Short Answer Type Questions II [3 Marks]

Question 1.

Prove that: $(\text{cosec}\,\theta - \sin\theta).(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$ [2010]

Solution:

$$\text{LHS} = (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta) = (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right) = \frac{(1-\sin^2\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta} = \frac{\cos^2\theta\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \sin\theta\cos\theta$$

$$\text{RHS} = \frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} = \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \sin\theta\cos\theta$$

Hence, LHS = RHS

Question 2.

Without using trigonometric table evaluate:

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ\,\text{cosec}\,52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ}\cot 72^\circ \cot 55^\circ$$

[2009]

Solution:

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ\,\text{cosec}\;52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ}\cot 72^\circ \cot 55^\circ$$

$$\frac{\cos 58^\circ}{\sin(90^\circ - 58^\circ)} + \frac{\sin 22^\circ}{\cos(90^\circ - 22^\circ)} - \frac{\cos 38^\circ\,\text{cosec}\,(90^\circ - 38^\circ)}{\tan 18^\circ \tan(90^\circ - 55^\circ)\sqrt{3}} \cdot \cot\;(90^\circ - 18^\circ)\cot\;55^\circ$$

$$\frac{\cos 58^\circ}{\cos 58^\circ} + \frac{\sin 22^\circ}{\sin 22^\circ} - \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 55^\circ(\sqrt{3})} \cdot \tan 18^\circ \cot 55^\circ$$

$$1 + 1 - \frac{1}{\sqrt{3}} = 2 - \frac{1}{\sqrt{3}}$$

Question 3.

Evaluate: $\frac{2}{3}\,\text{cosec}^2\,58^\circ - \frac{2}{3}\cot 58^\circ . \tan 32^\circ - \frac{5}{3}\tan 13^\circ . \tan 37^\circ . \tan 45^\circ . \tan 53^\circ . \tan 77^\circ$

[2009]

Solution:

$$\frac{2}{3}\operatorname{cosec}^2 58° - \frac{2}{3}\cot 58°\tan 32° - \frac{5}{3}\tan 13°\tan 37°\tan 45°\tan 53°\tan 77°$$

$$= \frac{2}{3}\{\operatorname{cosec}^2 58° - \cot 58°\cot(90° - 32°)\} - \frac{5}{3}\tan 13°.\tan 37° \times 1 \times \cot(90° - 53°) \times \cot(90° - 77°)$$

$$= \frac{2}{3}(\operatorname{cosec}^2 58° - \cot 58°.\cot 58°) - \frac{5}{3}\tan 13°.\tan 37°.\cot 37°.\cot 13°$$

$$= \frac{2}{3} \times 1 - \frac{5}{3}\tan 13°.\tan 37° \times \frac{1}{\tan 37°} \times \frac{1}{\tan 13°}$$

$$= \frac{2}{3} - \frac{5}{3} \times 1 = -1$$

Question 4.

If $\sin A = \cos A$, find the value of $2\tan^2 A + \sin^2 A + 1$. **[2014]**

Solution:

$\because \quad \sin A = \cos A$

$\Rightarrow \quad A = 45°$

Now, $\quad 2\tan^2 A + \sin^2 A + 1 = 2\tan^2 45° + \sin^2 45° + 1$

$$= 2 \times (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$$

$$= 2 + \frac{1}{2} + 1 = \frac{7}{2}.$$

Question 5.

Evaluate: $\dfrac{3\tan 25° \cdot \tan 40° \cdot \tan 50° \cdot \tan 65° - \frac{1}{2}\tan^2 60°}{4(\cos^2 29° + \cos^2 61°)}$ **[2014]**

Solution:

$$\frac{3\tan 25^\circ \tan 40^\circ \tan 50^\circ \tan 65^\circ - \frac{1}{2}\tan^2 60^\circ}{4(\cos^2 29^\circ + \cos^2 61^\circ)}$$

$$= \frac{3\tan 25^\circ \tan 40^\circ \tan(90^\circ - 40^\circ)\tan(90^\circ - 25^\circ) - \frac{1}{2}\times(\sqrt{3})^2}{4\{\cos^2 29^\circ + \cos^2(90^\circ - 29^\circ\}}$$

$$= \frac{3\tan 25^\circ \tan 40^\circ \cot 40^\circ \cot 25^\circ - \frac{(\sqrt{3})^2}{2}}{4(\cos^2 29^\circ + \sin^2 29^\circ)} = \frac{3 - \frac{3}{2}}{4} = \frac{6-3}{8} = \frac{3}{8}$$

Question 6.

Given $\tan A = \frac{5}{12}$, find the other trigonometric ratios of the angle A. **[2016]**

Solution:

Given: $\tan A = \frac{5}{12}$

As $\cot A = \frac{1}{\tan A} = \frac{12}{5}$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A = 1 + \left(\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}$$

$$\operatorname{cosec}^2 A = \frac{169}{25}$$

$\Rightarrow$ $\operatorname{cosec} A = \frac{13}{5}$

Now, $\operatorname{cosec} A = \frac{1}{\sin A} \Rightarrow \frac{13}{5} = \frac{1}{\sin A}$

$$\sin A = \frac{5}{13}$$

Also, $\cos^2 A = 1 - \sin^2 A = 1 - \frac{25}{169} = \frac{169-25}{169} = \frac{144}{169}$

$$\cos^2 A = \frac{144}{169}$$

$\therefore \quad \cos A = \frac{12}{13}$

As $\quad \cos A = \frac{1}{\sec A}$

$\therefore \quad \sec A = \frac{13}{12}$

Long Answer Type Questions [4 Marks]

Question 7.

If $\sin (A + B) = 1$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$ find the value of:

[2016]

1. $\tan A + \cot B$
2. $\sec A - \operatorname{cosec} B$

Solution:

$\sin (A + B) = 1$ (Given)

$\Rightarrow \quad \sin (A + B) = \sin 90°$ (As $\sin 90° = 1$)

$\Rightarrow \quad A + B = 90°$ ...(*i*)

Also $\quad \tan (A - B) = \frac{1}{\sqrt{3}}$ (Given)

$\tan (A - B) = \tan 30°$ (As $\tan 30° = \frac{1}{\sqrt{3}}$)

$\Rightarrow \quad A - B = 30°$ ...(*ii*)

Solving (*i*) and (*ii*) for A and B, we get $A = 60°$ and $B = 30°$

(*i*) $\tan A + \cot B = \tan 60° + \cot 30° = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

(*ii*) $\sec A - \operatorname{cosec} B = \sec 60° - \operatorname{cosec} 30° = 2 - 2 = 0$

Question 8.

Prove that: $\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$

[2016]

Solution:

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\operatorname{cosec}^2\theta}=\frac{\sin^3\theta}{\cos^3\theta}.\cos^2\theta+\frac{\cos^3\theta}{\sin^3\theta}.\sin^2\theta$$

$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}=\frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}=\frac{(\sin^2\theta)^2+(\cos^2\theta)^2}{\sin\theta\cos\theta}$$

$$=\frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}\ [\text{Using } a^2+b^2=(a+b)^2-2ab]$$

$$=\frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}-\frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Question 9.

If $A+B=90°$, prove that: $\sqrt{\frac{\tan A\tan B+\tan A\cot B}{\sin A\sec B}-\frac{\sin^2 B}{\cos^2 A}}=\tan A$ [2016]

Solution:

$$A+B=90°$$

$$\Rightarrow \quad A=90°-B$$

So, $\sqrt{\frac{\tan A\tan B+\tan A\cot B}{\sin A\sec B}-\frac{\sin^2 B}{\cos^2 A}}$

$$=\sqrt{\frac{\tan(90°-B).\tan B+\tan(90°-B)\cot B}{\sin(90°-B)\sec B}-\frac{\sin^2 B}{\cos^2(90°-B)}}$$

$$=\sqrt{\frac{\cot B.\tan B+\cot B.\cot B}{\cos B\sec B}-\frac{\sin^2 B}{\sin^2 B}}$$

$$\left[\begin{array}{l}\text{as } \tan(90°-\theta)=\cot\theta\\ \sin(90°-\theta)=\cos\theta\\ \cos(90°-\theta)=\sin\theta\end{array}\right]$$

$$=\sqrt{\frac{1+\cot^2 B}{1}-1}=\sqrt{1+\cot^2 B-1}=\sqrt{\cot^2 B}=\cot B=\cot(90°-A)=\tan A$$

Question 10.

Prove that:

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{1 - 2\cos^2 A}$$

[2015]

Solution:

$$\text{LHS} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{2(\sin^2 A + \cos^2 A)}{1 - \cos^2 A - \cos^2 A} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2}{1 - 2\cos^2 A} = \text{RHS}$$

Question 11.

If cosec A + cot A = m, show that $\frac{m^2-1}{m^2+1} = \cos A$. [2014]

Solution:

$$\text{LHS} = \frac{m^2 - 1}{m^2 + 1} \qquad [\because \text{cosec } A + \cot A = m]$$

$$= \frac{(\text{cosec } A + \cot A)^2 - 1}{(\text{cosec } A + \cot A)^2 + 1} = \frac{\text{cosec}^2 A + \cot^2 A + 2\,\text{cosec } A \cot A - 1}{\text{cosec}^2 A + \cot^2 A + 2\,\text{cosec } A \cot A + 1}$$

$$= \frac{(\text{cosec}^2 A - 1) + \cot^2 A + 2\,\text{cosec } A \cot A}{\text{cosec}^2 A + (1 + \cot^2 A) + 2\,\text{cosec } A \cot A}$$

$$= \frac{\cot^2 A + \cot^2 A + 2\,\text{cosec } A \cot A}{\text{cosec}^2 A + \text{cosec}^2 A + 2\,\text{cosec } A \cot A} = \frac{2\cot^2 A + 2\,\text{cosec } A \cot A}{2\,\text{cosec}^2 A + 2\,\text{cosec } A \cot A}$$

$$= \frac{2\cot A(\cot A + \text{cosec } A)}{2\,\text{cosec } A(\text{cosec } A + \cot A)} = \frac{\cot A}{\text{cosec } A} = \frac{\cos A}{\sin A\,\text{cosec } A} = \cos A = \text{RHS}$$

Hence, $\cos A = \frac{m^2-1}{m^2+1}$.

Question 12.

If $\text{cosec}\,\theta + \cot\theta = q$, show that $\text{cosec}\,\theta - \cot\theta = 1/q$ and hence find the values of $\sin\theta$ and $\sec\theta$

[2014]

Solution:

Given $\text{cosec}\,\theta + \cot\theta = q$ ...(*i*)

Now, $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$\Rightarrow q.(\text{cosec}\,\theta - \cot\theta) = 1$

$\Rightarrow \text{cosec}\,\theta - \cot\theta = \frac{1}{q}$ ...(*ii*)

On adding (*i*) and (*ii*), we get

$$2\text{cosec}\,\theta = q + \frac{1}{q} \Rightarrow \text{cosec}\,\theta = \frac{q^2+1}{2q} \Rightarrow \sin\theta = \frac{2q}{q^2+1}$$

On subtracting (*ii*) from (*i*), we get

$$2\cot\theta = q - \frac{1}{q} \Rightarrow \cot\theta = \frac{q^2-1}{2q} \Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{q^2-1}{2q}$$

$$\Rightarrow \cos\theta = \frac{(q^2-1)}{2q} \times \sin\theta \Rightarrow \cos\theta = \frac{(q^2-1)}{2q} \times \frac{2q}{(q^2+1)} = \frac{q^2-1}{q^2+1}$$

$$\sec\theta = \frac{q^2+1}{q^2-1}$$

Hence, $\sin\theta = \frac{2q}{q^2+1}$ and $\sec\theta = \frac{q^2+1}{q^2-1}$

Question 13.

If $\sec\theta + \tan\theta = p$, then find the value of $\text{cosec}\theta$ **[2013]**

Solution:

Here, $\sec\theta + \tan\theta = p$ ...(*i*)

Now, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow p.(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2\sec\theta = p + \frac{1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$$

On subtracting (*ii*) from (*i*), we get

$$2\tan\theta = p - \frac{1}{p} \Rightarrow \tan\theta = \frac{p^2-1}{2p}$$

Now, $$\frac{\sec\theta}{\tan\theta} = \frac{\frac{p^2+1}{2p}}{\frac{p^2-1}{2p}} \Rightarrow \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{p^2+1}{p^2-1}$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{p^2+1}{p^2-1} \Rightarrow \operatorname{cosec}\theta = \frac{p^2+1}{p^2-1}$$

Question 14.

If A + B = 90°, prove that

$$\sqrt{\frac{\tan A\tan B+\tan A\cot B}{\sin A\sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$$

[2012]

Solution:

$A + B = 90° \Rightarrow B = 90° - A$

Now, $$\text{LHS} = \sqrt{\frac{\tan A.\tan B+\tan A.\cot B}{\sin A.\sec B} - \frac{\sin^2 B}{\cos^2 A}}$$

$$= \sqrt{\frac{\tan A.\tan(90°-A)+\tan A.\cot(90°-A)}{\sin A.\sec(90°-A)} - \frac{\sin^2(90°-A)}{\cos^2 A}}$$

$$= \sqrt{\frac{\tan A.\cot A+\tan A.\tan A}{\sin A.\operatorname{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$$

$$= \sqrt{\frac{1+\tan^2 A}{1} - 1} = \sqrt{\tan^2 A} = \tan A = \text{RHS}$$

Question 15.

Prove that $$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$$

[2011]

Solution:

$$\text{Taking LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \frac{(\sqrt{1+\sin\theta})^2 + (\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin\theta} \times \sqrt{1+\sin\theta}} = \frac{1+\sin\theta+1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{2}{\sqrt{\cos^2\theta}} = \frac{2}{\cos\theta} = 2\sec\theta = \text{RHS}$$