SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a}$ and $\vec{b}$ $\sqrt{6}$

Ans. It is given that,

$|\vec{a}| = \sqrt{3}, \vec{b} = 2$ and, $\vec{a}.\vec{b} = \sqrt{6}$

Now, we know that $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, the angle between the given vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

Ans. The given vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Now, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k})$

$= 1.3 + (-2)(-2) + 3.1$

$= 3 + 4 + 3$

$= 10$

Also, we know tha $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \theta = \cos^{-1} = \left(\frac{5}{7}\right)$

3. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

Ans. $\vec{a} = \hat{i} - \hat{j}$ And $\hat{i} + \hat{j}$

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$\frac{1}{|\vec{b}|}(\vec{a}.\vec{b}) = \frac{1}{\sqrt{1+1}}\{1.1 + (-1)(1)\} = \frac{1}{\sqrt{2}}(1-1) = 0$

Hence, the projection of vector $\vec{a}$ on $\vec{b}$ is 0.

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

Ans. $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}(\vec{a}.\vec{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}}\{1(7) + 3(-1) + 7(8)\} = \frac{7 - 3 + 56}{\sqrt{49 + 1 + 64}} = \frac{60}{\sqrt{114}}$$

5. Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Ans. Let $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k},$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a}.\vec{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(\frac{-6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$

$$\vec{b}.\vec{c} = \frac{3}{7} \times \frac{6}{7} + \left(\frac{-6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(\frac{-3}{7}\right) = \frac{18}{49} - \frac{12}{49} + \frac{6}{49} = 0$$

$$\vec{c}.\vec{a} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(\frac{-3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$$

Hence, the given three vectors are mutually perpendicular to each other.

6. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$ *and* $|\vec{a}| = 8|\vec{b}|$

Ans. $(\vec{a}.\vec{b}).(\vec{a} - \vec{b}) = 8$

$\Rightarrow \vec{a}.\vec{b} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b} = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$ $\quad [|\vec{a}| = 8|\vec{b}|]$

$$\Rightarrow 64\left|\vec{b}\right|^2 - \left|\vec{b}\right|^2 = 8$$

$$\Rightarrow 63\left|\vec{b}\right|^2 = 8$$

$$\Rightarrow \left|\vec{b}\right|^2 = \frac{8}{63}$$

$$\Rightarrow \left|\vec{b}\right| = \sqrt{\frac{8}{63}} \qquad \text{Magnitude of a is non} - \text{negative}$$

$$\Rightarrow \left|\vec{b}\right| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$\left|\vec{a}\right| = 8\left|\vec{b}\right| = \frac{8\times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

7. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Ans. The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$

Now,

$$\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right) = \left(2-\lambda\right)\hat{i} + \left(2+2\lambda\right)\hat{j} + \left(3+\lambda\right)\hat{k}$$

$$\Rightarrow \left[\left(2-\lambda\right)\hat{i} + \left(2+2\lambda\right)\vec{j} + \left(3+\lambda\right)\hat{k}\right]\cdot\left(3\hat{i} + \vec{j}\right) = 0$$

$$\Rightarrow \left(2-\lambda\right)3 + \left(2+2\lambda\right)1 + \left(3+\lambda\right) = 0$$

$$\Rightarrow \left[\left(2-\lambda\right)\hat{i} + \left(2+2\lambda\right)\vec{j} + \left(3+\lambda\right)\hat{k}\right]\cdot\left(3\hat{i} + \vec{j}\right) = 0$$

$$\Rightarrow \left(2-\lambda\right)3 + \left(2+2\lambda\right)1 + \left(3+\lambda\right)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of λ is 8.

8. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$

Ans. $\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right)\cdot\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$

$=|\vec{a}|^2\vec{b}\cdot\vec{b}-|\vec{a}||\vec{b}|\vec{b}\cdot\vec{a}+|\vec{b}||\vec{a}|\vec{a}\cdot\vec{b}-|\vec{b}|^2\vec{a}\cdot\vec{a}$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{b}|^2|\vec{a}|^2$

$=0$

Hence, $|\vec{a}|\vec{b}+|\vec{b}|^2\vec{a}$ and $|\vec{a}|\vec{b}-|\vec{b}|^2\vec{a}$ are perpendicular to each other.

9. If $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$, then what can be concluded about the vector $\vec{b}$?

Ans. It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$

Now,

$\vec{a}\cdot\vec{a}=0\Rightarrow|\vec{a}|^2=0\Rightarrow|\vec{a}|=0$

$\therefore\vec{a}$ is a zero vector

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector.

10. If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}=\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

Ans. Considre $\vec{a}=2\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{a}=3\hat{i}+3\hat{j}+6\hat{k}$

Then,

$\vec{a}\cdot\vec{b}=2.3+4.3+3(-6)=6+12-18=0$

We now observe that:

$|\vec{a}|=\sqrt{2^2+4^2+3^2}=\sqrt{29}$

$\therefore\vec{a}\neq\vec{0}$

$|\vec{b}|=\sqrt{3^2+3^2+(-6)^2}=\sqrt{54}$

$\therefore\vec{b}\neq\vec{0}$

Hence, the converse of the given statement need not be true.

11. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find □ABC.[□ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$

Ans. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find □ABC.[□ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

$$\overrightarrow{BA}=\{1-(-1)\}\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}$$

$$\overrightarrow{BC}=\{0-(-1)\}\hat{i}+(1-0)\hat{j}+(2-0)\hat{k}=\hat{i}+\hat{j}+2\hat{k}$$

$$\therefore \overrightarrow{BA}\cdot\overrightarrow{BC}=(2\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+2\hat{k})=2\times1+2\times1+3\times2=2+2+6=10$$

$$\left|\overrightarrow{BA}\right|=\sqrt{2^2+2^2+3^2}=\sqrt{4+4+9}=\sqrt{17}$$

$$\overrightarrow{BC}=\sqrt{1+1+2^2}=\sqrt{16}$$

Now, it is known that:

$$\overrightarrow{BA}\cdot\overrightarrow{BC}=\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\cos(\angle ABC)$$

$$\therefore 10=\sqrt{17}\times\sqrt{6}\cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC)=\frac{10}{\sqrt{17}\times\sqrt{6}}$$

$$\Rightarrow \angle ABC=\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

12. Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, –1) are collinear.

Ans. The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, –1).

$$\therefore \overrightarrow{AB}=(2-1)\hat{i}+(6-2)\hat{j}+(3-7)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{BC}=(3-2)\hat{i}+(10-6)\hat{j}+(-1-3)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{BC}=(3-1)\hat{i}+(10-2)\hat{j}+(-1-7)\hat{k}=2\hat{i}+8\hat{j}-8\hat{k}$$

$$\left|\overrightarrow{AB}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$$\left|\overrightarrow{BC}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{2^2+8^2+8^2} = \sqrt{4+64+64} = \sqrt{132} = 2\sqrt{33}$$

$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Hence, the given points A, B, and C are collinear.

13. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.

Ans. Let vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ be position vectors of points A, B, and C respectively.

$$\text{i.e.,} \overrightarrow{OP} = 2i-\hat{j}+\hat{k}, \overrightarrow{OB} = \hat{i}-3j-5\hat{k} \text{ and } \overrightarrow{OC} = 3\hat{i}-4\hat{j}-4\hat{k}$$

Now, vectors $\overrightarrow{AB}.\overrightarrow{BC}, \text{and} \overrightarrow{AC}$ represent the sides of ΔABC.

$$\text{i.e.,} \overrightarrow{OP} = 2i-\hat{j}+\hat{k}, \overrightarrow{OB} = \hat{i}-3j-5\hat{k} \text{ and } \overrightarrow{OC} = 3\hat{i}-4\hat{j}-4\hat{k}$$

$$\therefore \overrightarrow{AB} = (1-2)\hat{i}+(-3+1)\hat{j}+(-5-1)\hat{k} = -\hat{i}-2\hat{j}-6\hat{k}$$

$$\overrightarrow{BC} = (3-1)\hat{i}+(-4+3)\hat{j}+(-4+5)\hat{k} = 2\hat{i}-j+\hat{k}$$

$$\overrightarrow{AC} = (2-3)\hat{i}+(-1+4)\hat{j}+(1+4)\hat{k} = -\hat{i}-3j+5\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2+(-2)^2+(-6)^2} = \sqrt{1+4+36} = \sqrt{41}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{2^2+(-1)^2+1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2+3^2+5^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$\therefore \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AC}\right| = 6+35 = 41 = \left|\overrightarrow{AB}\right|^2$$

Hence, ΔABC is a right-angled triangle.

14. If $\vec{a}$ is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then λ $\vec{a}$ is unit vector if

(A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = |\lambda|$ (D) $a = \frac{1}{|\lambda|}$

Ans. Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$.

Now,

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \qquad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \qquad [|\vec{a}| = a]$$

Hence, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$.

The correct answer is D.