SpeedLabs

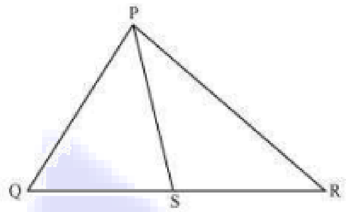
MATHS

CBSE 10th

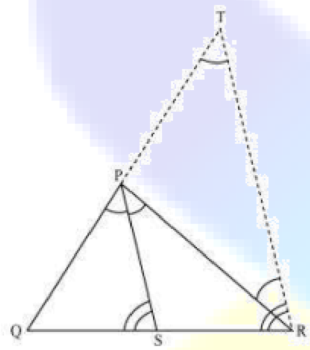
TEEVRA EDUTECH PVT. LTD.

Q.1 In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that

$$\frac{QS}{SR} = \frac{PQ}{PR}$$



Sol::



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR ... (1)

By construction, ∠SPR = ∠PRT (As PS || TR) ... (2)

∠QPS = ∠QTR (As PS || TR) ... (3)

Using these equations, we obtain

∠PRT = ∠QTR

∴ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

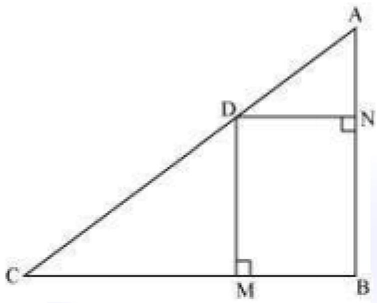
$$\frac{QS}{SR} = \frac{QP}{PT}$$

$$\frac{QS}{SR} = \frac{PQ}{PR} \qquad (PT = TR)$$

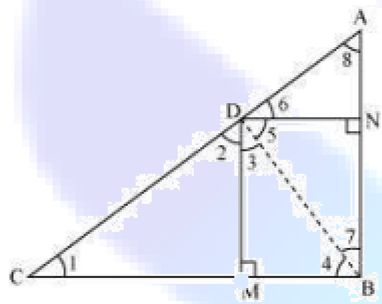
Q.2 In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$



Sol: (i)Let us join DB.



We have, DN || CB, DM || AB, and ∠B = 90°

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° ... (1)

In ΔCDM, ∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° ... (2)

In ΔDMB, ∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° ... (3)

From equation (1) and (2), we obtain

∠1 = ∠3

From equation (1) and (3), we obtain

∠2 = ∠4

In ΔDCM and ΔBDM,

∠1 = ∠3 (Proved above)

∠2 = ∠4 (Proved above)

∴ ΔDCM ~ ΔBDM (AA similarity criterion)

$\frac{BM}{DM} = \frac{DM}{MC}$

$\frac{DN}{DM} = \frac{DM}{MC}$ (BM = DN)

(Iii)In right triangle DBN,

$\angle 5 + \angle 7 = 90°$ ... (4)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ ... (5)

D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle ADB = 90° \Rightarrow \angle 5 + \angle 6 = 90°$ ... (6)

From equation (4) and (6), we obtain

$\angle 6 = \angle 7$

From equation (5) and (6), we obtain

$\angle 8 = \angle 5$

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ (Proved above)

$\angle 8 = \angle 5$ (Proved above)

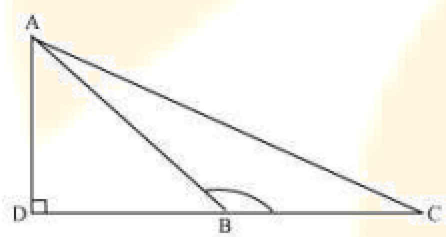
$\therefore \Delta DNA \sim \Delta BND$ (AA similarity criterion)

$\frac{AN}{DN} = \frac{DN}{NB}$

$\Rightarrow DN^2 = AN \times NB \Rightarrow DN^2 = AN \times DM$ (As NB = DM)

Q.3 In the given figure, ABC is a triangle in which $\angle ABC > 90°$ and $AD \perp CB$ produced.

Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Sol:

Applying Pythagoras theorem in ΔADB, we obtain

$AB2 = AD2 + DB2$ ... (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

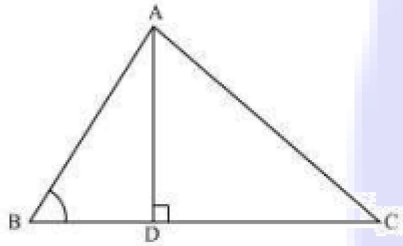
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$$AC^2 = AB^2 + BC^2 + 2DB \times BC \qquad \text{[Using equation (1)]}$$

Q.4 In the given figure, ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$.

Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Sol:

Applying Pythagoras theorem in ΔADB, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \qquad \ldots (1)$$

Applying Pythagoras theorem in ΔADC, we obtain

$$AD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2 \qquad \text{[Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

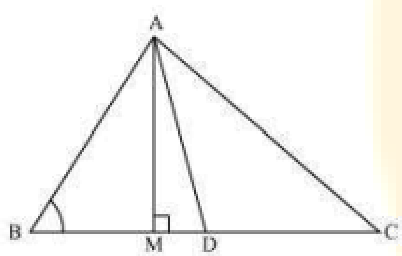
$$= AB^2 + BC^2 - 2BC \times BD$$

Q.5 In the given figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Sol:

(i) Applying Pythagoras theorem in ΔAMD, we obtain

AM2 + MD2 = AD2 ... (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC2^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD = AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii) Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ ... (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ ... (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB' + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

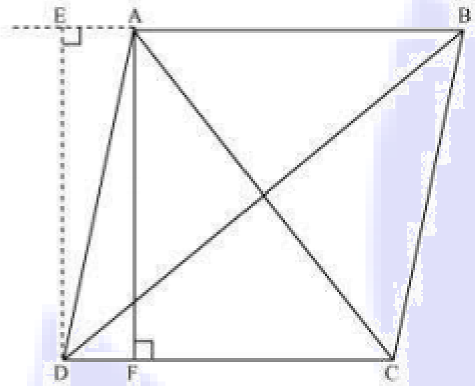
$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$2(AM^2 + MD^3) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Q.6 Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Sol:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in ΔDEA, we obtain

$DE^2 + EA^2 = DA^2$ ... (i)

Applying Pythagoras theorem in ΔDEB, we obtain

$DE^2 + EB^2 = DB^2$

$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$

$DA^2 + AB^2 + 2EA \times AB = DB^2$ ... (ii)

Applying Pythagoras theorem in ΔADF, we obtain

$AD2^2 = AF^2 + FD^2$

Applying Pythagoras theorem in ΔAFC, we obtain

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD2 - 2DC \times FD$

$= (AF^2 + FD^2) + DC^2 - 2DC \times FD\ AC^2$

$= AD^2 + DC^2 - 2DC \times FD$ ... (iii)

Since ABCD is a parallelogram,

AB = CD ... (iv)

And, BC = AD ... (v)

In ΔDEA and ΔADF,

∠DEA = ∠AFD (Both 90°)

∠EAD = ∠ADF (EA || DF)

AD = AD (Common)

$\therefore \Delta EAD\ \Delta FDA$ (AAS congruence criterion)

$\Rightarrow EA = DF$ ... (vi)

Adding equations (i) and (iii), we obtain

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$

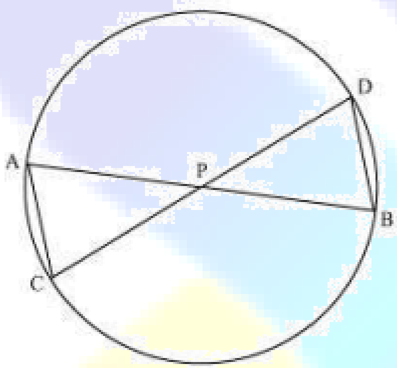
[Using equations (iv) and (vi)]

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

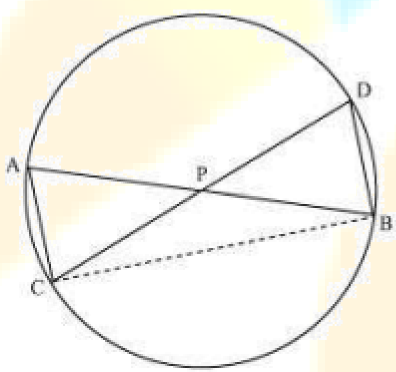
Q.7 In the given figure, two chords AB and CD intersect each other at the point P. prove that:

(i) $\Delta APC \sim \Delta DPB$

(ii) AP.BP = CP.DP



Sol: Let us join CB.



(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ (Vertically opposite angles)

$\angle CAP = \angle BDP$ (Angles in the same segment for chord CB)

$\Delta APC \sim \Delta DPB$ (By AA similarity criterion)

(ii) We have already proved that

$\Delta APC \sim \Delta DPB$

We know that the corresponding sides of similar triangles are proportional.

$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$

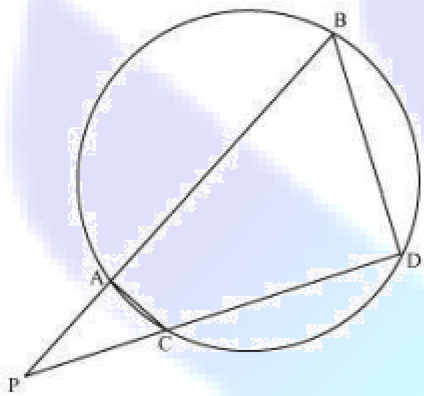
$\frac{AP}{DP} = \frac{PC}{PB}$

$\therefore AP.PB = PC.DP$

Q.8 In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$

(ii) PA.PB = PC.PD



Sol:

(i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ (Common)

$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is $\angle PCA = \angle PBD$ equal to the opposite interior angle)

$\therefore \Delta PAC \sim \Delta PDB$

(ii)We know that the corresponding sides of similar triangles are proportional.

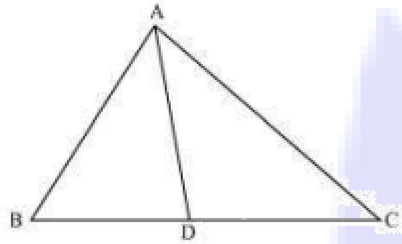
$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$

$\frac{PA}{PD} = \frac{PC}{PB}$

$PA.PB = PC.PD$

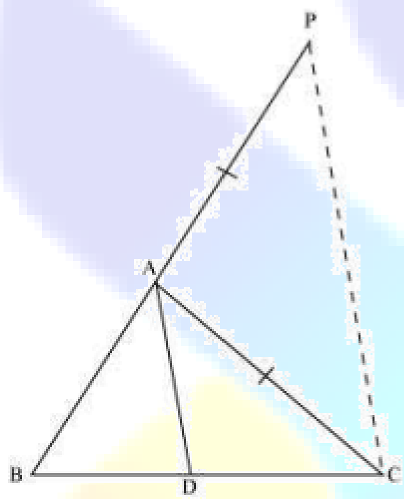
Q.9 In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$.

Prove that AD is the bisector of ∠BAC.



Sol:

Let us extend BA to P such that AP = AC. Join PC.



It is given that,

$$\frac{BD}{CD} = \frac{AB}{AC}$$

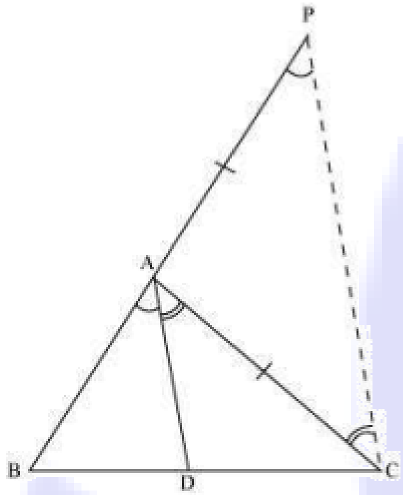
$$\frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

AD || PC

⇒ ∠BAD = ∠APC (Corresponding angles) … (1)

And, ∠DAC = ∠ACP (Alternate interior angles) … (2)



By construction, we have

AP = AC

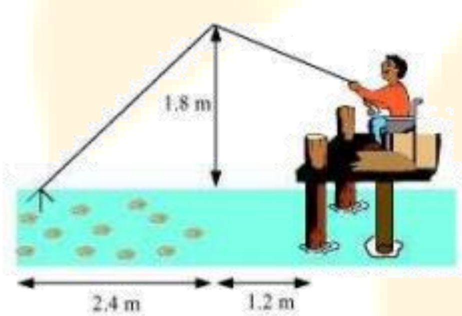
⇒ ∠APC = ∠ACP ... (3)

On comparing equations (1), (2), and (3), we obtain

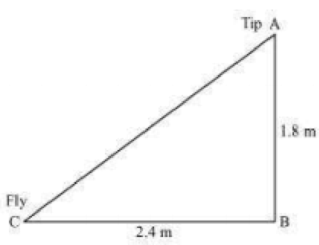
∠BAD = ∠APC

⇒ AD is the bisector of the angle BAC.

Q.10 Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, ho much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Sol:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC.

$AC^2 = AB^2 + BC^2$

$AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$

$AB^2 = (3.24 + 5.76)\text{m}^2$

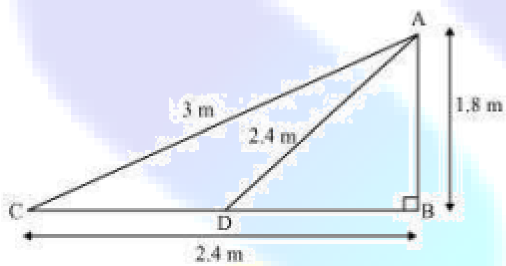
$AB^2 = 9.00 \text{ m}^2$

$AB = \sqrt{9} = 3 \text{ m}$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

AD = AC − String pulled by Nazima in 12 seconds

= (3.00 − 0.6) m

= 2.4 m

In ΔADB,

$AB^2 + BD^2 = AD^2$

$(1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$

$BD^2 = (5.76 - 3.24)\text{m}^2 = 2.52 \text{ m}^2$

BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m

= (1.587 + 1.2) m

= 2.787 m

= 2.79 m