1. Add: $5x^2 + 7y - 8$, $4y + 7 - 2x^2$ and $6 – 5y + 4x^2$.

Ans. Writing the given expressions in descending powers of x in the form of rows with like terms below each other and adding column wise;

$$\begin{array}{r} 5x^2 + 7y - 8 \\ -2x^2 + 4y + 7 \\ 4x^2 - 5y + 6 \\ \hline 7x^2 + 6y + 5 \\ \hline \end{array}$$

$= 7x^2 + 6y + 5$

2. Add: $8x^2 - 5xy + 3y^2$, $2xy - 6y^2 + 3x^2$ and $y^2 + xy - 6x^2$.

Ans. Arranging the given expressions in descending powers of x with like terms under each other and adding column wise;

$$\begin{array}{r} 8x^2 - 5xy + 3y^2 \\ 3x^2 - 2xy - 6y^2 \\ -6x^2 + xy + y^2 \\ \hline 5x^2 - 2xy - 2y^2 \\ \hline \end{array}$$

$= 5x^2 - 2xy - 2y^2$

3. Add: $11a^2 + 8b^2 - 9c^2$, $5b^2 + 3c^2 - 4a^2$ and $3a^2 - 4b^2 - 4c^2$.

Ans. Writing the terms of the given expressions in the same order in form of rows with like terms below each other and adding column wise;

$$\begin{array}{r} 11a^2 + 8b^2 - 9c^2 \\ -4a^2 + 5b^2 + 3c^2 \\ 3a^2 - 4b^2 - 4c^2 \\ \hline 10a^2 + 9b^2 - 10c^2 \\ \hline \end{array}$$

$= 10a^2 + 9b^2 - 10c^2$

4. Add the $3x + 2y$ and $x + y$.

Ans. Horizontal Method:

$(3x + 2y) + (x + y)$

Arrange the like terms together, then add.

Thus, the required addition

$= 3x + 2y + x + y$

$= 3x + x + 2y + y$

$= 4x + 3y$

Column Method:

Solution:

Arrange expressions in lines so that the like terms with their signs are one below the other i.e. like terms are in same vertical column and then add the different groups of like terms.

$3x + 2y$

$+ x + y$

$4x + 3y$

5. Add: $x + y + 3$ and $3x + 2y + 5$

Ans. Horizontal Method:

$(x + y + 3) + (3x + 2y + 5)$

$= x + y + 3 + 3x + 2y + 5$

Arrange the like terms together, then add.

Thus, the required addition

$= x + 3x + y + 2y + 3 + 5$

$= 4x + 3y + 8$

Column Method:

Solution:

Arrange expressions in lines so that the like terms with their signs are one below the other i.e. like terms are in same vertical column and then add the different groups of like terms.

$x + y + 3$

$+ 3x + 2y + 5$

$4x + 3y + 8$

6. Subtract $4a + 5b - 3c$ from $6a - 3b + c$

Ans.

$$\begin{array}{l} \ 6a - 3b + c \\ +\ 4a + 5b - 3c \\ (-)\quad (-)\quad (+) \\ \hline \ 2a - 8b + 4c \\ \hline \end{array}$$

7. Subtract $3x^2 - 6x - 4$ from $5 + x - 2x^2$.

Ans. Arranging the terms of the given expressions in descending powers of x and subtracting column-wise;

$$\begin{array}{l} -2x^2 + x + 5 \\ +\ 3x^2 - 6x - 4 \\ (-)\quad (+)\quad (+) \\ \hline -5x^2 + 7x + 9 \\ \hline \end{array}$$

8. Subtract $3x + y – 3z$ from $9x – 5y + z$

Ans.

$$\begin{array}{l} \ 9x - 5y + z \\ +\ 3x + y - 3z \\ (-)\quad (-)\quad (+) \\ \hline \ 7x - 6y + 4z \\ \hline \end{array}$$

9. Find the product of:

(i) $6xy$ and $-3x^2y^3$

(ii) $7ab^2$, $-4a^2b$ and $-5abc$

Ans. (i) $6xy$ and $-3x^2y^3 (6xy) \times (-3x^2y^3)$

Solution:

$= \{6 \times (-3)\} \times \{xy \times x^2y^3\}$

$= -18x1+2\ y1+3 = -18x^3y^4$.

(ii) $7ab^2$, $-4a^2b$ and $-5abc$

Solution:

$(7ab^2) \times (-4a^2b) \times (-5abc)$

$= \{7 \times (-4) \times (-5)\} \times \{ab^2 \times a^2b \times abc\}$

$= 140\ a1+2+1\ b2+1+1\ c$

$= 140a^4b^4c$.

10. Find each of the following products:

(i) $5a^2b^2 \times (3a^2 - 4ab + 6b^2)$

(ii) $(-3x^2y) \times (4x^2y - 3xy^2 + 4x - 5y)$

Ans. (i) $5a^2b^2 \times (3a^2 - 4ab + 6b^2)$

Solution:

$5a^2b^2 \times (3a^2 - 4ab + 6b^2)$

$= (5a^2b^2) \times (3a^2) + (5a^2b^2) \times (-4ab) + (5a^2b^2) \times (6b^2)$

$= 15a^4b^2 - 20a^3b^3 + 30a^2b^4$.

(ii) $(-3x^2y) \times (4x^2y - 3xy^2 + 4x - 5y)$

Solution:

$(-3x^2y) \times (4x^2y - 3xy^2 + 4x - 5y)$

$= (-3x^2y) \times (4x^2y) + (-3x^2y) \times (-3xy^2) + (-3x^2y) \times (4x) + (-3x^2y) \times (-5y)$

$= -12x^4y^2 + 9x^3y^3 - 12x^3y + 15x^2y^2$.

11. Multiplication of Two Binomials

(i) Multiply $(3x + 5y)$ and $(5x - 7y)$.

(ii) Multiply $(3x^2 + y^2)$ by $(2x^2 + 3y^2)$

Ans. (i) Multiply $(3x + 5y)$ and $(5x - 7y)$.

Solution:

$(3x + 5y) \times (5x - 7y)$

$= 3x \times (5x - 7y) + 5y \times (5x - 7y)$

$= (3x \times 5x - 3x \times 7y) + (5y \times 5x - 5y \times 7y)$

$= (15x^2 - 21xy) + (25xy - 35y^2)$

$= 15x^2 - 21xy + 25xy - 35y^2$

$= 15x^2 + 4xy - 35y^2$.

Column wise multiplication

The multiplication can be performed column wise as shown below.

$3x + 5y$

$\times (5x - 7y)$

____

$15x^2 + 25xy$ ⇐ multiplication by 5x.

$- 21xy - 35y^2$ ⇐ multiplication by -7y

____

$15x^2 + 4xy - 35y^2$ ⇐ multiplication by (5x - 7y)

____

(ii) Multiply $(3x^2 + y^2)$ by $(2x^2 + 3y^2)$

Solution:

Horizontal method,

$= 3x^2 (2x^2 + 3y^2) + y^2 (2x^2 + 3y^2)$

$= (6x^4 + 9x^2y^2) + (2x^2y^2 + 3y^4)$

$= 6x^4 + 9x^2y^2 + 2x^2y^2 + 3y^4$

$= 6x^4 + 11x^2y^2 + 3y^4$

Column methods,

$3x^2 + y^2$

$\times\ (2x^2 + 3y^3)$

$6x^4 + 2x^2y^2$ ⇐ multiplication by $2x^2$

$+ 9x^2y^2 + 3y^4$ ⇐ multiplication by $3y^3$.

$6x^4 + 11x^2y^2 + 3y^4$ ⇐ multiplication by $(2x^2 + 3y^3)$.

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12. Multiplication by Polynomial

(i) Multiply $(5x^2 - 6x + 9)$ by $(2x - 3)$

(ii) Multiply $(2x^2 - 5x + 4)$ by $(x^2 + 7x - 8)$

(iii) Multiply $(2x^3 - 5x^2 - x + 7)$ by $(3 - 2x + 4x^2)$

Ans. (i) Multiply $(5x^2 - 6x + 9)$ by $(2x - 3)$

$5x^2 - 6x + 9$

$\times\ (2x - 3)$

$10x^3 - 12x^2 + 18x$ ⇐ multiplication by 2x.

$- 15x^2 + 18x - 27$ ⇐ multiplication by -3.

$10x^3 - 27x^2 + 36x - 27$ ⇐ multiplication by (2x - 3).

Therefore, $(5x^2 - 6x + 9)$ by $(2x - 3)$ is $10x^3 - 27x^2 + 36x - 27$

(ii) Multiply $(2x^2 - 5x + 4)$ by $(x^2 + 7x - 8)$

Solution: By column method

$$
\begin{array}{rl}
2x^2 - 5x + 4 & \\
\times\ (x^2 + 7x - 8) & \\
\hline
2x^4 - 5x^3 + 4x^2 \qquad\qquad & \Leftarrow \text{multiplication by } x^2. \\
+ 14x^3 - 35x^2 + 28x \qquad & \Leftarrow \text{multiplication by } 7x. \\
- 16x^2 + 40x - 32 & \Leftarrow \text{multiplication by } -8. \\
\hline
2x^4 - 9x^3 - 47x^2 + 68x - 32 & \Leftarrow \text{multiplication by } (x^2 + 7x - 8). \\
\hline
\end{array}
$$

Therefore, $(2x^2 - 5x + 4)$ by $(x^2 + 7x - 8)$ is $2x^4 - 9x^3 - 47x^2 + 68x - 32$.

(iii) Multiply $(2x^3 - 5x^2 - x + 7)$ by $(3 - 2x + 4x^2)$

Solution:

Arranging the terms of the given polynomials in descending power of x and then multiplying,

$$
\begin{array}{rl}
2x^3 - 5x^2 - x + 7 & \\
\times\ (3 - 2x + 4x^2) & \\
\hline
8x^5 - 20x^4 - 4x^3 + 28x^2 \qquad\qquad & \Leftarrow \text{multiplication by } 3. \\
- 4x^4 + 10x^3 + 2x^2 - 14x \qquad & \Leftarrow \text{multiplication by } -2x. \\
+ 6x^3 - 15x^2 - 3x + 21 & \Leftarrow \text{multiplication by } 4x^2. \\
\hline
8x^5 - 24x^4 + 12x^3 + 15x^2 - 17x + 21 & \Leftarrow \text{multiplication by } (3 - 2x + 4x^2). \\
\hline
\end{array}
$$

12. Divide $(10x^4 + 17x^3 - 62x^2 + 30x - 3)$ by $(2x^2 + 7x - 1)$.

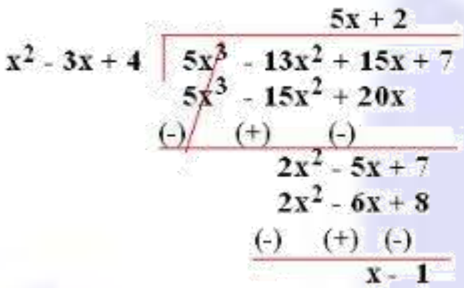
Ans. The terms of the dividend and that of the divisor are in descending order. So, we divide them as;

$$
\begin{array}{r|l}
 & 5x^2 - 9x + 3 \\
\hline
2x^2 + 7x - 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
 & 10x^4 + 35x^3 - 5x^2 \\
 & (-) \quad (-) \quad (+) \\
\hline
 & -18x^3 - 57x^2 + 30x - 3 \\
 & -18x^3 - 63x^2 + 9x \\
 & (+) \quad (+) \quad (-) \\
\hline
 & 6x^2 + 21x - 3 \\
 & 6x^2 + 21x - 3 \\
 & (-) \quad (-) \quad (+) \\
\hline
 & 0
\end{array}
$$

$(10x^4 + 17x^3 - 62x^2 + 30x - 3) \div (2x^2 + 7x - 1) = (5x^2 - 9x + 3)$.

13. Find the quotient and remainder when $(7 + 15x - 13x^2 + 5x^3)$ is divided by $(4 - 3x + x^2)$.

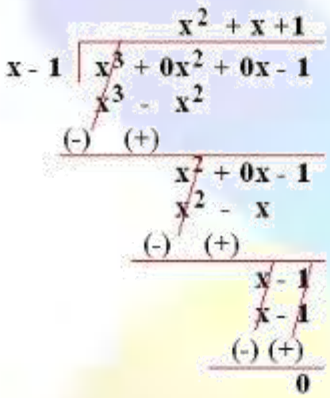
Ans. Arranging the terms of dividend and divisor in descending order and then dividing,



Therefore, quotient is $(5x + 2)$ and remainder is $(x - 1)$.

14. Using division, show that $(x - 1)$ is a factor of $(x^3 - 1)$.

Ans.



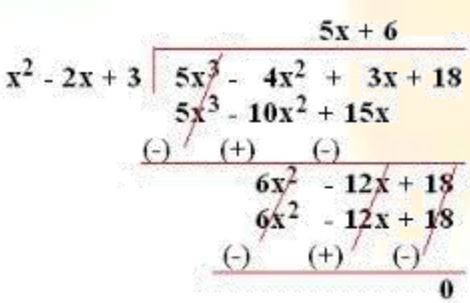
$(x - 1)$ completely divides $(x^3 - 1)$.

Hence, $(x - 1)$ is a factor of$(x^3 - 1)$.

15. Divide $(5x^3 - 4x^2 + 3x - 18)$ by $(3 - 2x + x^2)$.

Ans. The terms of the dividend are in descending order.

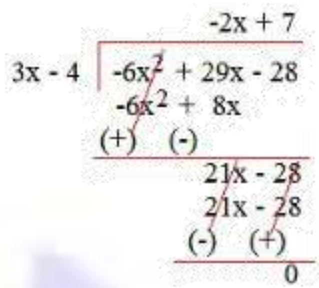
Arranging the terms of the divisor in descending order and then dividing,



Therefore, $5x^3 - 4x^2 + 3x - 18) \div (x^2 - 2x + 3) = (5x + 6)$.

16. Divide $(29x - 6x^2 - 28)$ by $(3x - 4)$.

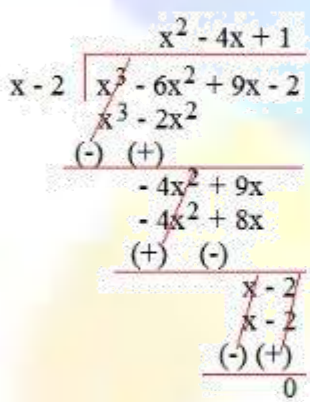
Ans. Arranging the terms of the dividend and divisor in descending order and then dividing,



Therefore, $(29x - 6x^2 - 28) \div (3x - 4) = (-2x + 7)$.

17. Divide $9x - 6x^2 + x^3 - 2$ by $(x - 2)$.

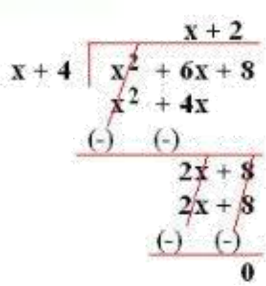
Ans. Arranging the terms of the dividend and divisor in descending order and then dividing,



Therefore, quotient = $(x^2 - 4x + 1)$ and remainder = 0.

18. Divide $x^2 + 6x + 8$ by $(x + 4)$.

Ans.



Therefore, Dividend = $x^2 + 6x + 8$

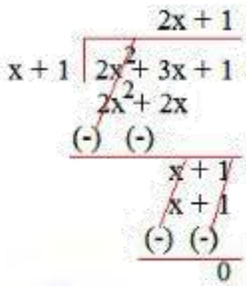
Divisor = $x + 4$

Quotient = x + 2 and

Remainder = 0.

19. Divide $2x^2 + 3x + 1$ by $(x + 1)$.

Ans.



Therefore, quotient = (2x + 1) and remainder = 0.

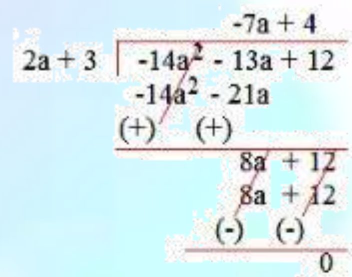
20. Divide $12 - 14a^2 - 13a$ by $(3 + 2a)$.

Ans. $12 - 14a^2 - 13a$ by $(3 + 2a)$.

Write the terms of the polynomial (dividend and divisor both) in decreasing order of exponents of variables.

So, dividend becomes $-14a^2 - 13a + 12$ and divisor becomes $2a + 3$.

Divide the first term of the dividend by the first term of the divisor which gives first term of the quotient.



Multiply the divisor by the first term of the quotient and subtract the product from the dividend which gives the remainder.

Now, this remainder is treated as, new dividend but the divisor remains the same.

Now, we divide the first term of the new dividend by the first term of the divisor which gives second term of the quotient.

Now, multiply the divisor by the term of the quotient just obtained and subtracts the product from the dividend.

Thus, we conclude that divisor and quotient are the factors of dividend if the remainder is zero.

Quotient = -7a + 4

Remainder = 0

Verification:

Dividend = divisor × quotient + remainder

$= (2a + 3)(-7a + 4) + 0$

$= 2a(-7a + 4) + 3(-7a + 4) + 0$

$= -14a^2 + 8a - 21a + 12 + 0$

$= -14a^2 - 13a + 12$