| Class – 9th | Topic – Congruent Triangles |
|---|---|

1. **In the given figure, we have AO = BO and CO = DO**

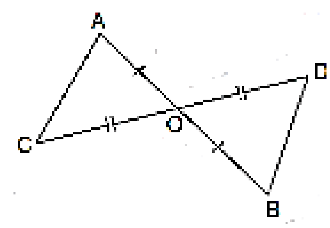
   **Prove that:**

   **(i) $\Delta AOC = \Delta BOD$**

   **(ii) AC=BD**

**Solution:**

(i) In $\Delta AOC$ and $\Delta BOD$




| | |
|---|---|
| $AO = BO$ | (given) |
| $CO = DO$ | (given) |
| $\angle AOC = \angle BOD$ | (vertically opposite angles) |
| $\Delta AOC \cong \Delta BOD$ | (S. A. S) |

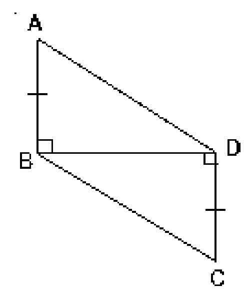
(ii) $AC = BD$ (C. P. C. T)

2. **In the given figure, AB $\perp$ BD and AB = CD.**

   **Prove that:**

   **(i) $\Delta ABD \cong \Delta CDB$**

   **(ii) $AB = CB$**




**Solution:**

(i) In $\Delta ABD$ and $\Delta CDB$

| | |
|---|---|
| $AB = CD$ | (given) |
| $BD = BD$ | (common) |
| $\angle ABD = \angle CDB$ | (each 90°) |
| $\therefore \Delta ABD \cong \Delta CDB$ | (S. A. S) |

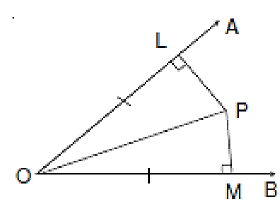
(ii) $AD = CB$ (C. P. C. T)

3. **In the given figure, PL $\perp$ OA nd PM $\perp$ OB such that OL = OM. Prove that.**

   **(i) $\Delta OLP \cong \Delta OMP$**

   **(ii) $PL = PM$**

   **(iii) $\angle LOP = \angle MOP$**



**Solution:**

(i) In the figure, $\angle L = \angle M = 90°, OL = OM$

Now, in two right triangles

$\Delta OLP$ and $\Delta OMP$

$OP = OP$ (common)

$OL = OM$ (given)

$\angle L = \angle M = 90°$

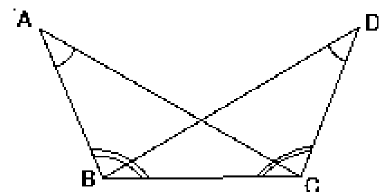
$\therefore \Delta OLP \cong \Delta OMP$ (by RHS axiom)

(ii) $\therefore PL = PM$ (C. P. C. T)

(iii) and $\angle LOP = \angle MOP$ (C. P. C. T)

**4. In the adjoining diagram, $\angle BAC = \angle BDC$ and $\angle ACB = \angle DBC$**

**Prove that : $AC = BO$**




**Solution:**

In $\Delta ABC$ and $\Delta BDC$

$\angle BAC = \angle BDC$ (given)

$\angle ACB = \angle DBC$ (given)

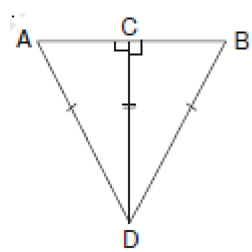
$BC = BC$ (common)

$\therefore \Delta ABC \cong \Delta DCB$ (By AAS axiom of congruency)

$\therefore AC = BD$ (corresponding parts of congruent traingles)

**5. In the given figure, we have $AC \perp CD$, $BC \perp CD$ and $DA = DB$. Prove that CA = CB.**

**Solution:**




In $\Delta ACD$ and $\Delta BCD$

$AD = BD$ (given)

$\angle ACD = \angle BCD$ (each 90°)

$CD = CD$ (common)

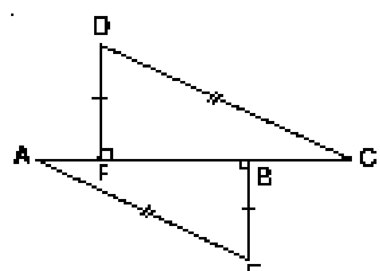
$\therefore \Delta ACD \cong \Delta BCD$

$\therefore CA = CB$

**6. In the figure given alongside prove that**

**(i) AB=FC**

**(ii) AF=BC**



**Solution:**

In $\Delta ABE$ and $\Delta DFC$

$\angle B = \angle F$ (each $90°$)

$AE = DC$ (given)

$BE = DF$ (given)

$\therefore \Delta ABE \cong \Delta CFD$ (R. H. S congruence rule)

(i) $AB = FC$ (corresponding parts of congruent triangles)

(ii) As $AB = FC$

$\Rightarrow AF + FB = FB + BC$

$\Rightarrow AF + FB - FB = BC$

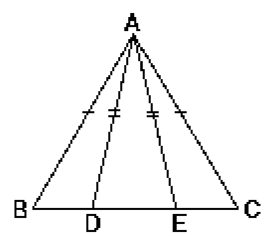
$\Rightarrow AF = BC$

**7. In the adjoining figure, AB = AC and AD = AE. Prove that:**

**(i) $\angle ADB = \angle AEC$**

**(ii) $\Delta ABC \cong \Delta ACE$**

**(iii) BE = DC**




**Solution:**

(i) In the given figure,

$AD = AE$ (given)

$\therefore \angle ADE = \angle AED$ (corresponding angles opposite to equal sides)

Now, $\angle ADE + \angle ADB = \angle AED + \angle AEC$ (Linear pair angles)

As $\angle ADE = \angle AED$

$\therefore \angle ADB = \angle AEC$

(ii) In $\Delta ABD$ and $\Delta AEC$

$AB = AC$, so, $\angle B = \angle C$

$AD = AE$

$\angle ADB = \angle AEC$ then, $\angle BAD = \angle EAC$ (Prove above)

$\therefore \Delta ABD \cong \Delta ACE$ (SAS congruence rule)

(iii) Since $\Delta ABD \cong \Delta ACE$

$BD = EC$ (by C. P. C. T)

$\Rightarrow BD + DE = EC + DE$

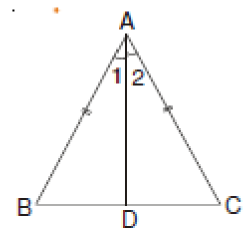
$\Rightarrow BE = CD$

8. **In the given figure; $\angle 1 = \angle 2$ and AB = AC. Prove that:**

   **(i) $\angle B = \angle C$**

   **(ii) BD = DC**

   **(iii) AD is perpendicular to BC.**




**Solution:**

In $\Delta ADB$ and $\Delta ADC$,

$AB = AC$ (given)

$\angle 1 = \angle 2$ (given)

$AD = AD$ (common)

$\therefore \Delta ADB \cong \Delta ADC$ (SAS axiom)

(i) Hence $\angle B = \angle C$ (C. P. C. T)

(ii) $BD = DC$ (C. P. C. T)

(iii) $\angle ADB = \angle ADC$ (C. P. C. T)

But, $\angle ADB + \angle ADC = 180°$ (linear pair angles)

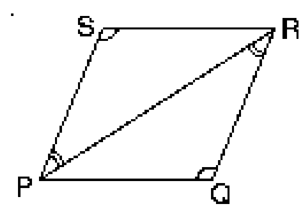
$\therefore \angle ADB = \angle ADC = 90°$

Hence, AD is perpendicular to BC.

9. **In the given figure prove that:**

   **(i) PQ = RS**

   **(ii) PS = QR**




**Solution:**

(i) In $\Delta PQR$ and $\Delta PSR$

$PR = PR$ (common)

$\angle PRQ = \angle RPS$ (given)

$\angle PQR = \angle PSR$ (given)

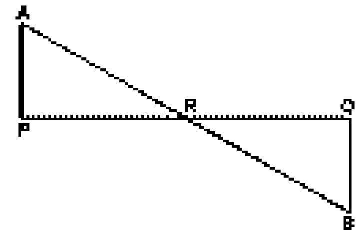
$\therefore \Delta PQR \cong \Delta RSP$ (A. A. S axiom)

$PQ = RS$ (C. P. C. T)

(ii) $QR = PS$ (C. P. C. T)

or $PS = OR$

**10. In the figure, AP and BQ are perpendiculars to PQ and AP = BQ, prove that R is the midpoint of PQ and AB.**




**Solution:**

In $\Delta APR$ and $\Delta BQR$,

$AP = BQ$ (Given)

$\angle ARP = \angle BRQ$ (Vertically opposite angles)

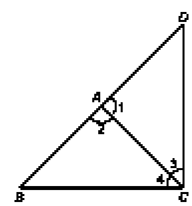
$\angle APR = \angle BQR$ (Each 90°)

$\Delta APR \cong \Delta BQR$ (RHS Criterion)

$\therefore PR = RQ$ and $AR = RB$ (C.P.C.T)

Hence R is the mid-point of AB and PQ.

**11. ΔABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Prove that ∠BCD = 90°.**




**Solution:**

In $\Delta ABC$, $AB = AC$

$\therefore$ ÐB $= \angle C = \angle 4$ ...... (i)

Since, $AB = AC$ (Given)

And $AB = AD$ (Produced)

$\therefore AD = AC$

Now, in $\Delta ACD$, $AD = AC$

$\therefore \angle D = \angle C = \angle 3$ ...... (ii)

Adding eqn (i) and eqn (ii), we get

$\angle B + \angle D = \angle 4 + \angle 3$

$\Rightarrow \angle B + \angle D = \angle BCD$

Now in $\Delta BCD$, we have

$\angle B + \angle BCD + \angle D = 180°$

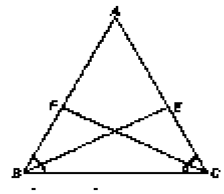
$\Rightarrow (\angle B + \angle D) + \angle BCD = 180°$

$\Rightarrow \angle BCD + \angle BCD = 180°$

$\Rightarrow 2\,\angle BCD = 180°$

$$\angle BCD = \frac{180°}{2} = 90°$$

**12. In $\Delta ABC$, if $AB = AC$ and BE, CF are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\Delta EBC \cong \Delta FCB$ and $BE = CF$.**




**Solution:**

Since in $\Delta ABC$, $AB = AC \Rightarrow \angle ABC = \angle ACB$ ... (i)

Since CF and BE are angle bisectors of $\angle C$ and $\angle B$,

We get $\angle ABE = \angle EBC$ ... (ii)

And $\angle ACF = \angle FCB$ ... (iii)

Now from equations (i), (ii) and (iii), we get

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

ĐEBC $= \angle FCB$ ...(iv)

Now in $\Delta FBC$ and $\Delta ECB$,

We have $\angle FBC = \angle ECB$ $(\angle B = \angle C)$

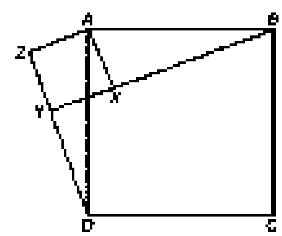
$BC = BC$ (Common)

$\angle FCB = \angle EBC$ [From (iv)]

$\Delta EBC \cong \Delta FCB$

$BE = CF$ (C.P.C.T.)

**13. In the figure x is a point in the interior of square ABCD, AXYZ is also a square. Prove that $BX = DZ$.**




**Solution:**

Since ABCD and AXYZ both are squares

$\angle AZY = \angle AXY$

$= \angle AXB$ ... (each 90°) and $AX = AZ$ and $AB = AD$

Now in $\Delta ABX$ and $\Delta ADZ$,

$\angle AZD = \angle AXB$ (Each 90°)

$AZ = AX$ (Sides of a square)

$AB = AD$ (Sides of a square)

$\Delta ABX \cong \Delta ADZ$

$BX = DZ$ (C.P.C.T.)

**14. In the figure, the sides AB and BC of square ABCD are produced to P and Q respectively so that BP = CQ. Prove that DP and AQ are perpendicular to each other.**

**Solution:**

Since ABCD is a square, AB = BC

Also BP = CQ (Given)

AB + BP = BC + CQ

AP = BQ

Now in ΔAPD and ΔBQA,

AP = BQ (Proved above)

∠ABQ = ∠DAP (each 90°)

And AB = AD

ΔAPD ≅ ΔBQA

∠APD = ∠BQA (C.P.C.T.)

And ∠ADP = ∠QAP (C.P.C.T.)

Also ∠DAQ = ∠AQB

∠DAO = ∠APO

Now in ΔAOD and ΔAOP,

∠ADO = ∠OAP,

And ∠DAO = ∠APO

3rd ∠DOA = 3rd ∠AOP [Since, two angles of DAOD and DAOP are equal, the third angle is also equal]

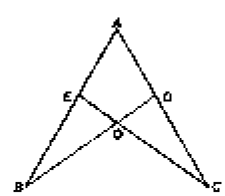
But ∠DOA + ∠AOP = 180°

2 ∠DOA = 180°⇒∠DOA = 90°

DO is perpendicular to AO or DP is perpendicular to AQ.

**15. In the figure, ∠B = ∠C and AB = AC. Prove that BD = CE.**

**Solution:**




In ΔABD and ΔACE,

AB = AC (Given)

∠B = ∠C (Given)

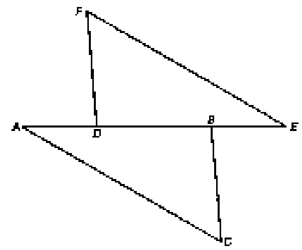
∠A = ∠A (Common)

ΔABD ≅ ΔACE (By ASA criterion)

BD = CE (C.P.C.T.)

**16. In the figure, AD = BE, BC = DF and ∠ABC = ∠EDF. Prove that AC ïï EF and AC = EF**

**Solution:**




Since AD = BE

AD + DB = BE + DB

⇒ AB = DE

Now in ΔABC and ΔEDF

AB = DE (Proved above)

BC = DF (Given)

And ∠ABC = ∠EDF (Given)

ΔABC ≅ ΔEDF

AC = EF (C.P.C.T.)

And ∠BAC = ∠DEF

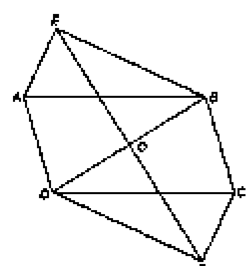
But these are alternate interior angles of AC and EF with transversal AE

AC || EF

**17. Given ABCD is a parallelogram, BC is produced to F and BD is produced to E and AE=CF Prove that:**

**(i) BE || DF**

**(ii) BD and EF bisect each other.**




**Solution:**

In ΔABE and ΔCDF

AB = DC (Opposite sides of parallelogram)

AE = CF and BE = DF (Given)

ΔABE ≅ ΔCDF

∠ABE = ∠CDF...... (i) (C.P.C.T.)

Now since AB ||DC and DB is a transversal

∠ABD = ∠BDC...... (ii) (Alternate interior angles)

Adding equations (i) and (ii), we get

∠ABE + ∠ABD = ∠CDF + ∠BDC

⇒ ∠EBD = ∠BDF ⇒ BE|| DF

Now in ΔOBE and ΔODF,

BE = DF (Given)

∠EBO = ∠FDO (Proved above)

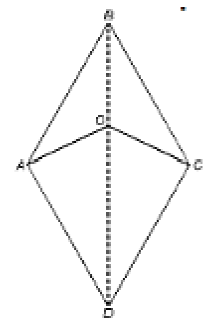
And ∠BOE = ∠DOF (Vertically opposite angles)

ΔOBE ≅ ΔODF

OE = OF and OB = OD (C.P.C.T.)

Therefore O is the mid-point of EF and DB.

**18. O is a point in the interior of a rhombus ABCD. If OA = OC then prove that DOB is a straight line.**




**Solution:**

Given: A rhombus ABCD and a point O in it such that OA = OC

To Prove: DOB is a straight line.

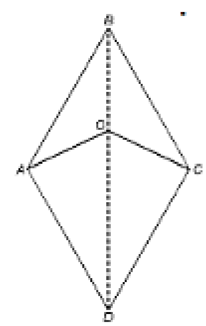
Construction: Join OB and OD

Proof: In ΔAOD and ΔCOD

AO = CO (Given)

OD = OD (Common)

AD = CD (Sides of a rhombus)




ΔAOD ≅ ΔCOD

∠AOD = ∠COD...... (i)

Similarly ΔAOB ≅ ΔCOB

∠AOB = ∠COB...... (ii)

Adding eqns (i) and (ii), we get

∠AOD + ∠AOB = ∠COD + ∠COB

But ∠AOD + ∠AOB + ∠COD + ∠COB = 360° (Angles at a point)

∠AOD + ∠AOB + ∠AOD + ∠AOB = 360°

⇒ 2 (∠AOD + ∠AOB) = 360°

⇒ ∠AOD + ∠AOB = 180° but it is a linear pair

OD and OB are in a line

Hence DOB is a straight line.