



**SpeedLabs**

**MATHS**

**CBSE 12<sup>th</sup>**

**TEEVRA EDUTECH PVT. LTD.**

# Vector Algebra

## Exercise - 10.4

1. Find  $|\vec{a} \times \vec{b}|$ , if  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

**Ans.** We have,

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = 2 \times (19) = 19\sqrt{2}$$

2. Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$$

**Ans.** We have,

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-8) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\therefore |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$

$$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$$

$$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$$

3. If either  $\vec{a} = \vec{0}$  or  $\vec{b} = \vec{0}$ , then  $\vec{a} \times \vec{b} = \vec{0}$ . Is the converse true? Justify your answer with an example.

**Ans.** Take any parallel non-zero vectors so that  $\vec{a} \times \vec{b} = \vec{0}$

$$\text{Let } \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}.$$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

4. Find the area of the triangle with vertices A (1,1,2), B(2,3,5) and C(1,5,5).

**Ans.** The triangle with vertices ABC are given as A (1,1,2), B (2,3,5) and C (1,5,5).

The adjacent sides  $\vec{AB}$  and  $\vec{BC}$  of  $\triangle ABC$  are given as:

$$\vec{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} |\vec{AB} \times \vec{BC}|$$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\therefore |\vec{AB} \times \vec{BC}| = \sqrt{(-6)^2 + (3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$$

Hence, The area of  $\triangle ABC$  is  $\frac{\sqrt{61}}{2}$  square units.

5. Find the area of the parallelogram whose adjacent sides are determined by the vector

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

**Ans.** The area of the parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$

Adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Hence, the area of the given parallelogram is  $15\sqrt{2}$  square units.

6. Let the vectors  $\vec{a}$  and  $\vec{b}$  be such that  $|\vec{a}|=3$  and  $|\vec{b}|=\frac{\sqrt{2}}{3}$ , then  $\vec{a} \times \vec{b}$  is a unit vector, if

the angle between  $\vec{a} \times \vec{b}$  is

(A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{2}$

**Ans.** It is given that  $|\vec{a}|=3$  and  $|\vec{b}|=\frac{\sqrt{2}}{3}$ .

We know that  $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$ , where  $\hat{n}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ .

Now,  $\vec{a} \times \vec{b}$  is a unit vector if  $|\vec{a} \times \vec{b}|$

$$|\vec{a} \times \vec{b}|$$

$$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 1$$

$$\Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1 \Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence,  $\vec{a} \times \vec{b}$  is a unit vector if the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$

The correct answer is B.

7.  $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$  and  $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$  respectively is

(A)  $\frac{1}{2}$  (B) 1 (C) 2 (D) 4

**Ans.** The position vectors of vertices A, B, C, and D of rectangle ABCD are given as:

$$\overrightarrow{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \overrightarrow{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \overrightarrow{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \overrightarrow{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

The adjacent sides  $\overrightarrow{AB}$  and  $\overrightarrow{CB}$  of the given rectangle are given as:

$$\overrightarrow{AB} = (1+1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\overrightarrow{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\therefore \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are

$\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$ .

Hence, the area of the given rectangle is  $|\overrightarrow{AB} \times \overrightarrow{CD}| = 2$  Square units.

The correct answer is C.