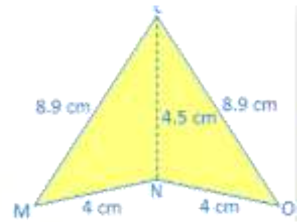
1. In the given figure, apply SSS congruence condition and state the result in the symbolic form.



Ans. In ΔLMN and ΔLON

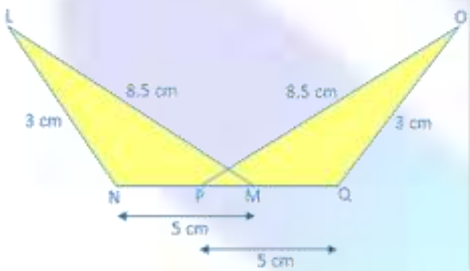
LM = LO = 8.9cm

MN = NO = 4cm

LN = NL = 4.5 cm

Therefore, ΔLMN ≅ ΔLON, by side side side (SSS) congruence condition

2. In the adjoining figure, apply S-S-S congruence condition and state the result in the symbolic form.



Ans. In ΔLNM and ΔOQP

LN = OQ = 3 cm

NM = PQ = 5cm

LM = PO = 8.5cm

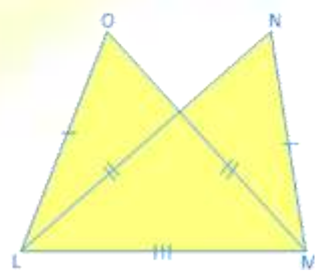
Therefore, ΔLNM ≅ ΔOQP, by Side Side Side (SSS) congruence condition

3. ΔOLM and ΔNML have common base LM, LO = MN and OM = NL. Which of the following are true?

(i) ΔLMN ≅ ΔLMO

(ii) ΔLMO ≅ ΔLNM

(iii) ΔLMO ≅ ΔMLN



Ans. LO = MN and OM = NL → given

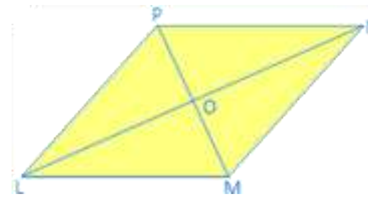
LM = LM → common

Thus, ΔMLN ≅ ΔLMO, by SSS congruence condition

Therefore, statement (iii) is true. So, (i) and (ii) statements are false.

4. By Side Side Side congruence prove that 'Diagonal of the rhombus bisects each other at right angles'.

Ans. Diagonal LN and MP of the rhombus LMNP intersect each other at O.



It is required to prove that LM ⊥ NP and LO = ON and MO = OP.

Proof: LMNP is a rhombus.

Therefore, LMNP is a parallelogram.

Therefore, LO = ON and MO = OP.

In ΔLOP and ΔLOM; LP = LM, [Since, sides of a rhombus are equal]

Side LO is common

PO = OM, [Since diagonal of a parallelogram bisects each other]

Therefore, ΔLOP ≅ ΔLOM, [by SSS congruence condition]

But, ∠LOP + ∠MOL = 2 rt. angle

Therefore, 2∠LOP = 2 rt. angle

or, ∠LOP = 1 rt. angle

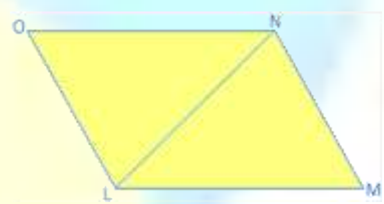
Therefore, LO ⊥ MP

i.e., LN ⊥ MP (Proved)

[Note: Diagonals of a square are perpendicular to each other]

5. If the opposite sides of a quadrilateral are equal, prove that the quadrilateral will be parallelogram.

Ans. LMNO is a parallelogram quadrilateral, whose sides LM = ON and LO = MN. It is required to prove that LMNO is a parallelogram.



Construction: Diagonal LN is drawn.

Proof: In ΔLMN and ΔNOL,

LM = ON and MN = LO, [By hypothesis]

LN is common side.

Therefore, ΔLMN ≅ ΔNOL, [by Side Side Side congruence condition]

Therefore, ∠MLN = ∠LNO, [Corresponding angles of congruent triangles]

Since, LN cuts LM and ON and the both alternate angles are equal.

Therefore, LM ∥ ON

Again, ∠MNL = ∠OLN [Corresponding angles of congruent triangles]

But LN cuts LO and MN, and the alternate angles are equal.

Therefore, LO ∥ MN

Therefore, In quadrilateral LMNO,

LM ∥ ON and

LO ∥ MN.

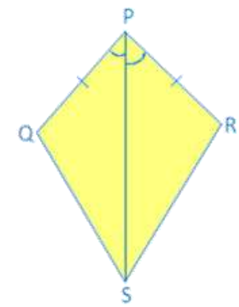
Therefore, LMNO is a parallelogram. [Proved]

[Note: Rhombus is parallelogram.]

6. In the kite shown, PQ = PS and ∠QPR = ∠SPR.

(i) Find the third pair of corresponding parts to make Δ PQR ≅ ΔPSR by SAS congruence condition.

(ii) Is ∠QRP = ∠SRP?



Ans. (i) In Δ PQR and Δ PSR

PQ = PS → given

∠QPR = ∠SPR → given

PR = PR → common

Therefore, ΔPQR ≅ ΔPSR by SAS congruence condition

(ii) Yes, ∠QRP = ∠SRP (corresponding parts of congruence triangle).

7. Identify the congruent triangle:

Ans. In ΔLMN,

65° + 45° + ∠L = 180°

110° + ∠L = 180°

∠L = 180° - 110°

Therefore, ∠L = 70°

Now in ΔXYZ and ΔLMN

∠X = ∠L (given in the picture)

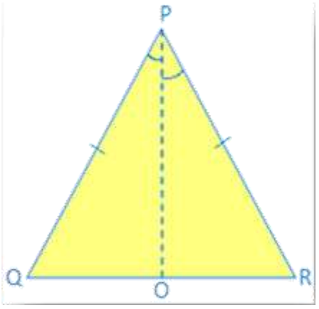
XY = LM (given in the picture)

XZ = NL (given in the picture)

Therefore, ΔXYZ ≅ ΔLMN by SAS congruence axiom

8. By using SAS congruency proof that, angles opposite to equal side of an isosceles triangle are equal.

Ans. Given: ΔPQR is isosceles and PQ = PR



Construction: Draw PO, the angle bisector of ∠P, PO meets QR at O.

Proof: In ΔQPO and ΔRPO

PQ = PR (given)

PO = PO (common)

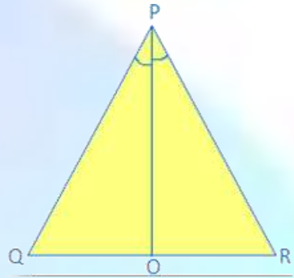
∠QPO = ∠RPO (by construction)

Therefore, ΔQPO ≅ ΔRPO (by SAS congruence)

Therefore, ∠PQO = ∠PRO (by corresponding parts of congruent triangle)

9. Show that bisector of the vertical angle of an isosceles triangle bisects the base at right angle.

Ans. Given: ΔPQR is isosceles, and PO bisects ∠P



Proof: In ΔPOQ and ΔPOR

PQ = PR (isosceles triangle)

∠QPO = ∠RPO (PO bisects ∠P)

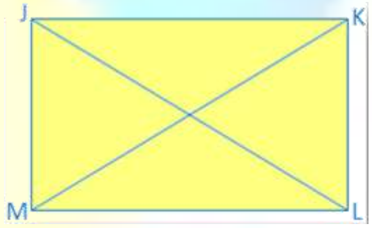
PO = PO (common)

Therefore, Δ POQ ≅ Δ POR (by SAS congruence axiom)

Therefore, ∠POQ = ∠POR (by corresponding parts of congruent triangle)

10. Diagonals of a rectangle are equal.

Ans. In the rectangle JKLM, JL and KM are the two diagonals.



It is required to prove that JL = KM.

Proof: In ΔJKL and ΔKLM,

JK = ML [Opposite of a parallelogram]

KL = KL [Common side]

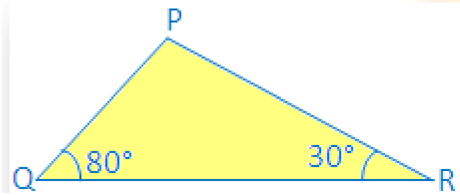
∠JKL = ∠KLM [Both are right angle]

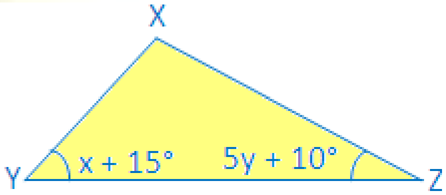
Therefore, ΔJKL ≅ ΔKLM [By Side Angle Side Congruence]

Therefore, JL = KM [Corresponding parts of congruence triangle]

Note: Diagonals of a square are equal to one another.

11. ΔPQR ≅ ΔXYZ by ASA congruence condition. Find the value of x and y.





Ans. We know Δ PQR ≅ ΔXYZ by ASA congruence.

Therefore ∠Q = ∠Y i.e., x + 15 = 80° and ∠R = ∠Z i.e., 5y + 10 = 30°.

Also, QR = YZ.

Since, x + 15 = 80°

Therefore x = 80 – 15 = 65°

Also, 5y + 10 = 30°

So, 5y = 30 – 10

Therefore, 5y = 20

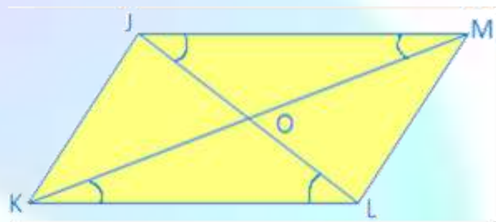
⇒ y = 20/5

⇒ y = 4°

Therefore, the value of x and y are 65° and 4°.

12. Prove that the diagonals of a parallelogram bisect each other.

Ans. In a parallelogram JKLM, diagonal JL and KM intersect at O



It is required to prove that JO = OL and KO = OM

Proof: In ΔJOM and ΔKOL

∠OJM = ∠OLK [since, JM ∥ KL and JL is the transversal]

JM = KL [opposite sides of a parallelogram]

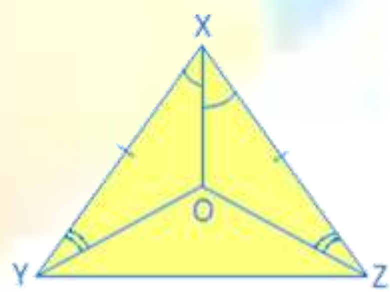
∠OMJ = ∠OKL [since, JM ∥ KL and KM is the transversal]

Therefore, ΔJOM and ΔKOL [Angle-Side-Angel]

Therefore, JO = OL and KO = OM [Sides of congruent triangle]

13. ΔXYZ is an equilateral triangle such that XO bisects ∠X.

Also, ∠XYO = ∠XZO. Show that ΔYXO ≅ ΔZXO



Ans. Δ XYZ is an equilateral

Therefore, XY = YZ = ZX

Given: XY bisects ∠X.

Therefore, ∠YXO = ∠ZXO

Given: ∠XYO = ∠XZO

Given: XY = XZ

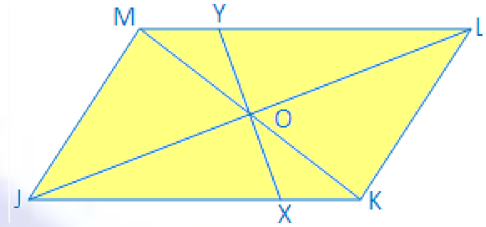
Therefore, ΔYXO ≅ ΔZXO by ASA congruence condition

14. The straight line drawn through the intersection of the two diagonals of a parallelogram divide it into two equal parts.

Ans. O is the point of intersection of the two diagonals JL and KM of the parallelogram JKLM.

Straight line XOY meets JK and LM at the point X and Y respectively.

Proof: In ΔJXO and ΔLYO, JO = OL [diagonals of a parallelogram bisect each other]



∠OJX= alternate ∠OLY

∠JOX = ∠LOY

Therefore, Δ JOX ≅ Δ LOY [by angle side angle congruence]

Therefore, JX = LY

Therefore, KX = MY [since, JK = ML]

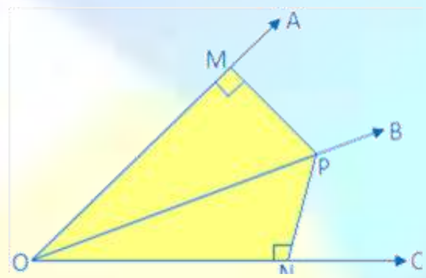
Now in quadrilaterals JXYM and LYXK, JX = LY; XY = YX, YM = XK and MJ = KL and ∠MJX = ∠KLY

Hence it is proved that in the two quadrilaterals the sides are equal to each other and the included angles of two equal sides are also equal.

Therefore, quadrilateral JXYM equal to quadrilateral XKLY.

15. OB is the bisector of ∠AOC, PM ⊥ OA and PN ⊥ OC. Show that ΔMPO ≅ ΔNPO.

Ans. In ΔMPO and ΔNPO



PM ⊥ OM and PN ⊥ ON

Therefore ∠PMO = ∠PNO = 90°

Also, OB is the bisector of ∠AOC

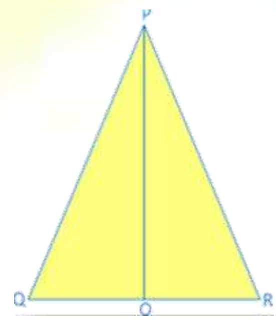
Therefore ∠MOP = ∠NOP

OP = OP common

Therefore, ΔMPO ≅ ΔNPO by AAS congruence condition.

16. ΔPQR is an isosceles triangle such that PQ = PR, prove that the altitude PO from P on QR bisects PQ.

Ans. In the right triangles POQ and POR,



∠POQ = ∠POR = 90°

PQ = PR [since, ΔPQR is an isosceles. Given PQ = PR]

PO = OP [common]

Therefore Δ POQ ≅ Δ POR by RHS congruence condition

So, QO = RO (by corresponding parts of congruence triangles)

17. ΔXYZ is an isosceles triangle such that $XY = XZ$, prove that the altitude XO from X on YZ bisects YZ.

Ans. In the right triangles XOY and XOZ,

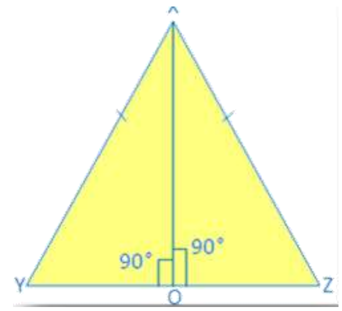
$\angle XOY = \angle XOZ = 90°$

$XY = XZ$ [since, ΔXYZ is an isosceles. Given $XY = XZ$]

$XO = OX$ [common]

Therefore $\Delta\ XOY \cong \Delta\ XOZ$ by RHS congruence condition

So, $YO = ZO$ (by corresponding parts of congruence triangles)



18. In the adjoining figure, given that $AB = BC$, $YB = BZ$, $BA \perp XY$ and $BC \perp XZ$. Prove that $XY = XZ$.

Ans. In right triangles YAB and BCZ we get,

$YB = BZ$ [given]

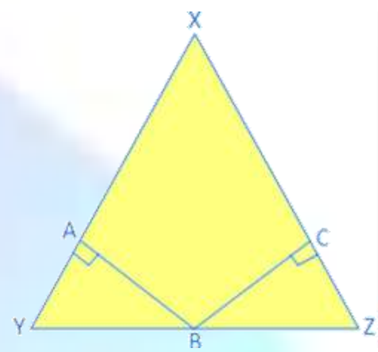
$AB = BC$ [given]

So, by RHS congruence condition

$\Delta\ YAB \cong \Delta\ BCZ$

$\angle Y = \angle Z$ (since by corresponding parts of congruence triangles are equal)

$XZ = XY$ (since sides opposite to equal angles are equal)



19. $LM = NO$ and $LO = MN$. Show that $\Delta\ LON \cong \Delta\ NML$.

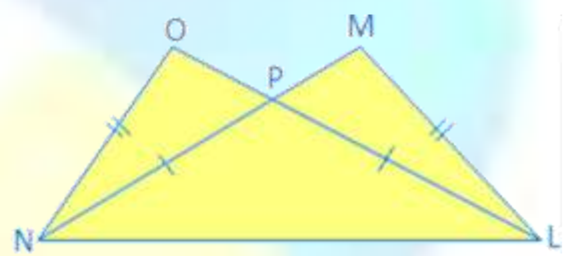
Ans. In ΔLON and ΔNML

$LM = NO$ → given

$LO = MN$ → given

$LN = NL$ → common

Therefore, $\Delta\ LON \cong \Delta\ NML$, by side-side-side (SSS) congruence condition .



20. In a quadrilateral LMNP, $LM = LP$ and $MN = NP$. Prove that $LN \perp MP$ and $MO = OP$

[O is the point of intersection of MP and LN]

Ans. In ΔLMN and ΔLPN,

$LM = LP$,

$MN = NP$,

$LN = NL$

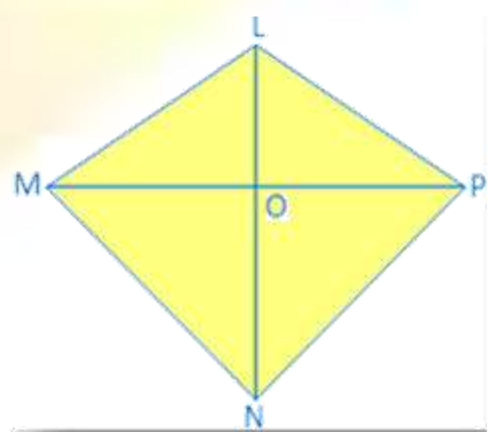
Therefore, $\Delta LMN \cong \Delta LPN$, [by SSS congruence condition]

Therefore, $\angle MLN = \angle PLN$ -------- (i)

Now in ΔLMO and ΔLPO,

$LM = LP$;

LO is common and



$\angle MLO = \angle PLO$

$\Delta LMO \cong \Delta LPO$, [by SAS congruence condition]

Therefore, $\angle LOM = \angle LOP$ and

$MO = OP$, [Proved]

But $\angle LOM + \angle LOP = 2$ rt. angles.

Therefore, $\angle LOM = \angle LOP = 1$ rt. angles.

Therefore, $LO \perp MP$

i.e., $LN \perp MP$, [Proved]