SpeedLabs

MATHS

CBSE 10th

TEEVRA EDUTECH PVT. LTD.

Q.1 In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Sol: Applying Pythagoras theorem for ΔABC, we obtain

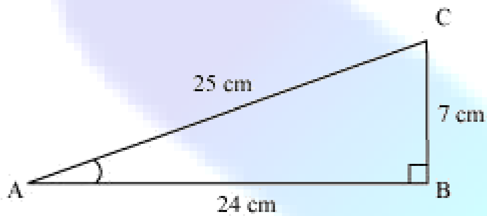
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= (576 + 49)\text{cm}^2$

$= 625\text{ cm}^2$

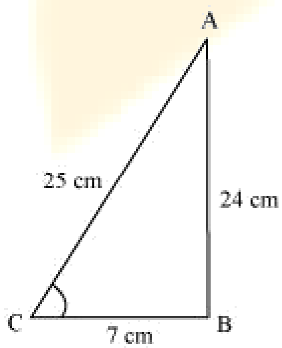
∴ AC = cm = 25 cm



(i) $\sin A = \dfrac{\text{side opposite to } \angle A}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7}{25}$

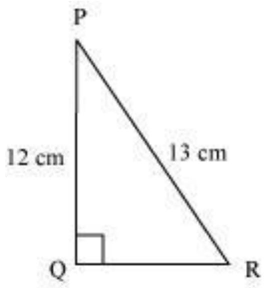
$\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$



(ii) $\sin C = \dfrac{\text{side opposite to } \angle C}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

$\cos C = \dfrac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{7}{25}$

Q.2 In the given figure find $\tan P - \cot R$



Sol:

Applying Pythagoras theorem for ΔPQR, we obtain

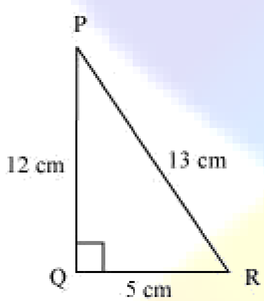
$PR^2 = PQ^2 + QR^2$

$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$

$169 \text{ cm}^2 = 144 \text{ cm}^2 + QR^2$

$25 \text{ cm}^2 = QR^2$

$QR = 5 \text{ cm}$



$$\tan P = \frac{\text{Side opposite } \angle P}{\text{Side adjacent } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

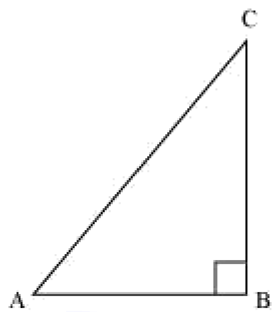
$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Q.3 If sin A = 3/4, calculate cos A and tan A.

Sol: Let ΔABC be a right-angled triangle, right-angled at point B.

Given that,



Let BC be 3k. Therefore, AC will be 4k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

$AB = \sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

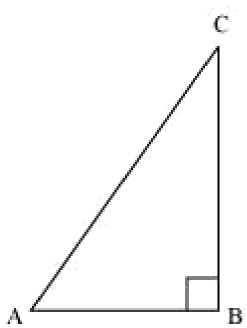
$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}k}$$

Q.4 Given 15 cot A = 8. Find sin A and sec A

Sol: Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \frac{AB}{BC}$$

It is given that,

$$\cot A = \frac{8}{15}$$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8k. Therefore, BC will be 15k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$$AC = 17k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

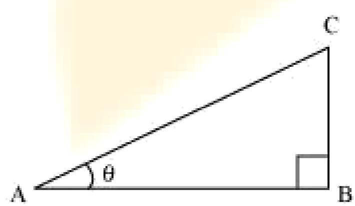
$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

$$= \frac{AC}{AB} = \frac{17}{8}$$

Q.5 Given sec θ = 13/12, calculate all other trigonometric ratios.

Sol:

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta} = \frac{13}{12} = \frac{AC}{AB}$$

If AC is 13k, AB will be 12k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + BC^2$$

$$25k^2 = BC^2$$

$BC = 5k$

$\sin\theta = \dfrac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{5k}{13k} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{12k}{13k} = \dfrac{12}{13}$

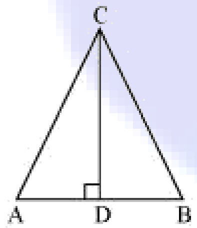
$\tan\theta = \dfrac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \dfrac{BC}{AB} = \dfrac{5k}{12k} = \dfrac{5}{12}$

$\cot\theta = \dfrac{\text{Side adjacent to } \angle\theta}{\text{side opposite to } \angle\theta} = \dfrac{AB}{BC} = \dfrac{12k}{5k} = \dfrac{12}{5}$

$\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{side opposite to } \angle\theta} = \dfrac{AC}{BC} = \dfrac{13k}{5k} = \dfrac{13}{5}$

Q.6 If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Sol: Let us consider a triangle ABC in which $CD \perp AB$.

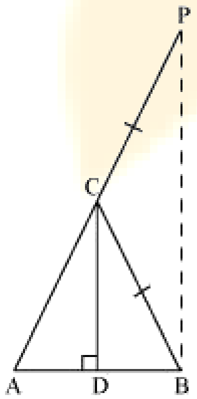


It is given that

$\cos A = \cos B$

$\dfrac{AD}{AC} = \dfrac{BD}{BC}$ ……… (1)

We have to prove $\angle A = \angle B$. To prove this, let us extend AC to P such that $BC = CP$.



From equation (1), we obtain

$\dfrac{AD}{BD} = \dfrac{AC}{BC}$

$\dfrac{AD}{BD} = \dfrac{AC}{CP}$ (By contruction, we have $BC = CP$) …..(2)

By using the converse of B.P.T,

CD||BP

$\Rightarrow \angle ACD = \angle CPB$ (Corresponding angles) … (3)

And, $\angle BCD = \angle CBP$ (Alternate interior angles) … (4)

By construction, we have BC = CP.

$\therefore \angle CBP = \angle CPB$ (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

$\angle ACD = \angle BCD$ … (6)

In ΔCAD and ΔCBD,

$\angle ACD = \angle BCD$ [Using equation (6)]

$\angle CDA = \angle CDB$ [Both 90°]

Therefore, the remaining angles should be equal.

$\therefore \angle CAD = \angle CBD$

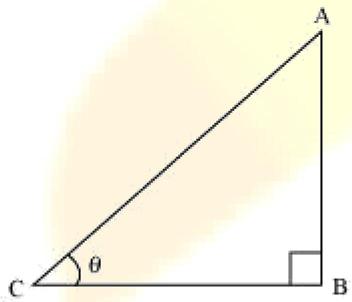
$\Rightarrow \angle A = \angle B$

Q.7 If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Sol: Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is 7k, then AB will be 8k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$AC = \sqrt{113}k$

$\sin\theta = \dfrac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

$= \dfrac{8k}{\sqrt{113}k} = \dfrac{8}{\sqrt{113}}$

$\cos\theta = \dfrac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7k}{\sqrt{113}k} = \dfrac{7}{\sqrt{113}}$

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$= \dfrac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \dfrac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$\dfrac{\frac{49}{113}}{\frac{64}{113}} = \dfrac{49}{64}$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}$

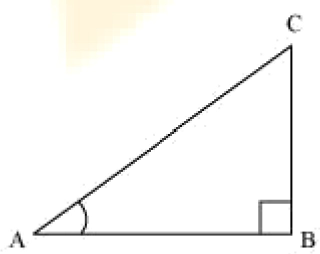
Q.8 If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol:

It is given that $3\cot A = 4$

or, $\cot A = \dfrac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$\cot A = \dfrac{\text{side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \dfrac{AB}{BC} = \dfrac{4}{3}$

If AB is 4k, then BC will be 3k, where k is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$AC = 5k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

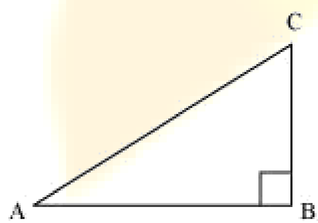
$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

Q.9 In ΔABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) $\sin A \cos A + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Sol:



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is k, then AB will be $\sqrt{3}$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

$\therefore AC = 2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) $\sin A \cos A + \cos A \sin C$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

(iii) $\cos A \cos C - \sin A \sin C$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q.10 In ΔPQR, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of sin P, cos P and tan P.

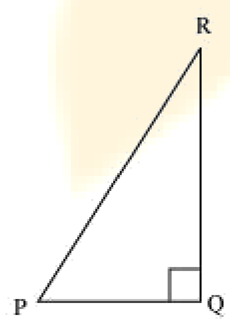
Sol:

Given that, $PR + QR = 25$

$PQ = 5$

Let PR be x.

Therefore, $QR = 25 – x$



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q.11 State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = 12/5 for some value of angle A.

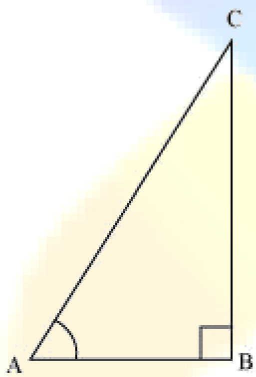
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = 4/3, for some angle θ.

Sol:

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{12}{5}$$

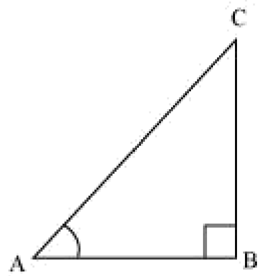
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be 12k, AB will be 5k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides AC = 12k and AB = 5k,

BC should be such that,

$AC - AB < BC < AC + AB$

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\ k$

However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A. Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A. Hence, the given statement is false.

(v) $\sin\theta = 4/3$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible. Hence, the given statement is false