SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. Compute the magnitude of the following vectors:

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 3\hat{i} + 7\hat{j} - 3\hat{k}$ $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

Ans. The given vectors are:

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 3\hat{i} + 7\hat{j} - 3\hat{k}$ $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

$$\left|\vec{a}\right| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$\left|\vec{b}\right| = \sqrt{(1)^2 + (-7)^2 + (3)^2}$$

$$= \sqrt{4 + 49 + 9}$$

$$= \sqrt{62}$$

$$\left|\vec{c}\right| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$$

2. Write two different vectors having same magnitude.

Ans. Consider $\vec{a} = \left(\hat{i} + \hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

It can be observed that $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$ and

$\left|\vec{b}\right| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$.

Hence $|\vec{a}|$ and $\left|\vec{b}\right|$, are two different vectors having the same magnitude. The vectors are different because they have different directions.

3. $= 8\left(\frac{5\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{30}}\right)$

$$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

4. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} - 6\hat{j} + 8\hat{k}$ are collinear.

Ans. Let $\vec{a} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

It is observed that $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = -2\vec{a}$

$\therefore\ \vec{b} = \lambda\vec{a}$

Where,

$\lambda = -2$

Hence, the given vectors are collinear.

5. Find the direction cosines of the vector $\hat{i} + 2\hat{j} - 3\hat{k}$

Ans. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\therefore\ |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

Hence, the direction cosines of $\vec{a}$ are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

6. Find the sum of the vectors $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \hat{i} + 6\hat{j} - 7\hat{k}$

Ans. The given vectors are $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \hat{i} + 6\hat{j} - 7\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k}$

$= 0.\hat{i} + 4\hat{j} - 1.\hat{k}$

$= -4\hat{j} - \hat{k}$

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Ans. The unit vector $\hat{a}$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$. is given by $\hat{a} = \frac{\vec{a}}{|a|}$

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+1+9} = \sqrt{6}$

$$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Ans. The given points are P (1, 2, 3) and Q (4, 5, 6).

$\therefore \overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3 + 3\hat{j} + 3\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$

Hence, the unit vector in the direction of $\overrightarrow{PQ}$ is

$$\frac{PQ}{PQ} = \frac{3\hat{i} + 3\hat{j} - 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

9. For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Ans. The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$

$\therefore\ \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the unit vector in the direction of $\vec{a}+\vec{b}$ is

$$\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=\frac{\hat{\imath}+\hat{k}}{\sqrt{2}}=\frac{1}{2}\hat{\imath}+\frac{1}{\sqrt{2}}\ \hat{k}$$

10. Find a vector in the direction of vector $5\hat{\imath}+\hat{\jmath}-2\hat{k}$ which has magnitude 8 units.

Ans. Let $\vec{a}\ =\ 5\hat{\imath}+\hat{\jmath}-2\hat{k}$

$\therefore\ |\vec{a}|=5^2+(-1)^2\ +\ 2^2\ =\sqrt{25+1+4}=\sqrt{30}$

$$\therefore\ \hat{a}\ =\frac{\vec{a}}{|\vec{a}|}=\frac{5\hat{\imath}+\hat{\jmath}-2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{\imath}+\hat{\jmath}-2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a}=8\left(\frac{5\hat{\imath}+\hat{\jmath}-2\hat{k}}{\sqrt{30}}\right)=\frac{40}{\sqrt{30}}\ \hat{\imath}-\frac{8}{\sqrt{30}}\hat{\jmath}+\frac{16}{\sqrt{30}}\ \hat{k}$$

$$=8\left(\frac{5\vec{\imath}-\vec{\jmath}+2\vec{k}}{\sqrt{30}}\right)$$

$$=\frac{40}{\sqrt{30}}\hat{\imath}-\frac{8}{\sqrt{30}}\hat{\jmath}+\frac{16}{\sqrt{30}}\ \hat{k}$$

11. Show that the vectors $2\hat{\imath}-3\hat{\jmath}+4\hat{k}$ and $-4\hat{\imath}+6\hat{\jmath}-8\hat{k}$ are collinear.

Ans. Let $\vec{a}=\ -2\hat{\imath}-3\hat{\jmath}+4k$ and $\vec{b}-4\hat{\imath}+6\hat{\jmath}-8\hat{k}$.

It is observed that $\vec{b}-4\hat{\imath}+6\hat{\jmath}-8\hat{k}=\ -2\left(2\hat{\imath}+3\hat{\jmath}-4\hat{k}\right)=-2\vec{a}$

$\therefore\ \vec{b}=\ \lambda\vec{a}$

Where,

$\lambda=\ -2$

Hence, the given vectors are collinear.

12. Find the direction cosines of the vector $\hat{\imath}+2\hat{\jmath}-3\hat{k}$

Ans. Let $\hat{\imath}+2\hat{\jmath}-3\hat{k}$.

$\therefore\ |\vec{a}|=\sqrt{1^2+2^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$

Hence, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$.

13. Find the direction cosines of the vector joining the points A (1, 2, –3) and B (–1, –2, 1) directed from A to B.

Ans. The given points are A (1, 2, –3) and B (–1, –2, 1).

$\therefore\ \overrightarrow{AB}=(-1-1)\ \hat{\imath}+(-2-2)\hat{\jmath}+\ -1(-3)\}\ \hat{k}$

$\Longrightarrow\overrightarrow{AB}=\ -2\hat{\imath}-4\hat{\jmath}+4\hat{k}$

$\therefore\ \left|\overrightarrow{AB}\right|=\sqrt{(-2)^2+(-4)^2+4^2}=\sqrt{4+16+16}=\sqrt{36}=6$

Hence, the direction cosines of $\overrightarrow{AB}$ are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.

Ans. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Then,

$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of $\vec{a}$ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α, β, and γbe the angles formed by with the positive directions of x, y, and z axes.

Then, we have $\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} - \hat{k}$ respectively, in the ration 2:1

(i) internally

(ii) externally

Ans. The position vector of point R dividing the line segment joining two points P and Q in the ratio m: n is given by

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m + n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m - n}$$

Position vectors of P and Q are given as:

$\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k}$ and $OQ = -\hat{i} + \hat{j} + \hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\overrightarrow{OR} = \frac{2\left(-\hat{i} + \hat{j} - \hat{k}\right) + (\hat{i} + 2\hat{j} - \hat{k})}{2 + 1} = \frac{\left(-2\hat{i} + 2\hat{j} + 2\hat{k}\right) + (\hat{i} + 2\hat{j} - \hat{k})}{3}$$

$$= \frac{-\hat{i} + 4\hat{j} - \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\overrightarrow{OR} = \frac{2\left(-\hat{i} + \hat{j} + \hat{k}\right) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2 - 1} = \left(-2\hat{i} + 2\hat{j} - 2\hat{k}\right) - (\hat{i} + 2\hat{j} - \hat{k})$$

$= -3\hat{i} + 3\hat{k}$

16. Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, – 2).

Ans. The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, – 2) is given by,

$$\overrightarrow{OR} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2}\ 3\hat{i} + 2\hat{j} + \hat{k}$$

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

Ans. Position vectors of points A, B, and C are respectively given as:

$\vec{a} = 3\hat{i} - 4\hat{j}\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$,

$\therefore\ \overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$

$\therefore\ |\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

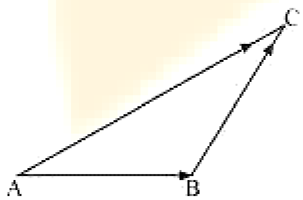
$|\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

$|\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

$\therefore\ |\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 36 + 6 = 41 = |\overrightarrow{CA}|^2$

Hence, ABC is a right-angled triangle

18. In triangle ABC which of the following is not true:



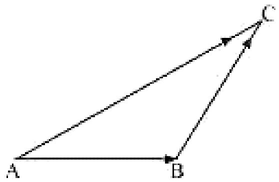
$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Ans. On applying the triangle law of addition in the given triangle, we have:



$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ ... (1)

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ ... (2)

∴ The equation given in alternative A is true.

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

∴ The equation given in alternative B is true.

From equation (2), we have:

$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

∴ The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$ ... (3)

From equations (1) and (3), we have:

$\overrightarrow{AB} = \overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$

$\Rightarrow 2\,\overrightarrow{AC} = \vec{0}$

$\Rightarrow \overrightarrow{AC} = \vec{0}$, which is not true.

Hence, the equation given in alternative C is incorrect.

The correct answer is C.

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are **incorrect:**

A. $\vec{b} = \lambda\vec{a}$, for some scalar λ

B. $\vec{a} = \pm\,\vec{b}$

C. the respective components of $\vec{a}$ and $\vec{b}$ are proportional

D. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Ans. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have:

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{b} = \lambda\vec{a}$.

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda b_1)\hat{i} + (\lambda b_2)\hat{j} + (\lambda b_3)\hat{k}$

$\Rightarrow b_1 = \lambda a_1, b_2, = \lambda a_2, b_3 = \lambda a_3$

$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3}$