SpeedLabs

MATHS

CBSE 10th

TEEVRA EDUTECH PVT. LTD.

Q.1 Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol:

We know that,

$\text{cosec}^2\text{A} = 1 + \cot^2\text{A}$

$\frac{1}{\text{cosec}^2\text{A}} = \frac{1}{\cot^2\text{A}}$

$\sin^2\text{A} = \frac{1}{1+\cot^2\text{A}}$

$\sin\text{A} = \pm\frac{1}{\sqrt{1+\cot^2\text{A}}}$

$\sqrt{1+\cot^2\text{A}}$ will always be positive quantities.

Therefore, $\sin\text{A} = \frac{1}{\sqrt{1+\cot^2\text{A}}}$

we know that, $\tan\text{A} = \frac{\sin\text{A}}{\cos\text{A}}$

however, $\cot\text{A} = \frac{\cos\text{A}}{\sin\text{A}}$

$\tan\text{A} = \frac{1}{\cot\text{A}}$

Also, $\sec^2\text{A} = 1 + \tan^2\text{A}$

$= 1 + \frac{1}{\cot^2\text{A}}$

$= \frac{\cot^2\text{A}+1}{\cot^2\text{A}}$

$\sec\text{A} = \frac{\sqrt{1+\cot^2\text{A}}}{\cot\text{A}}$

Q.2 Write all the other trigonometric ratios of ∠A in terms of sec A.

Sol: We know that,

$\cos\text{A} = \frac{1}{\sec\text{A}}$

also, $\sin^2\text{A} + \cos^2\text{A} = 1$

$\sin^2 A = 1 - \cos^2 A$

$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$

$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}}$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}}$

$= \frac{1}{\sqrt{\sec^2 A - 1}}$

$\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

For cot A,

$\cot A = \frac{1}{\tan A}$

$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$

Q.3 solve

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Sol:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \dfrac{\sin^2 63° + \sin^2(90° - 63°)}{\cos^2(90° - 73°) + \cos^2 73°}$

$= \dfrac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$ $[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta]$

$\dfrac{1}{1} = 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

(iii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$\sin 25° . \cos(90 - 25) + \cos 25 . \sin(90 - 25)$

$= \sin 25 . \sin 25 + \cos 25 . \text{coc}\ 25$

$[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta] \sin^2 25 + \cos^2 25 = 1$

$[\because \sin^2\theta + \cos^2\theta]$

Q.4 Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) sec A

(B) sin A

(C) cosec A

(D) cos A

(iv) $\frac{1 + \tan^2 A}{1 + \cot A} =$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Sol:

(i) $9\sec^2 A - 9\tan^2 A =$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \mathrm{cosec}\,\theta) =$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta . \sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta}$

$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta}$

$[\sin^2\theta + \cos^2\theta = 1]$

$= \frac{2\cos\theta\sin\theta}{\cos\theta . \sin\theta} = 2$

(iii) $(\sec A + \tan A)(1 - \sin A) =$

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\frac{1 - \sin A}{\cos A}\right)(1 - \sin A)$

$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$

$= \cos A$ $\qquad [1 - \sin^2 A = \cos^2 A]$

(iv) $\frac{1+\tan^2 A}{1+\cot A} =$

$= \frac{\sec^2 A}{\mathrm{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

Q.5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Sol:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LH.S$=(\operatorname{cosec}\theta - \cot\theta)^2$

$= \operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta$

$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2 \times \frac{1}{\sin\theta}.\frac{\cos\theta}{\sin\theta}$

$= \frac{1+\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}$

$= \frac{1+\cos^2\theta - 2\cos\theta}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$ $[a^2 + b^2 - 2ab = (a-b)^2]$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta} = \text{RH. S}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

$$\text{L.H.S}=\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$

$$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{1+1+2\sin A}{(1+\sin A)\cos A} \qquad [\sin^2\theta+\cos^2\theta=1]$$

$$=\frac{2+2\sin A}{(1+\sin A)\cos A}=\frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$=\frac{2}{\cos A}$$

$$=2\sec A=\text{R.H.S}$$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$

$$\text{L.H.S}\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta\,(\cos\theta-\sin\theta)}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta\,(\cos\theta-\sin\theta)}$$

$$=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$[a^3-b^3=(a-b)(a^2+b^2+ab)]$$

$$=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}\,[\sin^2\theta+\cos^2\theta=1]$$

$$=\frac{1}{\sin\theta\cos\theta}+1=1+\frac{1}{\sin\theta\cos\theta}$$

$$=1+\sec\theta\,\mathrm{cosec}\,\theta$$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S} = \frac{\cos A}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A$$

$$= \frac{1 - \cos^2 A}{1 - \cos A}$$

$$= \frac{\sin^2 A}{1 - \cos A} = \text{R.H.S.}$$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\text{L.H.S.} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Dividing all terms by sin A,

$$= \frac{\cot A + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\cot A + \operatorname{cosec} A) + (\cot^2 A - \operatorname{cosec}^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\cot A + \operatorname{cosec} A) + (\cot^2 A - \operatorname{cosec}^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \frac{(\cot A + \operatorname{cosec} A)(1 + \cot A - \operatorname{cosec} A)}{(1 + \cot A - \operatorname{cosec} A)}$$

$$= \cot A + \operatorname{cosec} A = \text{R.H.S.}$$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A} = \sec A + \tan A}$

$$\text{L.H.S.} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} \times \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \qquad [(a+b)(a-b) = a^2 - b^2]$$

$$= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \qquad [1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \text{R.H.S}$$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,[2(1 - \sin^2\theta)]} \qquad [1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\;(2 - 2\sin^2\theta - 1)}$$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\;(1 - 2\sin^2\theta)} = \frac{\sin\theta}{\cos\theta}$$

$= \tan\theta = \text{R.H.S.}$

(viii) $(\sin A + \text{cosec}\, A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$\text{L.H.S} = (\sin A + \text{cosec}^2 A)^2 + (\cos A + \sec A)^2$

$$= \left(\sin A + \frac{1}{\sin A}\right)^2 + \left(\cos A + \frac{1}{\cos A}\right)^2$$

$$= \sin^2 A + \frac{1}{\sin^2 A} + 2.\sin A.\frac{1}{\sin A} + \cos^2 A + \frac{1}{\cos^2 A} + 2.\cos A\frac{1}{\cos A}$$

$$= 2 + 2 + \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$$= 4 + 4 + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$$

$= 5 + \text{cosec}^2 A + \sec^2 A$

$= 5 + 1 + \cot^2 A + 1 + \tan^2 A \qquad [\text{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta\theta = 1 + \tan^2\theta]$

$= 7 + \tan^2 A + \cot^2 A = \text{R.H.S}$

(ix) $(\text{cosec}\, A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

$$\left(\frac{1}{\sin A} - \sin^2 A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A.\cos A$$

$$= \frac{\sin A.\cos A}{\sin^2 A + \cos^2 A}\,[\sin^2\theta + \cos^2\theta = 1]$$

Divining all the terms by $\sin A.\cos A$

$$\frac{\frac{\sin A \,.\, \cos A}{\sin A \,.\, \cos A}}{\frac{\sin^2 A}{\sin A.\cos A + \frac{\cos^2 A}{\sin A.\cos A}}}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A} = \text{R. H. S.}$$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$[1 + \tan^2\theta = \sec^2\theta, 1 + \cot^2\theta = \text{cosec}^2\theta]$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{R. H. S.}$$

now middle side

$$= \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right) = (-\tan A)^2$$

$$= \tan^2 A = \text{R. H. S.}$$