SpeedLabs

MATHS

CBSE 9th

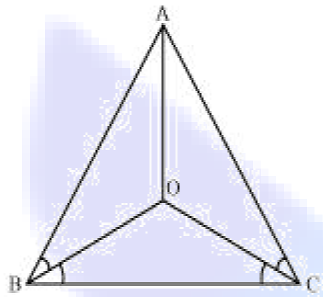
TEEVRA EDUTECH PVT. LTD.

Q.1 In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that:

(i) OB = OC

(ii) AO bisects ∠ A.

Ans. **(i)** ABC is an isosceles triangle in which AB = AC.



∴ ∠ C = ∠ B [Angles opposite to equal sides]

⇒ ∠ OCA + ∠ OCB = ∠ OBA + ∠ OBC

∴ OB bisects ∠ B and OC bisects ∠ C

∴ ∠ OBA = ∠ OBC and ∠ OCA = ∠OCB

⇒ ∠ OCB + ∠ OCB = ∠ OBC + ∠ OBC

⇒ 2 ∠ OCB = 2 ∠ OBC

⇒ ∠ OCB = ∠ OBC

Now in OBC,

∠ OCB = ∠ OBC [Prove above]

∴ OB = OC [Sides opposite to equal sides]

(ii) In Δ AOB and Δ AOC,

AB = AC [Given]

∠ OBA = ∠ OCA [Given]

And ∠ B = ∠ C

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

∠ OBA = ∠OCA

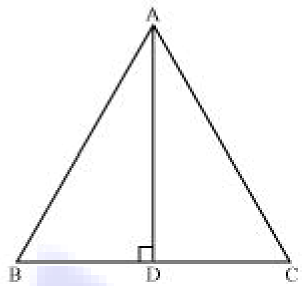
⇒ OB = OC [Prove above]

∴ Δ AOB ≅ Δ AOC [By SAS congruency]

⇒ ∠ OAB = ∠ OAC [By C.P.C.T.]

Hence AO bisects ∠ A.

Q.2 In Δ ABC, AD is the perpendicular bisector of BC (See figure). Show that Δ ABC is an isosceles triangle in which AB = AC.



Ans. In Δ AOB and Δ AOC,

BD = CD [AD bisects BC]

∠ ADB = ∠ ADC = 90∘ [AD ⊥ BC]

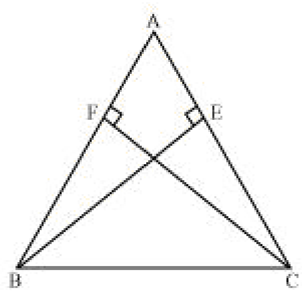
AD = AD [Common]

∴ Δ ABD ≅ Δ ACD [By SAS congruency]

⇒ AB = AC [By C.P.C.T.]

Therefore, ABC is an isosceles triangle.

Q.3 ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal.



Ans. In Δ ABE and Δ ACF,

∠ A = ∠ A [Common]

∠ AEB = ∠ AFC = 90° [Given]

AB = AC [Given]

∴ Δ ABE ≅ Δ ACF [By ASA congruency]

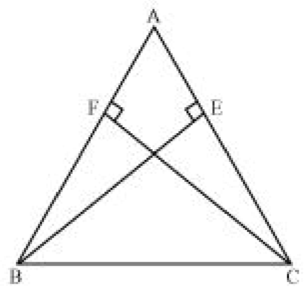
⇒ BE = CF [By C.P.C.T.]

⇒ Altitudes are equal.

Q.4 ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:

(i) Δ ABE ≅ Δ ACF

(ii) AB = AC or Δ ABC is an isosceles triangle.



Ans. **(i)** In Δ ABE and Δ ACF,

∠ A = ∠ A [Common]

∠ AEB = ∠ AFC = 90° [Given]

BE = CF [Given]

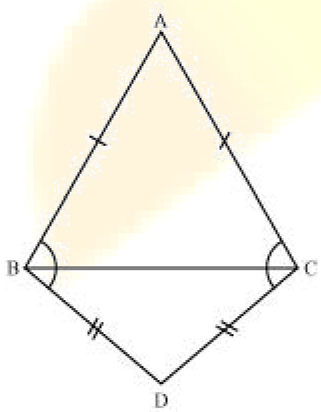
∴ Δ ABE ≅ Δ ACF [By ASA congruency]

(ii) Since ABE ACF

⇒ BE = CF [By C.P.C.T.]

⇒ ABC is an isosceles triangle.

Q.5 ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that ∠ ABD = ∠ ACD.



Ans. In isosceles triangle ABC,

AB = AC [Given]

∠ ACB = ∠ ABC ……. (i) [Angles opposite to equal sides]

Also, in Isosceles triangle BCD.

BD = DC

∴ ∠ BCD = ∠ CBD ………. (ii) [Angles opposite to equal sides]

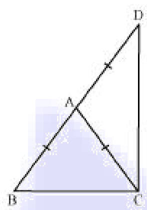
Adding eq. (i) and (ii),

$\angle ACB + \angle BCD = \angle ABC + \angle CBD$

$\therefore\ \angle ACD = \angle ABD$

Or $\angle ABD = \angle ACD$

Q.6 ABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle BCD$ is a right angle (See figure).



Ans. In isosceles triangle ABC,

AB = AC [Given]

$\angle ACB = \angle ABC$ ……. (i) [Angles opposite to equal sides]

Now AD = AB [By construction]

But AB = AC [Given]

$\therefore$ AD = AB = AC

$\Rightarrow$ AD = AC

Now in triangle ADC,

AD = AC

$\Rightarrow$ ADC = ACD ………(ii) [Angles opposite to equal sides]

Since $\angle BAC + \angle CAD = 180°$………(iii) [Linear pair]

And Exterior angle of a triangle is equal to the sum of its interior opposite angles.

$\therefore$ In ABC,

$\angle CAD = \angle ABC + \angle ACB = \angle ACB + \angle ACB$ [Using (i)]

$\Rightarrow\ \angle CAD = 2\angle ACB$ ……….(iv)

Similarly, for Δ ADC,

$\angle BAC = \angle ACD + \angle ADC = \angle ACD + \angle ACD = 2\angle ACD$ ……….(v)

From eq. (iii), (iv) and (v),

$2\angle ACB + 2\angle ACD = 180°$

$2(\angle ACB + \angle ACD) = 180°$

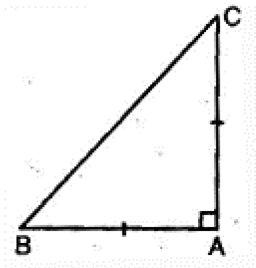
$ACB + ACD = 90°$

$\angle BCD = 90°$

Hence BCD is a right angle.

Q.7 ABC is a right-angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.

Ans. ABC is a right triangle in which,



∠ A = 90° And AB = AC

In Δ ABC,

AB = AC

⇒ ∠ C = ∠ B ....(i)

We know that, in ABC,

∠ A + ∠ B + ∠ C = 180° [Angle sum property]

⇒ 90° + ∠B + ∠B = 180°

[∠ A = 90° + (given) and ∠ B = ∠ C (from eq. (i)]

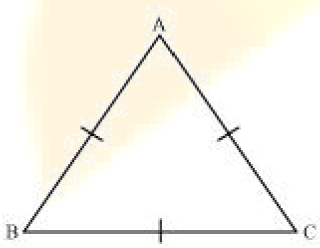
⇒ 2 ∠ B = 90°

⇒ ∠ B = 45°

Also ∠ C = 45° [∠ B = ∠ C]

Q.8 Show that the angles of an equilateral triangle are 60° each.

Ans. Let ABC be an equilateral triangle.



∴ AB = BC = AC

⇒ AB = BC

⇒ C = A .... (i)

Similarly, AB = AC

⇒ ∠ C = ∠ B .... (ii)

From eq. (i) and (ii),

∠ A = ∠ B = ∠ C .... (iii)

Now in Δ ABC

$\angle A + \angle B + \angle C = 180°$ ………. (iv)

$\Rightarrow \angle A + \angle A + \angle A = 180°$

$\Rightarrow 3\angle A = 180°$

$\Rightarrow A = 60°$

Since $\angle A = \angle B = \angle C$ [From eq. (iii)]

$\therefore \angle A = \angle B = \angle C = 60°$

Hence each angle of equilateral triangle is 60°