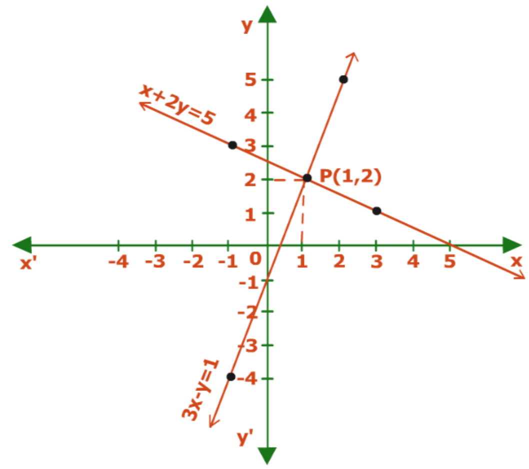
Class – 9th | **Topic – Graphical solution**

1. Solve graphically the following systems of equations.

$x + 2y = 5$
$3x - y = 1$

Solution



$x + 2y = 5$
$3x - y = 1$
$x + 2y = 5$
$\Rightarrow y = \frac{5-x}{2}$
$\therefore$ if $x = -1, y = \frac{5+1}{2} = 3$
If $x = 1, y = \frac{5-1}{2} = 2$
If $x = 3, y = \frac{5-3}{2} = 1$
On the basis of the above, the following points.

X	-1	1	3
y	3	2	1

Plotting (-1, 3), (1, 2), (3, 1) and joining them, we get a straight line.
Now, $3x - y = 1$
$\Rightarrow y = 3x - 1$
If $x = -1$,
$y = -3 - 1 = -4$
$If\ x = 1$
$y = 3 - 1 = 2$
If $x = 2, y = 6 - 1 = 5$

X	-1	1	2
Y	-4	2	5

Plotting (-1,-4), (1, 2), (2, 5) and joining them, we get another straight line.
These lines intersect at the point P (1, 2) and therefore, the solution of the equation is x = 1, y =2

2. Show graphically that these system of equations has no solution

$x - 2y = 4$
$2x - 4y = 5$

Solution

$x - 2y = 4$

$$\Rightarrow y = \frac{x-4}{2}$$

$\therefore$ If $x = 0, y = -2$

If $x = 2, y = -1$

If $x = 4, y = 0$

We now get the following points.

X	0	2	4
Y	0	-1	0

Plotting (0, -2), (2, -1) (4, 0) and joining them, we get the required line.

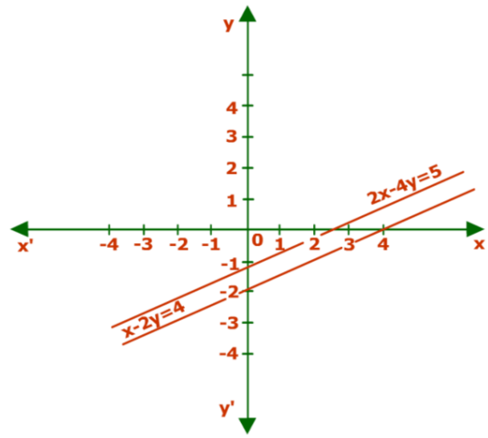
2x - 4y = 5

$$\Rightarrow y = \frac{2x-5}{4}$$

$\therefore$ If $x = -\frac{1}{2}, y = -\frac{3}{2}$

If $x = \frac{1}{2}, y = -1$

If $x = 2, y = -\frac{1}{4}$



We now have the following points.

X	$-\frac{1}{2}$	$\frac{1}{2}$	2
y	$-\frac{3}{2}$	-1	$-\frac{1}{4}$

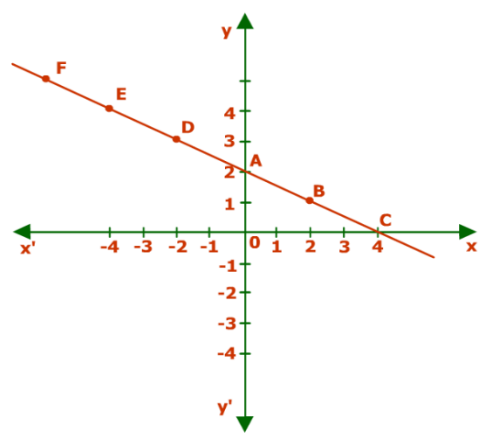
Plotting, $\left(-\frac{1}{2}, -\frac{3}{2}\right), \left(\frac{1}{2}, -1\right), \left(2, -\frac{1}{4}\right)$ and joining them, we get the required line.

3. Show graphically that the given system of equations has infinite number of solutions.

$x + 2y = 4$

$3x + 6y = 12$

Solution



$x + 2y = 4$

$$\Rightarrow y = \frac{4-x}{2}$$

$\therefore$ If $x = 0, y = 2$

If $x = 2, y = 1$

If $x = 4, y = 0$

X	0	2	4
y	2	1	0

Plotting A (0, 2), B (2, 1), C (4, 0) and joining them, we get the required line.

Now, $3x + 6y = 12$

$$\Rightarrow y = \frac{12-3x}{6} = \frac{4-x}{2}$$

If $x = -2, y = 3$

If $x = -4, y = 4$
If $x = -6, y = 5$
Plotting D (-2, 3), E (-4, 4), F (-6, 5) and joining them, we get the required lines.

4. Determine whether or not (1, 0) is a solution to the system $\begin{cases} x - y = 1 \\ -2x + 3y = 5 \end{cases}$

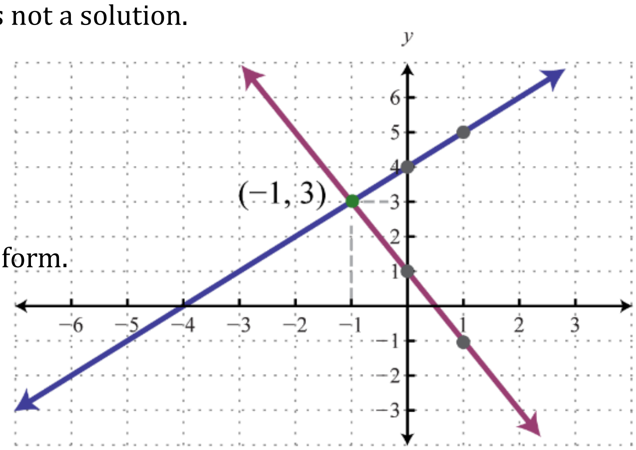
Solution

Substitute the appropriate values into both equations.

Check: (1, 0)	
Equation1: $x - y = 1$	Equation2: $-2x + 3y = 5$
(1)−(0)=1 1-0=1 1=1 ✓	-2(1) +3(0)=5 -2+0=5 -2=5 ✗

Since (1, 0) does not satisfy both equations, it is not a solution.

5. Solve by graphing: $\begin{cases} x - y = -4 \\ 2x + y = 1 \end{cases}$



Solution

Rewrite the linear equations in slope-intercept form.

$x - y = 4$ $-y = -x - 4$ $\frac{-y}{-1} = \frac{-x-4}{1}$ $y = x + 4$	$2x + y = 1$ $y = -2x + 1$

Write the equivalent system and graph the lines on the same set of axes.

$$\begin{cases} x - y = -4 \\ 2x + y = 1 \end{cases} \Rightarrow \begin{cases} y = x + 4 \\ y = -2x + 1 \end{cases}$$

Line 1: $y = x + 4$ $y -$ intercept: $(0, 4)$ slope: $m = 1 = \frac{1}{1} = \frac{\text{rise}}{\text{run}}$	Line 2: $y = -2x + 1$ $y -$ intercept: $(0, 1)$ slope: $m = -2 = \frac{-2}{1} = \frac{\text{rise}}{\text{run}}$

Use the graph to estimate the point where the lines intersect and check to see if it solves the original system. In the above graph, the point of intersection appears to be $(-1, 3)$.

Check: (-1, 3)	
Line 1: $x - y = -4$	Line 2: $-2x + y = 1$
(-1)−(-3)=-4 -1-3=-4 -4=-4 ✓	2(-1) +(3)=1 -2+3=1 1=1 ✓

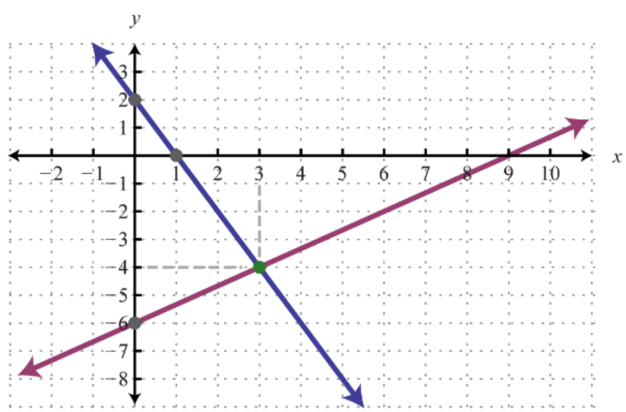
6. Solve by graphing: $\begin{cases} 2x + y = 2 \\ -2x + 3y = -18 \end{cases}$

Solution

We first solve each equation for y to obtain an equivalent system where the lines are in slope-intercept form.

$$\begin{cases} 2x + y = 2 \\ -2x + 3y = -18 \end{cases} \Rightarrow \begin{cases} y = -2x + 2 \\ y = \frac{2}{3}x - 6 \end{cases}$$

Graph the lines and determine the point of intersection.

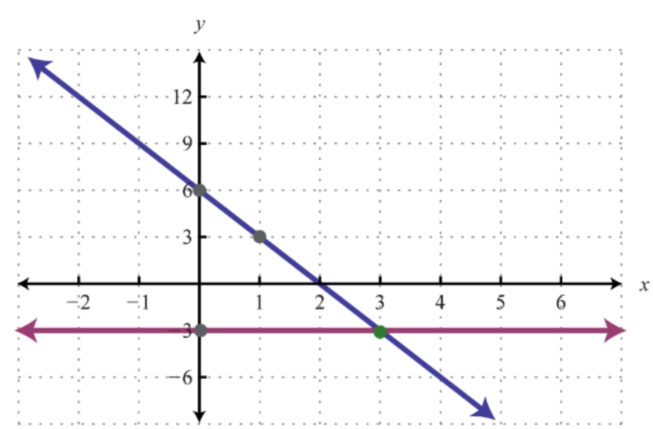


Check : (3, -4)	
$2x + y = 2$ $2(3) + (-4) = 2$ $6 - 4 = 2$ $2 = 2$	$-2x + 3y = -18$ $-2(3) + 3(-4) = -18$ $-6 - 12 = -18$ $-18 = -18$

7. Solve by graphing: $\begin{cases} 3x + y = 6 \\ y = -3 \end{cases}$

Solution

$$\begin{cases} 3x + y = 6 \\ y = -3 \end{cases} \Rightarrow \begin{cases} y = -3x + 6 \\ y = -3 \end{cases}$$

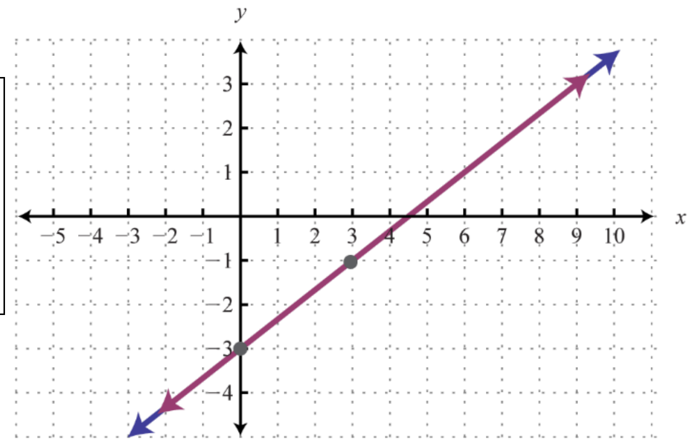


Check : (3, -3)	
$3x + y = 6$ $3(3) + (-3) = 6$ $9 - 3 = 6$ $6 = 6$ ✓	$y = -3$ $(-3) = -3$ $-3 = -3$ ✓

8. Solve by graphing: $\begin{cases} -2x + 3y = -9 \\ 4x - 6y = 18 \end{cases}$

Solution

Determine slope-intercept form for each linear equation in the system.



$-2x + 3y = -9$ $-2x + 3y = -9$ $3y = 2x - 9$ $y = \dfrac{2x - 9}{3}$ $y = \dfrac{2}{3}x - 3$	$4x - 6y = 18$ $4x - 6y = 18$ $-6y = -4x + 18$ $y = \dfrac{-4x + 18}{-6}$ $y = \dfrac{2}{3}x - 3$

$$\begin{cases} -2x + 3y = -9 \\ 4x - 6y = 18 \end{cases} \Rightarrow \begin{cases} y = \dfrac{2}{3}x - 3 \\ y = \dfrac{2}{3}x - 3 \end{cases}$$

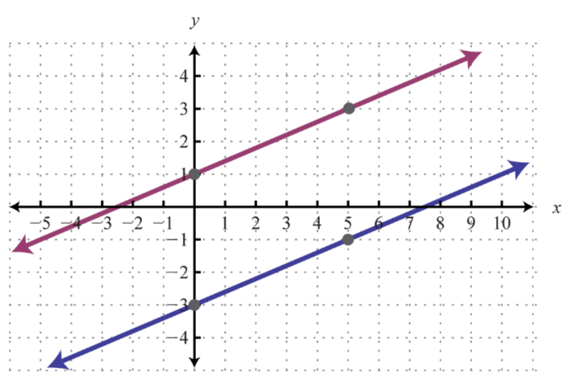
In slope-intercept form, we can easily see that the system consists of two lines with the same slope and same y-intercept. They are, in fact, the same line. And the system is dependent.

9. Solve by graphing: $\begin{cases} -2x + 5y = -15 \\ -4x + 10y = 10 \end{cases}$

Solution

Determine slope-intercept form for each linear equation.

$-2x + 5y = -15$ $-2x + 5y = -15$ $5y = 2x - 15$ $y = \dfrac{2x - 15}{5}$ $y = \dfrac{2}{5}x - 3$	$-4x + 10y = 10$ $-4x + 10y = 10$ $10y = 4x + 10$ $y = \dfrac{4x + 10}{10}$ $y = \dfrac{2}{5}x + 1$



$$\begin{cases} -2x + 5y = -15 \\ -4x + 10y = 10 \end{cases} \Rightarrow \begin{cases} y = \dfrac{2}{5}x - 3 \\ y = \dfrac{2}{5}x + 1 \end{cases}$$

In slope-intercept form, we can easily see that the system consists of two lines with the same slope and different y-intercepts. Therefore, the lines are parallel and will never intersect.