

MATHEMATICS

SOLVED EXAMPLES

1. Factorise by taking out the common factors: $2(2x - 5y)(3x + 4y) - 6(2x - 5y)(x - y)$

Solution :

Taking $(2x - 5y)$ common from both terms

$$\begin{aligned} &= (2x - 5y)[2(3x + 4y) - 6(x - y)] \\ &= (2x - 5y)(6x + 8y - 6x + 6y) \\ &= (2x - 5y)(8y + 6y) \\ &= (2x - 5y)(14y) \\ &= (2x - 5y)14y \end{aligned}$$

2. Factorise by grouping method: $a^3 + a - 3a^2 - 3$

Solution :

$$\begin{aligned} &= a^3 + a - 3a^2 - 3 \\ &= a(a^2 + 1) - 3(a^2 + 1) \\ &= (a^2 + 1)(a - 3) \end{aligned}$$

3. Factorise: $(x^2 - 3x)(x^2 - 3x - 1) - 20$

Solution :

$$\begin{aligned} &= (x^2 - 3x)(x^2 - 3x - 1) - 20 \\ &= (x^2 - 3x)[(x^2 - 3x) - 1] - 20 \\ &= a[a - 1] - 20 \quad \dots \text{(Taking } x^2 - 3x = a\text{)} \\ &= a^2 - a - 20 \\ &= a^2 - 5a + 4a - 20 \\ &= a(a - 5) + 4(a - 5) \\ &= (a - 5)(a + 4) \\ &= (x^2 - 3x - 5)(x^2 - 3x + 4) \end{aligned}$$

4. Give possible expressions for the length and the breadth of the rectangle whose area is $12x^2 - 35x + 25$

Solution :

$$\begin{aligned} &= 12x^2 - 35x + 25 \\ &= 12x^2 - 20x - 15x + 25 \\ &= 4x(3x - 5) - 5(3x - 5) \\ &= (3x - 5)(4x - 5) \end{aligned}$$

Thus, Length = $(3x - 5)$ and breadth = $(4x - 5)$ OR Length = $(4x - 5)$ and breadth = $(3x - 5)$

5. Factorise $(x - y)^3 - 8x^3$

Solution: $(x - y)^3 - 8x^3$

$$\begin{aligned} &= (x - y)^3 - (2x)^3 \\ &= (x - y - 2x)[(x - y)^2 + 2x(x - y) + (2x)^2] \\ &\quad [\text{Using identity } (a^3 - b^3) = (a - b)(a^2 + ab + b^2)] \\ &= (-x - y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2] \\ &= -(x + y)[7x^2 - 4xy + y^2] \end{aligned}$$

6. Factorise $x^2 + \frac{1}{x^2} - 11$

Solution : $x^2 + \frac{1}{x^2} - 11$

$$\begin{aligned} &= x^2 + \frac{1}{x^2} - 2 - 9 \end{aligned}$$

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$$\begin{aligned}&= \left(x^2 + \frac{1}{x^2} - 2\right) - 9 \\&= \left(x - \frac{1}{x}\right)^2 - 3^2 \\&= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right)\end{aligned}$$

7. Factorise $a(3a-2)-1$

Solution:

$$\begin{aligned}a(3a-2)-1 &= 3a^2 - 2a - 1 \\&= 3a^2 - 3a + a - 1 \\&= 3a(a-1) + 1(a-1) \\&= (3a+1)(a-1)\end{aligned}$$

8. Factorise $a^2 b^2 + 8ab - 9$

Solution:

$$\begin{aligned}a^2 b^2 + 8ab - 9 &= a^2 b + 9ab - ab - 9 \\&= ab(ab+9) - 1(ab+9) \\&= (ab+9)(ab-1)\end{aligned}$$

9. Factorise $3-a(4+7a)$

Solution:

$$\begin{aligned}3-a(4+7a) &= 3-4a-7a^2 \\&= 3-7a+3a-7a^2 \\&= 1(3-7a)+a(3-7a) \\&= (3-7a)(a+1)\end{aligned}$$

10. Factorise $(2a+b)^2 - 6a - 3b - 4$

solution:

$$\begin{aligned}(2a+b)^2 - 6a - 3b - 4 &= (2a+b)^2 - 3(2a+b) - 4 \\&\text{Assume that } 2a+b=x \\&\text{Therefore,} \\(2a+b)^2 - 6a - 3b - 4 &= x^2 - 3x - 4 \\&= x^2 - 4x + x - 4 \\&= 1(x-4) + x(x-4) \\&= (x+1)(x-4) \\&= (2a+b+1)(2a+b-4) \\&\text{(Resubstitute the value of } x)\end{aligned}$$

11. Factorise:

$$(x^2 - 3x)(x^2 - 3x - 1) - 20$$

Solution:

$$\begin{aligned}(x^2 - 3x)(x^2 - 3x - 1) - 20 &\\&= (x^2 - 3x)[(x^2 - 3x) - 1] - 20 \\&= a[a - 1] - 20 \dots \text{(Taking } x^2 - 3x = a) \\&= a^2 - a - 20 \\&= a^2 - 5a + 4a - 20 \\&= a(a - 5) + 4(a - 5) \\&= (a - 5)(a + 4) \\&= (x^2 - 3x - 5)(x^2 - 3x + 4)\end{aligned}$$

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12. Give possible expressions for the length and the breadth of the rectangle whose area is

$$12x^2 - 35x + 25$$

Solution:

$$\begin{aligned}12x^2 - 35x + 25 \\= 12x^2 - 20x - 15x + 25 \\= 4x(3x - 5) - 5(3x - 5)\end{aligned}$$

$$= (3x - 5)(4x - 5)$$

Thus,

Length = $(3x - 5)$ and breadth = $(4x - 5)$

OR

Length = $(4x - 5)$ and breadth = $(3x - 5)$

13. Factorise:

1. $A^3 - 27$

solution:

$$\begin{aligned}a^3 - 27 &= (a)^3 - (3)^3 \\&= (a-3)[(a)^2 + 3a + (3)^2] \quad [a^3 - b^3 = (a-b)(a^2 + ab + b^2)] \\&= (a-3)[a^2 + 3a + 9]\end{aligned}$$

14. Factorise $(x - y)^3 - 8x^3$

Solution:

$$\begin{aligned}(x - y)^3 - 8x^3 &= (x - y)^3 - (2x)^3 \\&= (x - y - 2x)[(x - y)^2 + 2x(x - y) + (2x)^2] \\&\quad [\text{Using identity } (a^3 - b^3) = (a - b)(a^2 + ab + b^2)] \\&= (-x - y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2] \\&= -(x + y)[7x^2 - 4xy + y^2]\end{aligned}$$