SpeedLabs

MATHS

CBSE 10th

TEEVRA EDUTECH PVT. LTD.

Q.1 Evaluate the following

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}(2 + 2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}(2\sqrt{6} - 2\sqrt{2})}{(2\sqrt{6} + 2\sqrt{2})}$$

$$= \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{24 - 8} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{16}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) $\frac{\sin 30°+\tan 45°-\text{cosec } 60°}{\sec 30°+\cos 60°+\cot 45°}$

$$=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}}=\frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$=\frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}}=\frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$$

$$=\frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)}=\frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-(4)^2}$$

$$=\frac{27+16-24\sqrt{3}}{27-16}=\frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

$$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\frac{5\left(\frac{1}{4}\right)+\left(\frac{16}{3}\right)-1}{\frac{1}{4}+\frac{3}{4}}$$

$$=\frac{\frac{15+64-12}{12}}{\frac{4}{4}}=\frac{67}{12}$$

Q.2 Choose the correct option and justify your choice.

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

a. $\sin 60°$

b. $\cos 60°$

c. $\tan 60°$

d. $\sin 30°$

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

a. tan90°

b. 1

c. sin45°

d. 0

(iii) sin2A = 2sinA is true when A =

a. 0°

b. 30°

c. 45°

d. 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

a. cos60°

b. sin60°

c. tan60°

d. sin30°

Sol:

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60° = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

$$\frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only A = 0° is correct.

As $\sin 2A = \sin 0° = 0$

$2\sin A = 2\sin 0° = 2(0) = 0$

Hence, (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

Out of the given alternatives, only $\tan 60° = \sqrt{3}$

Hence, (C) is correct.

Q.3 If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B \leq 90°$, $A > B$ find A and B.

Sol:

$\Rightarrow \tan(A + B) = \sqrt{3}$

$\Rightarrow \tan(A + B) = \tan 60$

$A + B = 60 \quad \ldots (1)$

$\tan(A - B) = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan(A - B) = \tan 30$

$\Rightarrow A - B = 30 \quad \ldots (2)$

On adding both equations, we obtain

$2A = 90$

$\Rightarrow A = 45$

From equation (1), we obtain

$45 + B = 60$

$B = 15$

Therefore, $\angle A = 45°$ and $\angle B = 15°$

Q.4 State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$

Sol:

(i) $\sin(A + B) = \sin A + \sin B$

Let $A = 30°$ and $B = 60°$

$\sin(A + B) = \sin(30° + 60°)$

$= \sin 90°$

$= 1$

$\sin A + \sin B = \sin 30° + \sin 60°$

$= \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{1 + \sqrt{3}}{2}$

Clearly, $\sin(A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin\theta$ increases as θ increases in the interval of $0° < \theta < 90°$ as $\sin 0° = 0$

$\sin 30° = \frac{1}{2} = 0.5$

$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$

$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$

$\sin 90° = 1$

Hence, the given statement is true.

(iii) $\cos 0° = 1$

$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60° = \frac{1}{2} = 0.5$

$\cos 90° = 0$

It can be observed that the value of cos θ does not increase in the interval of $0° < \theta < 90°$.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

$\sin 45° = \frac{1}{\sqrt{2}}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

$\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0} = \text{Undefined}$

Hence, the given statement is true.