SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

1. Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Ans. $\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$

Now , $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

2. Find the principal and general solutions of the equation $\sec x = 2$

Ans. $\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are x $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

Now , $\sec x = \sec\frac{\pi}{3}$

$= \cos x = \cos\frac{\pi}{3}$ $\qquad \left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3,}$ where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

3. Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Ans. $\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi\frac{\pi}{6}\right) = -\frac{\pi}{6} = -\cot\frac{\pi}{6} = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e, $\cot\frac{5\pi}{6}=-\sqrt{3}$ and $\cot\frac{11\pi}{6}=-\sqrt{3}$

Therefore, the principal solutions are $x=\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Now , $\cot x=\cot\frac{5\pi}{6}$

$\Rightarrow \tan x=\tan\frac{5\pi}{6}$

$\Rightarrow x=n\pi+\frac{5\pi}{6}$, where $n\in Z$

Therefore, the general solution is $x=n\pi+\frac{5\pi}{6}$, where $n\in Z$

4. Find the general solution of $\operatorname{cosec} x=-2$

Ans. $\operatorname{cosec} x=-2$

It is known that

$\operatorname{cosec}\left(\pi+\frac{\pi}{6}\right)=-\operatorname{cosec}\frac{\pi}{6}=-2$ and $\operatorname{cosec}\left(2\pi-\frac{\pi}{6}\right)=-\operatorname{cosec}\frac{\pi}{6}=-2$

i.e., $\operatorname{cosec}\frac{7\pi}{6}=-2$ and $\operatorname{cosec}\frac{11\pi}{6}=-2$

Therefore, the principal solutions are $x=\frac{7\pi}{6}$

$\Rightarrow \sin x=\sin\frac{7\pi}{6}$ $\left[\operatorname{cosec}x=\frac{1}{\sin x}\right]$

$\Rightarrow x=n\pi+(-1)^n\frac{7\pi}{6}$, where $n\in Z$

Therefore, the general solution is $x=n\pi+(-1)^n\frac{7\pi}{6}$, where $n\in Z$

5. Find the general solution of the equation $\cos 4x=\cos 2x$

Ans. $\cos 4x=\cos 2x$

$\Rightarrow \cos 4x-\cos 2x$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)=0$

$\left[\therefore \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x\sin x=0$

$\Rightarrow \sin 3x = 0$ or $\sin x = 0$

$\Rightarrow \therefore 3x = n\pi$ or $x = n\pi$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

6. Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Ans. $\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \cos 2x = 0$ or $\cos x = \frac{1}{2}$

$\therefore 2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \cos\frac{\pi}{3}$, where $n \in Z$

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

7. Find the general solution of the equation $\sin 2x + \cos x = 0$

Ans. $\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1$

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0$

Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2\sin x + 1 = 0$

$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in Z$

8. Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Ans. $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x\left(\tan 2x + 1\right) = 0$

$\Rightarrow \tan 2x = 0 \text{ or } \tan 2x + 1 = 0$

Now, $\tan 2x = 0$

$\Rightarrow \tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2}$, where $n \in Z$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$

9. Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Ans. $\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow \left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos\left(-2x\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x\cos 2x + \sin 3x = 0$

$\Rightarrow 3x\left(2\cos 2x + 1\right) = 0$

$\Rightarrow \sin 3x = 0 \text{ or } 2\cos 2x + 1 = 0$

Now, $\sin 3x = 0 \Rightarrow n\pi$, where $n \in Z$

i.e, $x = \frac{n\pi}{3}$, where $n \in Z$

$2\cos 2x + 1 = 0$

$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3}, n \in Z$