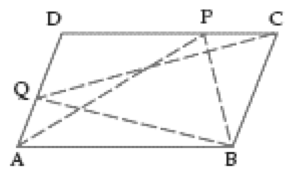
Board – ICSE	Class – IX	Topic – Circle

1. **P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that area of ΔAPB = area of ΔBQC.**

Solution:

Given a parallelogram ABCD, and P and Q are points lying on the sides DC and AD respectively as shown in the adjoining figure. As ΔAPB and || gm ABCD are on the same base and between the same parallels AB and DC,



area of ΔAPB $= \frac{1}{2}$ area of || gm ABCD ... (i)

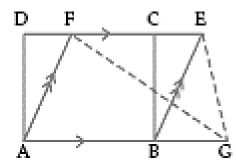
Also, as ΔBQC and || gm ABCD are on the same BC and between the same parallels AD and BC,

area of ΔBQC $= \frac{1}{2}$ area of || gm ABCD ... (ii)

From (i) and (ii), we get

Area of ΔAPB = area of ΔBQC.

2. In the adjoining figure, ABCD is a rectangle with sides AB = 8 cm and AD = 5 cm. Compute
   (i) Area of parallelogram ABEF
   (ii) Area of ΔEFG.



Solution:

(i) Area of || gm ABEF= area of rectangle ABCD (on the same base AB and between the same parallels AB and DE)

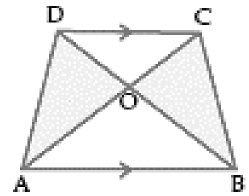
$= (8 \times 5)\ \text{cm}^2 = 40\ \text{cm}^2$.

(ii) Area of ΔEFG $= \frac{1}{2} \times$ area of || gm ABEF

(On the same base FE and between the same parallels FE and AG)

$= \left(\frac{1}{2} \times 40\right) \text{cm}^2 = 20\ \text{cm}^2$

3. ABCD is a trapezium with AB || DC, and diagonals AC and BD meet at O. Prove that area of Δ DAO = area of Δ OBC.

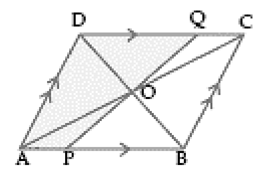


Solution:

Statements	*Reasons*
1. AB ∥ DC	1. Given.
2. Area of ΔABD = area of ΔABC	2. Δs on the same base AB and between the same parallels AB and CD are equal in area.
3. Area of ΔDAO + area of ΔOAB = area of ΔOBC + area of ΔOAB	3. Addition area axiom.
4. Area of ΔDAO = area of ΔOBC **Q.E.D.**	4. Subtracting same area from both sides.

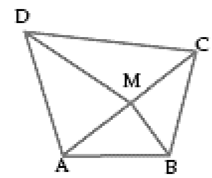
4. The diagonals of a parallelogram ABCD intersect at O. A straight line through O meets AB at P and the opposite side CD at Q. Prove that area of quad. **APQD** $= \frac{1}{2}$ Area of || gm ABCD.

Solution:



Statements	*Reasons*
1. Area of ΔACD = $\frac{1}{2}$ area of ∥ gm ABCD	1. Diagonal divides a ∥gm into two Δs of equal area.
In ΔOAP and ΔOCQ	
2. ∠OAP = ∠OCQ	2. Alt. ∠s.
3. ∠AOP = ∠COQ	3. Vert. opp. ∠s.
4. AO = OC	4. Diagonals bisect each other.
5. ΔOAP ≅ ΔOCQ	5. ASA rule of congruency.
6. Area of ΔOAP = area of ΔOCQ	6. Congruence area axiom.
7. Area of ΔOAP + area of quad. AOQD = area of ΔOCQ + area of quad. AOQD	7. Adding same area on both sides.
8. Area of quad. APQD = area of ΔACD	8. Addition area axiom.
9. Area of quad. APQD $= \frac{1}{2}$ area of ∥ gm ABCD **Q.E.D.**	9. From 8 and 1.

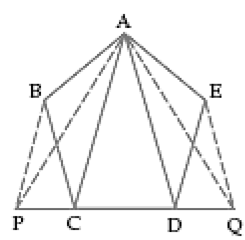
5. In quadrilateral ABCD, M is mid-point of the diagonal AC. Prove that area of quad. ABMD = area of quad. DMBC.



Solution:

Statements	*Reasons*
1. BM is median of ΔBCA	1. M is mid-point of AC (given).
2. Area of ΔABM = area of ΔMBC	2. Median divides a Δinto two Δs of equal area.
3. DM is median of ΔDAC	3. M is mid-point of AC (given).
4. Area of ΔDAM = area of ΔDMC	4. Median divides a triangle into two Δs of equal area.
5. Area of ΔABM + area of ΔDAM = area of ΔMBC + area of ΔDMC	5. Adding 2 and 4.
6. Area of quad. ABMD = area of quad. DMBC **Q.E.D.**	6. Addition area axiom.

6. In the adjoining figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that area of ABCDE = area of ΔAPQ.



Solution:

ΔPCA and ΔBCA are on the same base CA and between same parallels BP || AC.

∴ Area of ΔBCA = area of ΔPCA ... (i)

ΔEAD and ΔQAD are on the same base AD and between same parallels EQ || AD,

∴ Area of ΔEAD = area of ΔQAD ... (ii)

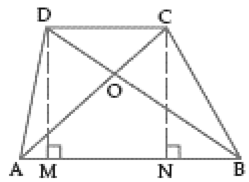
Also, area of ΔACD = area of ΔACD ... (iii)

On adding (i), (iii) and (ii), we get area of ΔBCA + area of ΔACD + area of ΔEAD

= area of ΔPCA + area of ΔACD + area of ΔQAD

$\Rightarrow$ Area of ABCDE = area of ΔAPQ.

7. The diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that area of Δ OAD = area of Δ OBC. Prove that ABCD is a trapezium.



Solution:

Draw DM $\perp$ AB and CN $\perp$ AB.

As DM and CN are both perpendiculars to AB, therefore, DM || CN.

Given area of Δ OAD = area of Δ OBC

$\Rightarrow$ Area of Δ OAD + area of Δ OAB = area of Δ OBC + area of Δ OAB

(adding same area on both sides)

$\Rightarrow$ Area of Δ ABD = area of Δ ABC

$\Rightarrow \frac{1}{2}AB \times DM = \frac{1}{2}AB \times CN$

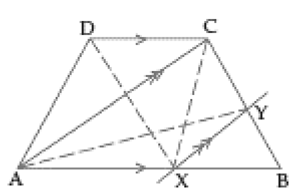
$\Rightarrow DM = CN.$

Thus DM || CN and DM = CN, therefore, DMNC is a parallelogram

$\Rightarrow$ D C || MN i.e. DC || AB.

Hence, ABCD is a trapezium.

8. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that: area of ΔADX = area of ΔACY.



Solution:

Join CX.

As triangles ADX and ACX have same base AX and are between the same

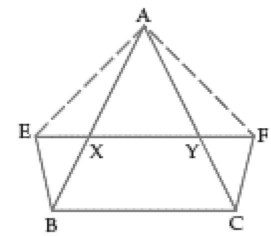
Parallels (AB || DC given, so, AX || DC),

∴ Area of ΔADX = area of ΔACX ... (i)

As triangles ACY and ACX have same base AC and are between the same parallels (XY || AC given),

∴ Area of ΔACY = area of ΔACX ... (ii)

From (i) and (ii), we get area of ΔADX = area of ΔACY.



9. XY is a line parallel to side BC of a triangle ABC. If BE || CA and FC || AB meet XY at E and F respectively, show that area of ΔABE = area of ΔACF.

Solution:

As ΔABE and || gm EBCY have the same base BE and are between the same parallels BE || CA (given),

∴ Area of $\Delta ABE = \frac{1}{2} \times$ Area of || gm EBCY ... (i)

As ΔACF and || gm XBCF have the same base CF and are between the same parallels FC || AB (given),

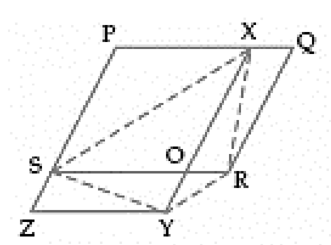
∴ Area of $\Delta ACF = \frac{1}{2} \times$ Area of || gm XBCF ... (ii)

But || gm EBCY and || gm XBCF have the same base BC and are between the same parallels (XY || BC given),

∴ Area of || gm EBCY = area of || gm XBCF

$\Rightarrow \frac{1}{2} \times$ Area of || gm EBCY $= \frac{1}{2} \times$ Area of || gm XBCF

⇒ Area of ΔABE = area of ΔACF



10. In the adjoining figure, PQRS and PXYZ are two parallelograms of equal SX is parallel to YR.

Solution:

Join XR, SY.

Given area of || gm PQSR = area of || gm PXYZ.

Subtract area of || gm PSOX from both sides.

∴ Area of || gm XORQ = area of || gm SZYO

⇒ Area of Δ

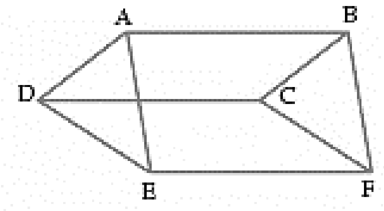
XOR = area of Δ SYO (because diagonal divides a || gm into two equal areas)

Adding area of Δ OYR to both sides, we get area of Δ XYR = area of Δ SYR.

Also the Δs XYR and SYR have the same base YR, therefore, these lie between the same Parallels ⇒ SX is parallel to YR.

11. In the adjoining figure, ABCD, DCFE and ABFE are parallelograms. Show that area of ΔADE = area of ΔBCF.

Solution:



As ABCD is a parallelogram, AD= B C (opp. sides of a || gm)

Similarly, DE = CF and AE = BF.

In ΔADE and ΔBCF,

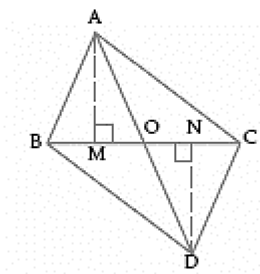
AD= B C, DE = CF and AE = BF

∴ ΔADE ≅ ΔBCF (by SSS rule of congruency)

∴ Area of ΔADE = area of ΔBCF (congruent figures have equal areas)

12. Triangles ABC and DBC are on the same base BC with A, D on opposite sides of BC. If area of Δ ABC = area of Δ DBC, prove that BC bisects AD.

Solution:



Let BC and AD intersect at O.

Draw AM ⊥ BC and DN ⊥ BC.

Given area of Δ ABC = area of Δ DBC

$\Rightarrow \frac{1}{2}$ BC × AM $= \frac{1}{2}$ BC × DN

⇒ AM = DN.

In ΔAMO and ΔDNO,

∠ AOM = ∠ DON (vert. opp. ∠ s)

∠ AMO = ∠ DNO (each angle = 90°)

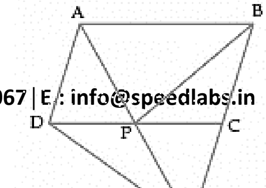
AM = DN (proved above)

∴ Δ AMO ≅ Δ DNO (AAS rule of congruency)

∴ A O = DO (c.p.c.t.)

Hence, BC bisects AD.

13. In the adjoining figure, ABCD is a parallelogram and BC is produced to a point



Q such that CQ = AD. If AQ intersects DC at P, show that area of ΔBPC = area of ΔDPQ.

Solution:

Join AC. A s triangles BPC and APC have same base PC and are between the same parallels (AB || DC i.e. AB || PC),

∴ Area of ΔBPC = area of ΔAPC … (i)

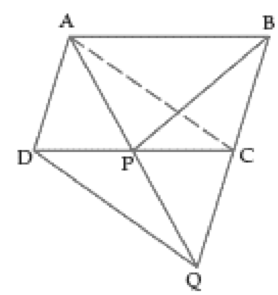
In quad. ADQC, AD || CQ

(∵ AD || BC, opp. sides of || gm ABCD)

AD= CQ (given)

∴ ADQC is a parallelogram, so its diagonals AQ and DC bisect each other

i.e. DP = PC and AP = PQ.



In ΔAPC and ΔQPD, PC = D P

AP = PQ

∠APC = ∠QPD (vert. opp. ∠s)

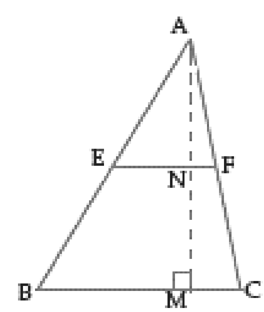
ΔAPC ≅ ΔQPD

∴ Area of ΔAPC = area of ΔDPQ … (ii)

From (i) and (ii), we get

Area of ΔBPC = area of ΔDPQ.

14. ABC is a triangle whose area is 50 cm². E and F are mid-points of the sides AB and AC respectively. Prove that EBCF is a trapezium. Also find its area.



Solution:

Since E and F are mid-points of the sides AB and AC respectively,

E F || BC and EF = 1

2 BC.

As EF || BC, EBCF is a trapezium.

From A, draw AM ⊥ BC.

Let AM meet EF at N.

Since EF || BC, ∠ ENA = ∠ BMN.

But ∠ BMN = 90° (∵AM ⊥ BC)

So ∠ ENA = 90° i.e. AN⊥ EF.

Also, as E is mid-point of AB and EN || BM, N is mid-point of AM.

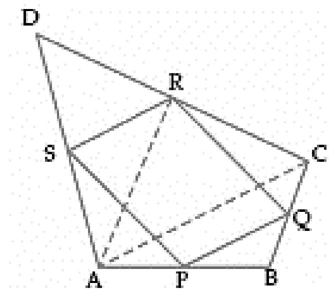
Now, area of Δ AEF $= \frac{1}{2}\text{EF} \times \text{AN} = \frac{1}{2}\left(\frac{1}{2}\text{ BC} \times \frac{1}{2}\text{AM}\right) = \frac{1}{4}\left(\frac{1}{2}\text{BC} \times \text{AM}\right) = \frac{1}{4}$ (area of Δ ABC)

$= \frac{1}{4}(50\text{cm}^2) = 12.5 \text{ cm}^2.$

$\therefore$ Area of trapezium EBCF = area of Δ ABC – area of Δ AEF

$= 50 \text{ cm}^2 - 12{\cdot}5 \text{ cm}^2 = 37{\cdot}5 \text{ cm}^2.$

15. Prove that the area of the quadrilateral formed by joining the mid-points of the adjacent sides of a quadrilateral is half the area of the given quadrilateral.



Solution:

A quadrilateral ABCD, and PQRS is the quadrilateral formed by joining mid-points of the sides AB, BC, CD and DA respectively.

To prove: Area of quad. PQRS $= \frac{1}{2}$ area of quad. ABCD.

Construction: Join AC and AR.

Proof:

Statements	*Reasons*
1. Area of ΔARD = $\frac{1}{2}$ area of ΔACD	1. Median divides a triangle into two triangles of equal area.
2. Area of ΔSRD = $\frac{1}{2}$ area of ΔARD	2. Same as in 1.
3. Area of ΔSRD = $\frac{1}{4}$ area of ΔACD	3. From 1 and 2.
4. Area of ΔPBQ = $\frac{1}{4}$ area of ΔABC	4. As in 3.
5. Area of ΔSRD + area of ΔPBQ = $\frac{1}{4}$(area of ΔACD + area of ΔABC)	5. Adding 3 and 4.
6. Area of ΔSRD + area of ΔPBQ = $\frac{1}{4}$ area of quad. ABCD	6. Addition area axiom.
7. Area of ΔAPS + area of ΔQCR = $\frac{1}{4}$ area of quad. ABCD	7. Same as in 6.
8. Area of ΔAPS + area of ΔPBQ + area of ΔQCR + area of ΔSRD = $\frac{1}{2}$ area of quad. ABCD	8. Adding 6 and 7.
9. Area of ΔAPS + area of ΔPBQ + area of ΔQCR + area of ΔSRD + area of quad. PQRS = area of quad. ABCD	9. Addition area axiom.
10. Area of quad. PQRS = $\frac{1}{2}$ area of quad. ABCD Q.E.D.	10. Subtracting 8 from 9.