



SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Trigonometric Functions

Exercise- 3.3

1. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

Ans. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$

$$\text{L.H.S.} = \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

R.H.S

2. Prove that $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

Ans. L.H.S = $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$$

= R.H.S

3. Prove that $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} 3\tan^2 \frac{\pi}{6} = 6$

Ans. L.H.S = $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} 3\tan^2 \frac{\pi}{6}$

$$= (\sqrt{3})^2 + \operatorname{cosec}^2\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

= R.H.S

4. Prove that $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{3} + 2\sec^2 \frac{\pi}{3} = 10$

Ans. L.H.S = $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{3} + 2\sec^2 \frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin \frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

$$=$$

5. Find the value of: (i) $\sin 75^\circ$ (ii) $\tan 15^\circ$

Ans. (i) $\sin 75^\circ = \sin (45^\circ + 30^\circ)$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

[$\sin (x + y) = \sin x \cos y + \cos x \sin y$]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15^\circ = \tan (45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \quad \left[\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}$$

6. Prove that: $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$

Ans. $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$

$$\frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\left[\begin{array}{l} \therefore 2\cos A \cos B = \cos(A + B) + \cos(A - B) \\ -2\sin A \sin B = \cos(A + B) - \cos(A - B) \end{array}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x + y)\right]$$

$$= \sin(x + y)$$

$$= \text{R.H.S}$$

7. Prove that: $\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$

Ans. It is known that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \text{ and } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\left[\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right]}{\left[\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right]} = \left(\frac{1 + \tan x}{1 - \tan x}\right) \left(\frac{1 - \tan x}{1 + \tan x}\right)^2 = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S}$$

$$8. \quad \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2$$

$$\text{Ans. L.H.S.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$\frac{-\cos^2 x}{-\sin^2 x}$$

$$= \cot^2 x$$

$$= \text{R.H.S}$$

$$9. \quad \cos\left(\frac{3\pi}{2} + x\right)\cos(2x + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2x + x)\right] = 1$$

Ans. It is known that

$$\text{L.H.S} = \cos\left(\frac{3\pi}{2} + x\right)\cos(2x + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$$

$$= \sin x \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right]$$

$$= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right]$$

$$= 1 = \text{R.H.S.}$$

10. Prove that $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$.

$$\text{Ans. L.H.S.} = \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x$$

$$= \frac{1}{2} [2\sin(n+1)x \sin(n+2)x + 2\cos(n+1)x \cos(n+2)x]$$

$$= \frac{1}{2} \left[\cos\{(n+1)x - (n+2)x\} - \cos\{(n+1)x + (n+2)x\} \right. \\ \left. + \cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \right]$$

$$\left[\begin{aligned} \therefore -2\sin A \sin B &= \cos(A+B) - \cos(A-B) \\ 2\cos A \cos B &= \cos(A+B) + \cos(A-B) \end{aligned} \right]$$

$$= \frac{1}{2} \times 2\cos\{(n+1)x - (n+2)x\}$$

$$= \cos(-x) = \cos x = \text{R.H.S}$$

11. Prove that $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

Ans. It is known that

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

$$= -2 \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\}$$

$$= -2 \sin\left(\frac{3\pi}{4}\right) \sin x$$

$$= -2 \sin\left(\pi \frac{\pi}{4}\right) \sin x$$

$$= -2 \sin \frac{\pi}{4} \sin x$$

$$= -2 \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\sqrt{2} \sin x$$

$$= \text{R.H.S}$$

11. Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Ans. It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \sin^2 6x - \sin^2 4x$$

$$= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x)$$

$$= \left[2 \sin\left(\frac{6x+4x}{2}\right) \cos\left(\frac{6x-4x}{2}\right)\right] \left[2 \cos\left(\frac{6x+4x}{2}\right) \sin\left(\frac{6x-4x}{2}\right)\right]$$

$$= (2 \sin 5x \cos x) (2 \cos 5x \sin x)$$

$$= (2 \sin 5x \cos 5x) (2 \sin x \cos x)$$

$$= \sin 10x \sin 2x$$

$$= \text{R.H.S.}$$

13. Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Ans. It is known that

$$\cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right), \cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S.} = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[2 \cos \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] \left[-2 \sin \left(\frac{2x+6x}{2} \right) \sin \left(\frac{2x-6x}{2} \right) \right]$$

$$= \left[2 \cos 4x \cos(-2x) \right] \left[-2 \sin 4x \sin(-2x) \right]$$

$$= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$$

$$= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$$

$$= \sin 8x \sin 4x$$

$$= \text{R.H.S.}$$

14. Prove that $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$.

Ans. L.H.S. = $\sin 2x + 2 \sin 4x + \sin 6x$

$$= [\sin 2x + \sin 6x] + 2 \sin 4x$$

$$= \left[2 \sin \left(\frac{2x+6x}{2} \right) \cos \left(\frac{2x-6x}{2} \right) \right] + 2 \sin 4x$$

$$\left[\therefore \sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$= 2 \sin 4x \cos(-2x) + 2 \sin 4x$$

$$= 2 \sin 4x \cos 2x + 2 \sin 4x$$

$$= 2 \sin 4x (\cos 2x + 1)$$

$$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$$

$$= 2 \sin 4x (2 \cos^2 x)$$

$$= 4 \cos^2 x \sin 4x = \text{R.H.S.}$$

15. Prove that $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Ans. L.H.S = $\cot 4x (\sin 5x + \sin 3x)$

$$= \frac{\cot 4x}{\sin 4x} \left[2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\therefore \sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$= 2 \cos 4x \cos x$$

$$\text{R.H.S.} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cot x}{\sin x} \left[2 \cos \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\therefore \sin A + \sin B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$\frac{\cos x}{\sin x} [2 \cos 4x \sin x]$$

$$= 2 \cos 4x \cdot \cos x$$

$$\text{L.H.S.} = \text{R.H.S.}$$

16. Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Ans. It is known that

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S.} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

$$= \frac{-2 \sin \left(\frac{9x+5x}{2} \right) \cdot \sin \left(\frac{9x-5x}{2} \right)}{2 \cos \left(\frac{17x+3x}{2} \right) \cdot \sin \left(\frac{17x-3x}{2} \right)}$$

$$= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= \text{R.H.S.}$$

17. Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Ans. It is known that

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right), \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2 \sin \left(\frac{5x+3x}{2} \right) \cdot \cos \left(\frac{5x-3x}{2} \right)}{2 \cos \left(\frac{5x+3x}{2} \right) \cdot \cos \left(\frac{5x-3x}{2} \right)}$$

$$= \frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x}$$

$$= \tan 4x$$

$$\begin{aligned}
 &= \frac{\sin 4x}{\cos 4x} \\
 &= \tan 4x = \text{R.H.S}
 \end{aligned}$$

18. Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

Ans. It is known that

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= 2 \cos \frac{\left(\frac{x+y}{2} \right) \cdot \sin \left(\frac{x-y}{2} \right)}{2 \cos \left(\frac{x+y}{2} \right) \cdot \cos \left(\frac{x-y}{2} \right)}$$

$$= \frac{\sin \left(\frac{x-y}{2} \right)}{\cos \left(\frac{x-y}{2} \right)}$$

$$= \tan \left(\frac{x-y}{2} \right) = \text{R.H.S.}$$

19. Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Ans. It is known that

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right), \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{2 \sin \left(\frac{x+3x}{2} \right) \cos \left(\frac{x-3x}{2} \right)}{2 \cos \left(\frac{x+3x}{2} \right) \cos \left(\frac{x-3x}{2} \right)}$$

$$\begin{aligned}
 &= \frac{\sin 2x}{\cos 2x} \\
 &= \tan 2x \\
 &= \text{R.H.S}
 \end{aligned}$$

20. Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

Ans. It is known that

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2 \cos \left(\frac{x+3x}{2} \right) \sin \left(\frac{x-3x}{2} \right)}{-\cos 2x}$$

$$= \frac{2 \cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2 \sin x = \text{R.H.S.}$$

21. Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

Ans. It is known that

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2 \cos \left(\frac{x+3x}{2} \right) \sin \left(\frac{x-3x}{2} \right)}{-\cos 2x}$$

$$= \frac{2 \cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

$$= 2 \sin x = \text{R.H.S.}$$

22. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

Ans. L.H.S. = $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$\frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2 \cos\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2 \sin\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right]$$

$$= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x (2 \cos x + 1)}{2 \sin 3x (2 \cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

23. Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Ans. L.H.S. = $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x)$$

$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \right]$$

$$= \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1 = \text{R.H.S.}$$

24. Prove that $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Ans. It is known that

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\therefore \text{L.H.S.} = \tan 4x = \tan 2(2x)$$

$$= \frac{2 \tan 2x}{1 - \tan^2 (2x)}$$

$$= \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$\begin{aligned}
&= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x}\right)}{1 - \left(\frac{4 \tan^2 x}{(1 - \tan^2 x)^2}\right)} \\
&= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x}\right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2}\right]} \\
&= \frac{4 \tan x(1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4 \tan^2 x} \\
&= \frac{4 \tan x(1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x} \\
&= \frac{4 \tan x(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x} = \text{R.H.S.}
\end{aligned}$$

25. Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Ans. L.H.S. = $\cos 4x$

$$\begin{aligned}
&= \cos 2(2x) \\
&= 1 - 2 \sin^2 2x \quad [\cos 2A = 1 - 2 \sin^2 A] \\
&= 1 - 2(2 \sin x \cos x)^2 \quad [\sin 2A = 2 \sin A \cos A] \\
&= 1 - 8 \sin^2 x \cos^2 x \\
&= \text{R.H.S.}
\end{aligned}$$

26. Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Ans. L.H.S. = $\cos 6x$

$$\begin{aligned}
&= \cos 3(2x) \\
&= 4 \cos^3 2x - 3 \cos 2x \quad [\cos^3 A = 4 \cos^3 A - 3 \cos A] \\
&= 4 [(2 \cos^2 x - 1)^3 - 3(2 \cos^2 x - 1)] \quad [\cos 2x = 2 \cos^2 x - 1] \\
&= 4 [(2 \cos^2 x)^3 - (1)^3 - 3(2 \cos^2 x)^2 + 3(2 \cos^2 x)] - 6 \cos^2 x + 3 \\
&= 4 [8 \cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3 \\
&= 32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3 \\
&= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1 \\
&= \text{R.H.S.}
\end{aligned}$$