SpeedLabs

MATHS

CBSE 10^{th}

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Q.1 Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Sol:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

$49 + 576 = 625$

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, $9 + 36 \neq 64$

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii) Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, $2500 + 6400 \neq 10000$

Or, $502^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv) Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, $144 + 25 = 169$

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

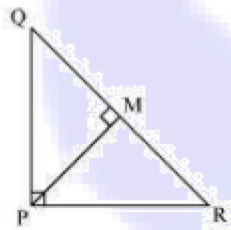
Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Q.2 PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM2 = QM \times MR$.

Sol:



Let $\angle MPR = x$

In $\Delta MPR, \angle MRP = 180° - 90° - x$

$\angle MRP = 90° - x$

Similarly, in ΔMPQ,

$\angle MPQ = 90° - \angle MPR$

$= 90° - x$

$\angle MQP = 180° - 90° - (90° - x)$

$\angle MQP = x$

In ΔQMP and ΔPMR,

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

$\therefore \Delta QMP \sim \Delta PMR$ (By AAA similarity criterion)

$$\frac{QM}{PM} = \frac{MP}{MR}$$

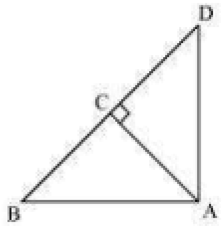
$PM^2 = QM \times MR$

Q.3 In the following figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC2^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Sol:

In ΔADB and ΔCAB

$\angle DAB = \angle ACB$ (Each 90°)

$\angle ADB = \angle CBA$ (Common angle)

$\Delta ADB \sim \Delta CAB$ (AAA similarity criterion)

$\frac{AB}{CB} = \frac{BD}{AB}$

$AB^2 = CB \times BD$

(ii) Let $\angle CBA = x$,

$\angle CBA = 180° - 90° - x$

$\angle CBA = 90° - x$

Similarly, in ΔCAD,

$\angle CAD = 90° - \angle CAB$

$= 90° - x$

$\angle CDA = 180° - 90° - (90° - x)$

$\angle CDA = x$

In ΔCBA and ΔCAD,

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA$ (Each 90°)

$\Delta CBA \sim \Delta CAD$ (AAA rule)

$AC^2 = DC \times BC$

In ΔDCA and ΔDAB,

$\angle DCA = \angle DAB$

$\angle CDA = \angle ADB$

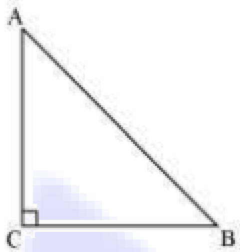
$\Delta DCA \sim \Delta DAB$ (AA Similarity criterion)

$\frac{DC}{DA} = \frac{DA}{DB}$

$AD^2 = BD \times CD$

Q.4 ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2AC^2$. .

Sol:



Given that ΔABC is an isosceles triangle.

$\therefore AC = CB$

Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

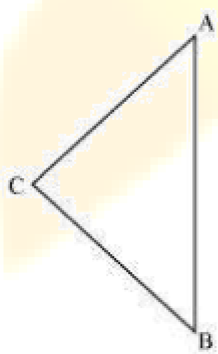
$AC^2 + CB^2 = AB^2$

$AC^2 + AC^2 = AB^2$ (AC = CB)

$2AC^2 = AB^2$

Q.5 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Sol:



Given that,

$AB^2 = 2\ AC^2$

$AB^2 = AC^2 + AC^2$

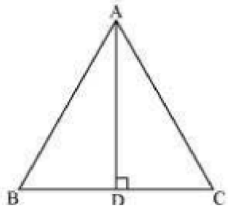
$AB^2 = AC^2 + BC^2$ (As AC = BC)

The triangle is satisfying the Pythagoras theorem.

Therefore, the given triangle is a right – angled triangle.

Q.6 ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Sol:



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

$\therefore BD = DC = a$

In ΔADB,

$\angle ADB = 90°$

Applying Pythagoras theorem, we obtain

$AD^2 + DB^2 = AB^2$

$AD^2 + a^2 = (2a)^2$

$AD^2 + a^2 = 4a^2$

$AD^2 = 3a^2$

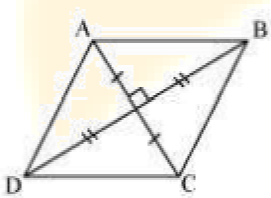
$AD = a\sqrt{3}$

In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Q.7 Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Sol:



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ ...1

$BC^2 = BO^2 + OC^2$ ...2

$CD^2 = CO^2 + OD^2$ ...3

$AD^2 = AO^2 + OD^2$ ...4

Adding all these equations, we obtain

$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$

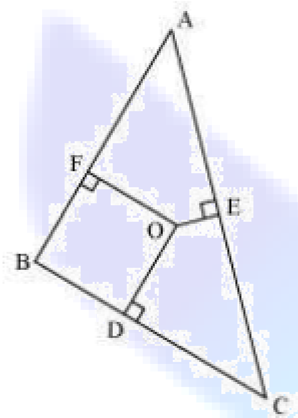
$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right) =$$

$$(AC)^2 + (BD)^2$$

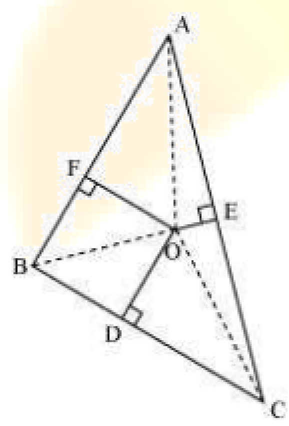
Q.8 In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that



(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Sol: Join OA, OB, and OC.



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$OA^2 = OF^2 + AF^2$

Similarly, in ΔBOD,

$OB^2 = OD^2 + BD^2$

Similarly, in ΔCOE,

$OC^2 = OE^2 + EC^2$

Adding these equations

$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$

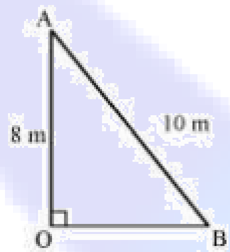
(ii) From the above result,

$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$

$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$

Q.9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol:



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$AB^2 = OA^2 + BO^2$

$(10\text{ m})^2 = (8\text{ m})^2 + OB^2$

$100\text{ m}^2 = 64\text{ m}^2 + OB^2$

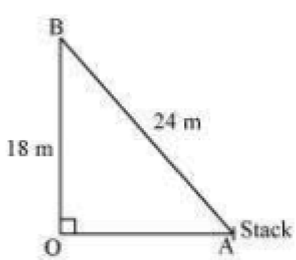
$OB^2 = 36\text{ m}^2$

$OB = 6\text{ m}$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Q.10 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

$AB^2 = OB^2 + OA^2$

$(24\text{ m})^2 = (18\text{ m})^2 + OA^2$

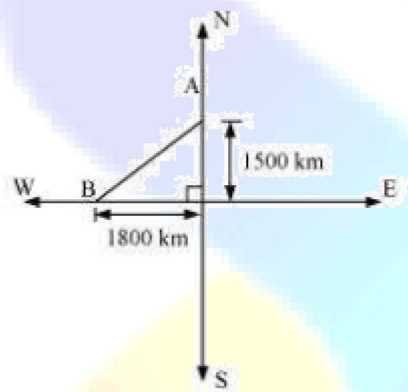
$OA^2 = (576 - 324)\text{m}^2 = 252\text{ m}^2$

$OA = \sqrt{252}\text{ m} = \sqrt{6 \times 6 \times 7}\text{ m} = 6\sqrt{7}\text{ m}$

Therefore, the distance from the base is $6\sqrt{7}$m

Q.11 An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Sol:



Distance travelled by the plane flying towards north in $1\frac{1}{2}$ hours

$= 1{,}000 \times 1\frac{1}{2} = 1{,}500\text{ km}$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hours

$= 1{,}200 \times 1\frac{1}{2} = 1{,}800\text{ km}$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}$ hours

$AB = \sqrt{OA^2 + OB^2}$

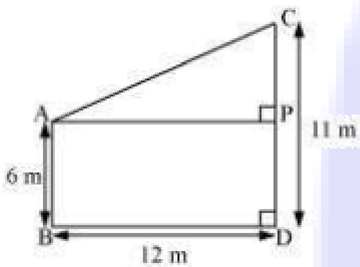
$= \left(\sqrt{(1{,}500)^2 + (1{,}800)^2}\right)\text{ km} = \left(\sqrt{2250000 + 3240000}\right)\text{km}$

$= \left(\sqrt{5490000}\right)\text{km} = \left(\sqrt{9 \times 610000}\right)\text{km} = 300\sqrt{61}\text{ km}$

Therefore, the distance between these planes will be $300\sqrt{61}$km after $1\frac{1}{2}$ hours

Q.12 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, $CP = 11 - 6 = 5$ m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain

$AP^2 + PC^2 = AC^2$

$(12 \text{ m})^2 + (5 \text{ m})^2 = AC^2$

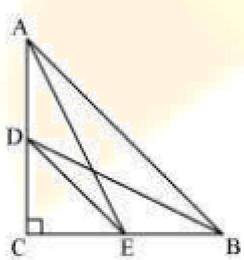
$AC^2 = (144 + 25)\text{m}^2 = 169 \text{ m}^2$

$AC = 13$ m

Therefore, the distance between their tops is 13 m.

Q.13 D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol:



Applying Pythagoras theorem in ΔACE, we obtain

$AC^2 + CE^2 = AE^2$ ...(1)

Applying Pythagoras theorem in ΔBCD, we obtain

$BC^2 + CD^2 = BD^2$ ...(2)

Using (1) and (2), we obtain

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ ...(3)

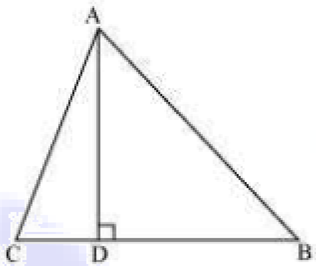
Applying Pythagoras theorem in ΔABC, we obtain

$AB^2 = AC^2 + CB^2$

Putting the values in eq. (3), we obtain

$DE^2 + AB^2 = AE^2 + BD^2$

Q.14 The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that $2\,AB^2 = 2\,AC^2 + BC^2$.



Sol: Applying Pythagoras theorem for ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

$AD^2 = AC^2 - DC^2$ (1)

Applying Pythagoras theorem in ΔABD, we obtain

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2$ (2)

From eq.(1) and (2), we obtain,

$AC^2 - DC^2 = AB^2 - DB^2$ (3)

It is given that 3DC = DB

$\therefore DC = \frac{BC}{4}$ and $DB = \frac{3BC}{4}$

Putting these values in eq. (3), we obtain

$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$

$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$

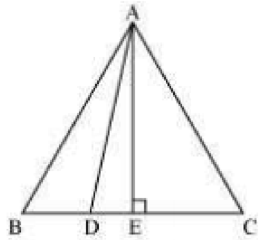
$16\,AC^2 - BC^2 = 16AB^2 - 9BC^2$

$16AB^2 - 16AC^2 = 8BC^2$

$2AB^2 = 2AC^2 + BC^2$

Q.15 In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Sol:



Let the side of the equilateral triangle be a, and AE be the altitude of ΔABC.

$\therefore\ BE = EC = \frac{BC}{2} = \frac{a}{2}$

And, $AE = \frac{a\sqrt{3}}{2}$

Given that, $BD = \frac{1}{3}BC$

$\therefore\ BD = \frac{a}{3}$

$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2\ AD^2$

$= \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$

$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$

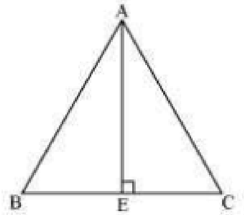
$= \frac{28\ a^2}{36}$

$= \frac{7}{9}\ AB^2$

$9AD^2 = 7AB^2$

Q.16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Sol:



Let the side of the equilateral triangle be a, and AE be the altitude of ΔABC.

$\therefore\ BE = EC = \frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - \frac{a^2}{4}$

$AE^2 = \frac{3a^2}{4}\ 4$

$AE^2 = 3a^2$

⇒ 4 × (Square of altitude) = 3 × (Square of one side)

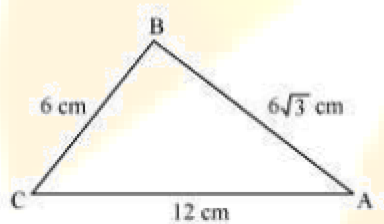
Q.17 Tick the correct answer and justify: In ΔABC, AB $=6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Sol:



Given that, AB $=6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$ And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

$\therefore \angle B = 90°$

Hence, the correct answer is (C).