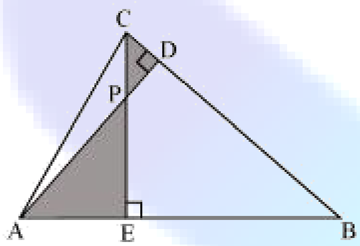
(i) ΔAEP ~ ΔCDP

(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(iv) ΔPDC ~ ΔBEC

Sol:



(i)

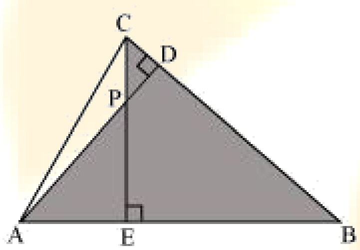
In ΔAEP and ΔCDP,

∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP



(ii)

In ΔABD and ΔCBE,

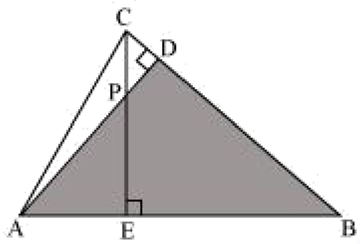
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

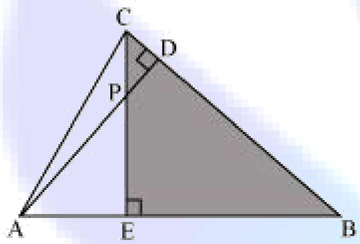
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)

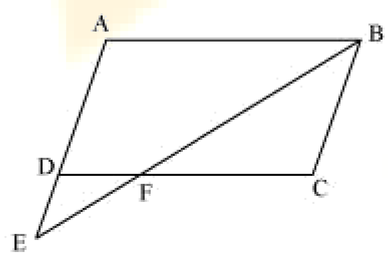
∠PCD = ∠BCE (Common angle)

Hence, by using AA similarity criterion,

ΔPDC ~ ΔBEC

Q.8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB

Sol:



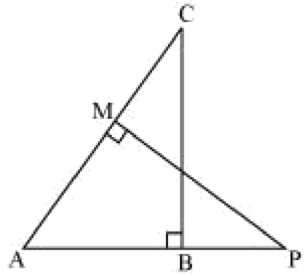
In ΔABE and ΔCFB,

∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

Q.9 In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:

In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle A = \angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AMP$ (By AA similarity criterion)

$\frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

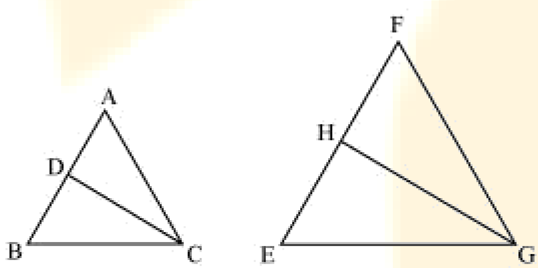
Q.10 CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Sol:



It is given that $\Delta ABC \sim \Delta FEG$.

$\therefore \angle A = \angle F$, $\angle B = \angle E$, and $\angle ACB = \angle FGE$

$\angle ACB = \angle FGE$

$\therefore \angle ACD = \angle FGH$ (Angle bisector)

And, $\angle DCB = \angle HGE$ (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$$\frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

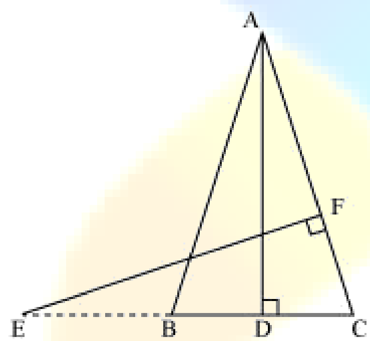
In ΔDCA and ΔHGF,

∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Q.11 In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Sol:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

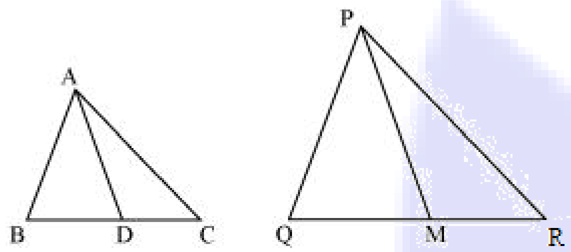
∠ADB = ∠EFC (Each 90°)

∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Q.12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that $\Delta ABC \sim \Delta PQR$.

Sol:



Median divides the opposite side.

$BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

$\therefore \Delta ABD \sim \Delta PQM$ (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

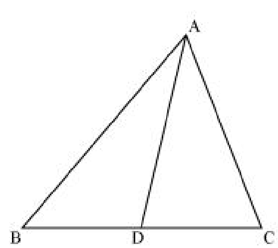
$\angle ABD = \angle PQM$ (Proved above)

$\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)

Q.13 D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

Sol:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

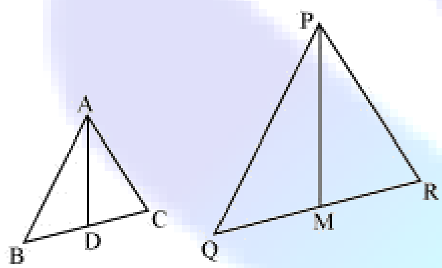
We know that corresponding sides of similar triangles are in proportion.

$$\frac{CA}{CB} = \frac{CD}{CA}$$

$$CA^2 = CB \times CD$$

Q.14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC~ΔPQR.

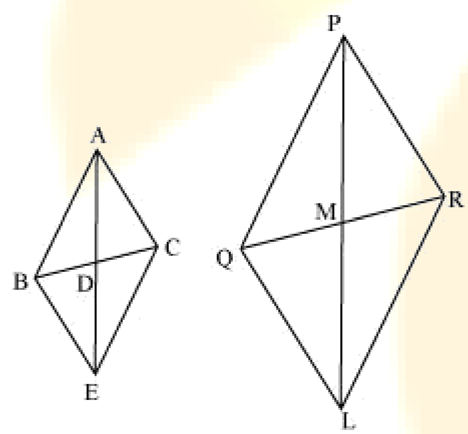
Sol:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

∴ $\Delta ABE \sim \Delta PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ $\angle BAE = \angle QPL$ ... (1)

Similarly, it can be proved that $\Delta AEC \sim \Delta PLR$ and

$\angle CAE = \angle RPL$ ... (2)

Adding equation (1) and (2), we obtain

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\Rightarrow \angle CAB = \angle RPQ$ ... (3)

In ΔABC and ΔPQR,

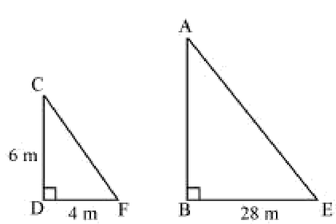
$$\frac{AB}{PQ} = \frac{AC}{PR}$$

$\angle CAB = \angle RPQ$ [Using equation (3)]

∴ $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)

Q.15 A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \Delta ABE \sim \Delta CDF$ (AAA similarity criterion)

$$\frac{AB}{CD} = \frac{BE}{DF}$$

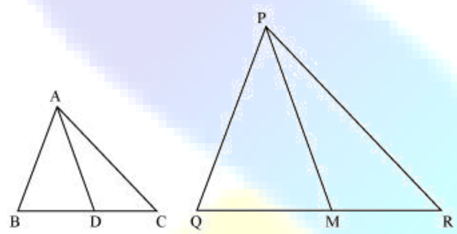
$$\frac{AB}{6\text{ m}} = \frac{28}{4}$$

$AB = 42$ m

Therefore, the height of the tower will be 42 metres.

Q.16 If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Sol:



It is given that $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \ldots (1)$$

Also, $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ... (2)

Since AD and PM are medians, they will divide their opposite sides.

$$BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \quad \ldots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \ldots (4)$$

In ΔABD and ΔPQM,

$\angle B = \angle Q$ [Using equation (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using equation (4)]

$\therefore \Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$