Class – IX	Topic – Expansion

1. **Without multiplying evaluate:**

   (i) $(\mathbf{101})^2$

   (ii) $(\mathbf{502})^2$

   (iii) $(\mathbf{97})^2$

   (iv) $(\mathbf{998})^2$

Solution:

(i) $(101)^2 = (100+1)^2 = (100)^2 + (1)^2 + 2 \times 100 \times 1 = 10000 + 1 + 200 = 10201$

(ii) $(502)^2 = (500+2)^2 = (500)^2 + (2)^2 + 2 \times 2 \times 500 = 250000 + 4 + 2000 = 252004$

(iii) $(97)^2 = (100-3)^2 = (100)^2 + (3)^2 - 2 \times 100 \times 3 = 10000 + 9 + 600 = 9409$

(iv) $(998)^2 = (1000-2)^2 = (1000)^2 + (2)^2 - 2 \times 1000 \times = 1000000 + 4 - 4000 = 996004$

2. **Solve:**

   (i) $\mathbf{If\ a + b = 7\ and\ ab = 10, find\ the\ value\ of\ (a - b)}$

   (ii) $\mathbf{If\ x - y = 5\ and\ xy = 24, find\ the\ value\ of\ (x + y)}$

Solution:

(i) $(a-b)^2 = (a+b)^2 - 4ab = (7)^2 - 4 \times 10 = 49 - 40 = 9 = (\pm 3)^2$

$\therefore (a-b) = \pm 3$

(ii) $(x+y)^2 = (x-y)^2 + 4xy = (5)^2 + 4 \times 24 = 25 + 96 = 121 = (\pm 11)^2$

$\therefore x + y = \pm 11$

3. **If $(\mathbf{a - b}) = \mathbf{0.9\ and\ ab} = \mathbf{0.36}$, find the values of**

   (i) $(\mathbf{a + b})$

   (ii) $(\mathbf{a^2 - b^2})$

Solution:

$a - b = 0.9$ and $ab = 0.36$

(i) $(a+b)^2 = (a-b)^2 + 4ab$ .....(i)

Putting values of (a+b) and (a-b) in equation (i), we get

$= (0.9)^2 + 4 \times 0.36 = 0.81 + 1.44 = 2.25 = (\pm 1.5)^2$

$\therefore a + b = \pm 1.5$

(ii) $a^2 - b^2 = (a+b)(a-b)$ .....(ii)

Putting values of $(a+b)$ and $(a-b)$ in eqn. (ii), we get

$= \pm 1.5 \times 0.9 = \pm 1.35$

4. If $a + \frac{1}{a} = 6$, find :

(i) $a - \frac{1}{a}$

(ii) $a^2 - \frac{1}{a^2}$

Solution:

(i) $a + \frac{1}{a} = 6$

We know that $\left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4$

Putting the value of $\left(a + \frac{1}{a}\right)$, we get

$(6)^2 - \left(a - \frac{1}{a}\right) = 4 \Rightarrow -\left(a - \frac{1}{a}\right)^2 = 4 - 36 \Rightarrow \left(a - \frac{1}{a}\right)^2 = 32$

Taking square root of both sides, we get

$\sqrt{\left(a - \frac{1}{a}\right)^2} = \pm\sqrt{32} \Rightarrow \sqrt{\left(a - \frac{1}{a}\right)^2} = \pm\sqrt{16 \times 2} \Rightarrow a - \frac{1}{a} = \pm 4\sqrt{2}$

(ii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

Here, $\left(a + \frac{1}{a}\right) = 6, \left(a - \frac{1}{a}\right) = \pm 4\sqrt{2}$

$a^2 - \frac{1}{a^2} = (6)(\pm 4\sqrt{2}) = \pm 24\sqrt{2}$

4. If $a = \frac{1}{(a-5)}$ where a≠5 and a≠0, find the values of:

(i) $a - \frac{1}{a}$

(ii) $\left(a + \frac{1}{a}\right)$

(iii) $\left(\mathbf{a^2 - \frac{1}{a^2}}\right)$

(iv) $\left(\mathbf{a^2 + \frac{1}{a^2}}\right)$

Solution:

$$a = \frac{1}{a-5} \Longrightarrow a - 5 = \frac{1}{a}$$

$$\Longrightarrow a - \frac{1}{a} = 5$$

(i) Hence $a - \frac{1}{a} = 5$

(ii) $\left(a + \frac{1}{a}\right)^2 = \left(a - \frac{1}{a}\right)^2 + 4$

$$= (5)^2 + 4$$
$$= 25 + 4 = 29$$

Hence, $a + \frac{1}{a} = \pm\sqrt{29}$

(iii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= \pm\sqrt{29} \times 5$$
$$= \pm 5\sqrt{29}$$

(iv) $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2 = (5)^2 + 2 = 25 + 2 = 27$

Hence, $a^2 + \frac{1}{a^2} = \sqrt{27} = \sqrt{9 \times 3} = \pm 3\sqrt{3}$

5. If $\mathbf{x - \frac{1}{x} = 5}$, find the value of $\mathbf{x^3 - \frac{1}{x^3}}$

Solution:

$$x - \frac{1}{x} = 5$$

Cubing both sides, we get

$$\Rightarrow x^3 = \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 5 = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 15 = 125$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 125 + 15 = 140$$

$$\text{Hence, } x^3 - \frac{1}{x^3} = 140$$

6. If $x + \frac{1}{x} = 4$ find the value of:

(i) $\left(x^3 + \frac{1}{x^3}\right)$

(ii) $\left(x - \frac{1}{x}\right)$

Solution:

(i) $x + \frac{1}{x} = 4$

Cubing both sides, we get

$$\left(x + \frac{1}{x}\right)^3 = (4)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 4 = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 12 = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 64 - 12 = 52$$

(ii) $x + \frac{1}{x} = 4$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = 16 \quad \text{[squaring both side]}$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 - 4 = 16 - 4 \text{ [subtracting 4 from both side]}$$

$$\Rightarrow x^2+\frac{1}{x^2}-2.x.\frac{1}{x}=12 \qquad \left[2 \text{ can be written as } 2\times x\times\frac{1}{x}\right]$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=12 \Rightarrow x-\frac{1}{x}=\sqrt{12}$$

7. If $\mathbf{a-\frac{1}{a}=\sqrt{5}}$, find the values of:

(i) $\left(\mathbf{a+\frac{1}{a}}\right)$

(ii) $\left(\mathbf{a^3+\frac{1}{a}}\right)$

Solution:

(i) $\left(a+\frac{1}{a}\right)^2=\left(a-\frac{1}{a}\right)^2+4=\left(\sqrt{5}\right)^2+4=5+4=9$

$$\therefore a+\frac{1}{a}=\pm\sqrt{9}=\pm3$$

(ii) Cubing both sides, we get

$$\left(a+\frac{1}{a}\right)^3=(\pm3)^3$$

$$\Rightarrow a^3+\frac{1}{a^3}+3\times a\times\frac{1}{a}\left(a+\frac{1}{a}\right)=\pm27$$

$$\Rightarrow a^3+\frac{1}{a^3}+3(\pm3)=\pm27$$

$$\Rightarrow a^3+\frac{1}{a^3}\pm9=\pm27$$

$$\therefore a^3+\frac{1}{a^3}=\pm27-(\pm9)=\pm18$$

8. If $\mathbf{x^2+\frac{1}{25x^2}=8\frac{3}{5}}$ find the value of:

(i) $\left(\mathbf{x+\frac{1}{5x}}\right)$

(ii) $\left(\mathbf{x^3+\frac{1}{125\,x^3}}\right)$

Solution:

(i) $\left(x+\frac{1}{5x}\right)^2 = x^2+\frac{1}{25\,x^2}+2\times x\times\frac{1}{5x} = x^2+\frac{1}{25x^2}+\frac{2}{5} = \frac{43}{5}+\frac{2}{5} = \frac{45}{5} = 9$

$\therefore x+\frac{1}{5x} = \pm\sqrt{9} = \pm 3$

(ii) Cubing both sides, we get

$\left(x+\frac{1}{5x}\right)^3 = (\pm 3)^3$

$\Rightarrow (x^3)+\left(\frac{1}{5x}\right)^3+3\times x\times\frac{1}{5x}\left(x+\frac{1}{5x}\right) = \pm 27$

$\Rightarrow x^3+\frac{1}{125\,x^3}+\frac{3}{5}\times(\pm 3) = \pm 27$

$\Rightarrow x^3+\frac{1}{125\,x^3} = \pm\frac{126}{5} = \pm 25\frac{1}{5}$

9. If $\left(x+\frac{1}{x}\right)^2 = 3$, show that $\left(x^3+\frac{1}{x^3}\right) = 0$

Solution:

$\left(x+\frac{1}{x}\right)^2 = 3$

$\therefore x+\frac{1}{x} = \pm\sqrt{3}$

Cubing both sides, we get

$\left(x+\frac{1}{x}\right)^3 = \left(\pm\sqrt{3}\right)^3 \Rightarrow x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right) = \pm 3\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3}+3\times\left(\pm\sqrt{3}\right) = \pm 3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3}+\left(\pm 3\sqrt{3}\right) = \pm 3\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3} = \pm 3\sqrt{3}-\left(\pm 3\sqrt{3}\right) = 0$

Hence, $x^3+\frac{1}{x^3} = 0$

10. If $x-\frac{1}{x} = 4$, find the values of:

(i) $\left(x^2+\frac{1}{x^2}\right)$

(ii) $\left(\mathbf{x^4+\frac{1}{x^4}}\right)$

Solution:

(i) $x-\frac{1}{x}=4 \Rightarrow \left(x-\frac{1}{x}\right)^2=4^2$

Squaring both sides, we get

$\Rightarrow x^2+\frac{1}{x^2}-2=16$

$\Rightarrow x^2+\frac{1}{x^2}=16+2=18$

$\Rightarrow x^2+\frac{1}{x^2}=18 \qquad ...(1)$

(ii) On squaring both sides of equation (1), we get

$\left(x^2+\frac{1}{x^2}\right)^2=(18)^2$

$\Rightarrow x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}=324$

$\Rightarrow x^4+\frac{1}{x^4}+2=324$

$\Rightarrow x^4+\frac{1}{x^4}=324-2=322$

$\therefore x^4+\frac{1}{x^4}=322$

11. If $\mathbf{a+\frac{1}{a}=p}$; **then show that:**$\mathbf{a^3+\frac{1}{a^3}=p\left(p^2-3\right)}$

Solution:

If $a+\frac{1}{a}=p$, show that, $a^3+\frac{1}{a^3}=p(p^2-3)$

$a+\frac{1}{a}=p \qquad$ [Given]

Cubing both sides, we get

$\therefore \left(a+\frac{1}{a}\right)^3=p^3$

$\Rightarrow a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)=p^3 \Rightarrow a^3+\frac{1}{a^3}=p^3-3(p)$

Hence, $a^3+\frac{1}{a^3}=p(p^2-3)$

12. If $\mathbf{a+\frac{1}{a}=4}$; find:

(i) $\mathbf{a^2+\frac{1}{a^2}}$

(ii) $\mathbf{a^4+\frac{1}{a^4}}$

Solution:

(i) If $a+\frac{1}{a}=4$

Squaring both sides, we get

$\Rightarrow \left(a+\frac{1}{a}\right)^2=(4)^2$

$\Rightarrow a^2+2a\times\frac{1}{a}+\frac{1}{a^2}=16$

$\Rightarrow a^2+2+\frac{1}{a^2}=16$

$\Rightarrow a^2+\frac{1}{a^2}=16-2$

$\Rightarrow a^2+\frac{1}{a^2}=14$

(ii) $a^4+\frac{1}{a^4}=\left(a^2+\frac{1}{a^2}\right)^2-2=(14)^2-2=194$ $\quad$ [Since $a^2+\frac{1}{a^2}=14$ from (i)]

13. If $\mathbf{2x-3y=10}$ and $\mathbf{xy=16}$; find the value of $\mathbf{8x^3-27y^3}$.

Solution:

$2x-3y=10$ and $xy=16$

Cubing both sides, we get

$\therefore (2x-3y)^3=(10)^3$

$\Rightarrow 8x^3-27y^3-3(2x)(3y)(2x-3y)=1000$

$\Rightarrow 8x^3-27y^3-18xy(2x-3y)=1000$

$\Rightarrow 8x^3-27y^3-18(16)(10)=1000$

$\Rightarrow 8x^3-27y^3-2880=1000 \Rightarrow 8x^3-27y^3=3880$

14. If $\mathbf{3a + 5b + 4c = 0}$, show that: $\mathbf{27a^3 + 125b^3 + 64c^3 = 180abc}$.

Solution:

$3a + 5b + 4c = 0$

$\Rightarrow 3a + 5b = -4c$

Cubing both sides, we get

$(3a + 5b)^3 = (-4c)^3$

$\Rightarrow (3a)^3 + (5b)^3 + 3 \times 3a \times 5b(3a + 5b) = -64c^2$

$\Rightarrow 27a^3 + 125b^3 - 180abc = -64c^3$

$\Rightarrow 27a^3 + 125b^3 + 64c^3 = 180\ abc$