SpeedLabs

MATHS

CBSE 10th

TEEVRA EDUTECH PVT. LTD.

Q.1 Let ΔABC~ΔDEF and their areas be, respectively, $64\ cm^2$ and $121\ cm^2$. If EF = 15.4 cm, find BC.

Sol: It is given that ΔABC~ΔDEF

$$\therefore \frac{ar(\Delta\ ABC)}{ar(\Delta\ DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

EF = 15.4 cm,

$ar(\Delta\ ABC) = 64\ cm^2$

$ar(\Delta\ DEF) = 121\ cm^2$

$$\therefore \frac{ar(\Delta\ ABC)}{ar(\Delta\ DEF)} = \left(\frac{BC}{EF}\right)^2$$

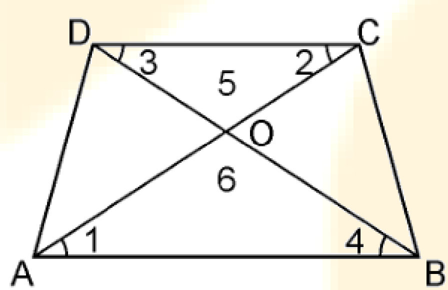
$$\left(\frac{64\ cm^2}{121\ cm^2}\right) = \frac{BC^2}{(15.4\ cm)^2}$$

$$\frac{BC}{15.4} = \left(\frac{8}{11}\right) cm$$

$$BC = \left(\frac{8 \times 15.4}{11}\right) cm = (8 \times 1.4) cm = 11.2\ cm$$

Q.2 Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Sol:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

$\therefore \Delta AOB \sim \Delta COD$ (By AAA similarity criterion)

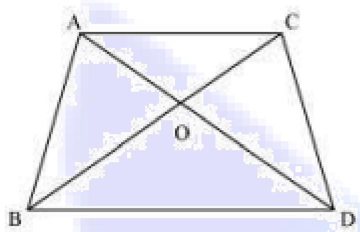
$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$

Since AB = 2 CD,

$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\ CD}{CD}\right)^2 = \frac{4}{1} = 4:1$

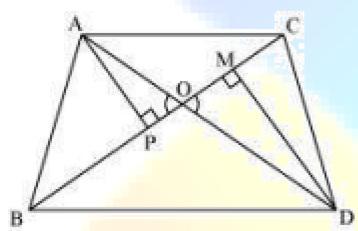
Q.3 In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O,

Show that $\frac{area\ (\Delta\ ABC)}{area(\Delta\ DBC)} = \frac{AO}{DO}$



Sol:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle =

$\frac{1}{2} \times \text{Base} \times \text{Height}$

$\therefore \frac{area\ (\Delta\ ABC)}{area(\Delta\ DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$

In ΔAPO and ΔDMO,

$\angle APO = \angle DMO$ (Each = 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\therefore \Delta APO \sim \Delta DMO$ (By AA similarity criterion)

$\frac{AP}{DM} = \frac{AO}{DO}$

$\frac{area\ (\Delta\ ABC)}{area(\Delta\ DBC)} = \frac{AO}{DO}$

Q.4 If the areas of two similar triangles are equal, prove that they are congruent.

Sol: Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$.

$$\frac{\text{area}\ (\Delta\ ABC)}{\text{area}(\Delta\ PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta\ ABC) =$ ar$(\Delta\ PQR)$

$$\frac{\text{area}\ (\Delta\ ABC)}{\text{area}(\Delta\ PQR)} = 1$$

Putting this value in eq. 1, we obtain

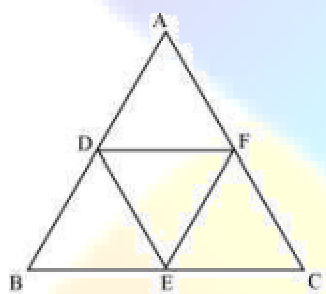
$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$AB = PQ, BC = QR,$ and $AC = PR$

$\Delta\ ABC \cong \Delta\ PQR$

Q.5 D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Sol:



D and E are the mid-points of ΔABC.

$\therefore$ DE||AC and $DE = \frac{1}{2} AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$

$\angle BDE = \angle BAC$

$\angle EBD = \angle CBA$

$\therefore \Delta BED \sim \Delta BCA$

$$\frac{\text{ar}\ (\Delta BED)}{\text{ar}\ (\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\frac{\text{ar}\ (\Delta BED)}{\text{ar}\ (\Delta BCA)} = \frac{1}{4}$$

$$\text{ar}\ (\Delta BED) = \frac{1}{4}\ \text{ar}\ (\Delta BCA)$$

Similarly, $\text{ar}(\Delta CFE) = \frac{1}{4}\text{ar}(\Delta CBA)$ and $\text{ar}(\Delta ADF) = \frac{1}{4}\text{ar}(\Delta ABC)$

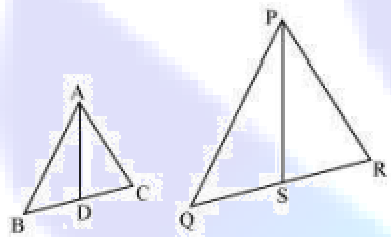
Also, $\text{ar}(\Delta DEF) = \text{ar}(\Delta ABC) - [\text{ar}(\Delta BED) + \text{ar}(\Delta CFE) + \text{ar}(\Delta ADF)]$

$\text{ar}(\Delta DEF) = \text{ar}(\Delta ABC) - \frac{3}{4}\text{ar}(\Delta ABC) = \frac{1}{4}\text{ar}(\Delta ABC)$

$$\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta DEF)} = \frac{1}{4}$$

Q.6 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol:



Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$. Let AD and PS be the medians of these triangles.

$\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \qquad (1)$$

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ... (2)

Since AD and PS are medians,

$\therefore\ BD = DC = \frac{BC}{2}$

And, $QS = SR = \frac{QR}{2}$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \qquad (3)$$

In ΔABD and ΔPQS,

$\angle B = \angle Q$ [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

$\therefore \Delta ABD \sim \Delta PQS$ (SAS similarity criterion)

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \qquad \ldots (4)$$

$$\frac{\text{area }(\Delta\text{ ABC})}{\text{area}(\Delta\text{ PQR})} = \left(\frac{\text{AB}}{\text{PQ}}\right)^2 = \left(\frac{\text{BC}}{\text{QR}}\right)^2 = \left(\frac{\text{AC}}{\text{PR}}\right)^2$$

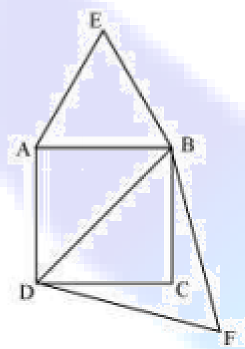
From equations (1) and (4), we may find that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} = \frac{AD}{PS}$ And hence,

$$\frac{\text{area }(\Delta\text{ ABC})}{\text{area}(\Delta\text{ PQR})} = \left(\frac{\text{AD}}{\text{PS}}\right)^2$$

Q.7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Sol:



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = a

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{area }(\Delta\text{ ABE})}{\text{area}(\Delta\text{ DBF})} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Q.8 ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

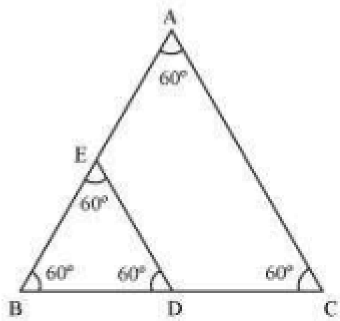
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Sol:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of $\Delta ABC = x$

Therefore, side of $\Delta ABD = \frac{x}{2}$

$$\frac{\text{area } (\Delta\ ABC)}{\text{area}(\Delta\ BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Q.9 Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Sol: If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

therefore, ratio between area of these triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).