Class – IX	Topic – Co-ordinate of geometry

1. **Is it possible to have a triangle with the following sides?**

   (i) 2 cm, 3 cm, 5 cm

   (ii) 3 cm, 6 cm, 7 cm

   (iii)6 cm, 3 cm, 2 cm

Solution:

We know that the sum of the length of any two sides of a triangle is greater than the length of the third side.

(i) $2\text{ cm} + 3\text{ cm} = 5\text{ cm} = 5\text{ cm}$

$3\text{ cm} + 5\text{ cm} = 8\text{ cm} > 2\text{ cm}$

Hence, it is not possible to have a triangle.

(ii) $3\text{ cm} + 6\text{ cm} = 9\text{ cm} > 7\text{ cm}$

$3\text{ cm} + 7\text{ cm} = 10\text{ cm} > 6\text{ cm}$

$6\text{ cm} + 7\text{ cm} = 13\text{ cm} > 3\text{ cm}$

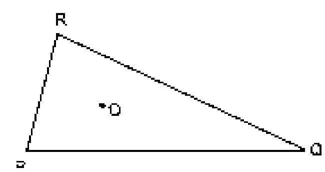
(iii) $6\text{ cm} + 3\text{ cm} = 9\text{ cm} > 2\text{ cm}$

$6\text{ cm} + 2\text{ cm} = 8\text{ cm} > 3\text{ cm}$

$2\text{ cm} + 3\text{ cm} = 5\text{ cm} < 6\text{ cm}$

Hence, it is not possible to have a triangle.

2. **Take any point O in the interior of a triangle PQR. Is**



   (i) $\mathbf{OP + OQ > PQ}$?

   (ii) $\mathbf{OQ + OR > QR}$?

   (iii) $\mathbf{OR + OP > RP}$?

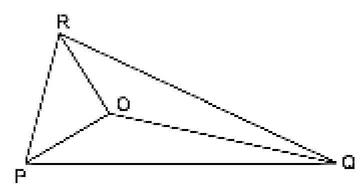
Solution:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(i) In ΔOPQ, yes, $OP + OQ > PQ$

(ii) In ΔORQ, yes, $OR + OQ > QR$

(iii) In ΔPOR, yes, $OR + OP > PR$



3. **AM is a median of a triangle ABC.**

   Is $AB + BC + CA > 2\ AM$? (Consider the sides of triangles ΔABM and ΔAMC).

Solution:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

In ΔAMB,

$AB + BM > AM$ (i)

In ΔAMC

$AC + CM > AM$ (ii)

Adding (i) and (ii), we have

$AB + BM + AC + CM > AM + AM$

or $AB + AC + (BM + CM) > 2\ AM$

or $AB + AC + BC > 2\ AM$

or $AB + BC + CA > 2AM$

4. **ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?**

Solution:

Given: In a quadrilateral ABCD, AC and BD are diagonals.

To prove: $AB + BC + CD + DA > AC + BD$

Proof: In ΔABD,

$AB + DA > BD$ (i)

In ΔBDC,

$BC + CD > BD$ (ii)

In ΔADC,

$DA + CD > AC$ (iii)

In ΔABC,

$AB + BC > AC$ (iv)

Adding (i), (ii), (iii) and (iv), we have

$AB + DA + BC + CD + DA + CD + AB + BC > BD + AC + AC + BD$

or $2\ [AB + BC + CD + DA] > 2[BD + AC]$

or $AB + BC + CD + DA > BD + AC$

5. **ABCD is a quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)?**

Solution:

Given: A quadrilateral ABCD in which AC and BD are diagonals.

To prove: $AB + BC + CD + DA < 2(AC + BD)$

Proof: In ΔAOB,

$OB + OA > AB$ (i)

In ΔBOC,

$OB + OC > BC$ (ii)

In ΔCOD,

$OD + OC > CD$ (iii)

In ΔAOD,

$OA + OD > DA$ (iv)

Adding (i), (ii), (iii) and (iv), we have

$OB + OA + OB + OC + OD + OC + OA + OD > AB + BC + CD + DA$

or $2OB + 2OD + 2OC + 2OA > AB + BC + CD + DA$

or $2(OB + OD) + 2(OC + OA) > AB + BC + CD + DA$

or $2(BD + AC) > AB + BC + CD + DA$

or $AB + BC + CD + AD < 2(BD + AC)$

6. **The lengths of two sides of a triangle are 12 cm and 15 cm. between what two measures should the length of the third side fall?**

Solution:

Let x cm be the length of the third side. We know that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$\therefore\ 12 + 15 > x, \text{so}, 27 > x$

$x + 12 > 15, \text{so}, x > 3$

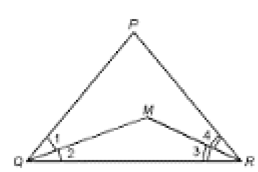
$x + 15 > 12, \text{so}, x > -3$

The numbers between 3 and 27 satisfy these.

$\therefore$ the length of the third side could be any length between 3 cm and 27 cm.

7. **In the figure PQ > PR, QM and RM are the bisectors of ∠Q and ∠R respectively. Prove that QM > RM.**

Solution:



Since QM and RM are angle bisectors of ∠Q and ∠R.

$\therefore \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Now in ΔPQR, $PQ > PR$

$\angle PRQ > \angle PQR$ (Angles opposite to larger side)

$\angle PRM + \angle MRQ > \angle PQM + \angle MQR$

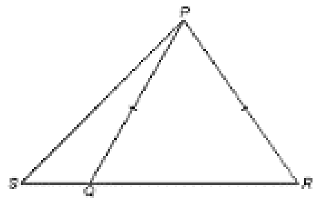
$\angle 4 + \angle 3 > \angle 1 + \angle 2$

$2\angle 3 > 2\angle 2$

$\Rightarrow \angle 3 > \angle 2$

$QM > RM.$

8. In the figure, PQ = PR, show that PS > PQ.



Solution:

In ΔPQR, $PQ = PR$

$\Rightarrow \angle PQR = \angle PRQ$... (i)

Now in ΔPSQ, $\angle PQR$ is the exterior angle

$\Rightarrow \angle PQR = \angle PSQ + \angle SPQ$

$\Rightarrow \angle PQR > \angle PSQ$

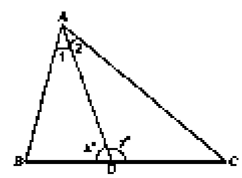
$\Rightarrow \angle PRQ > \angle PSQ$ $(\angle PQR = \angle PRQ)$

$\Rightarrow PS > PR$ (Side opposite to greater angle)

$PS > PQ$ $(PR = PQ)$

9. In ΔABC, AC > AB and AD is the bisector of ∠A show that y > x.

Solution:



In ΔABC, since $AC > AB$

$\therefore \angle B > \angle C$ (i)

Also $\angle 1 = \angle 2$ [Given]

Now, in ΔABD, we have

$\angle 1 + \angle B + \angle x = 180°$ (ii)

And in ΔADC, we have

$\angle 2 + \angle C + \angle y = 180°$ (iii)

Comparing equation (i) and (ii), we get

$\angle 1 + \angle B + \angle x = \angle 2 + \angle C + \angle y$

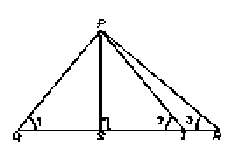
$\angle B + \angle x = \angle C + \angle y$

Now since $\angle B > \angle C$ [Proved above]

$\angle x < \angle y$

or $\angle y > \angle x$

10. In ΔPQR, PS ⊥ QR and SR > SQ, show that PR > PQ.



Solution:

$PR > PQ$

Construction: Draw PT such that $SQ = ST$

Proof: In ΔPQS and ΔPTS,

$PS = PS$ [Common]

$SQ = ST$ [By construction]

$\angle PSQ = \angle PST$ [Each $90°$]

$\Delta PQS \cong \Delta PTS$ [SAS criterion]

$\angle 1 = \angle 2$ [c.p.c.t.]

Now in ΔPRT, we have the exterior angle $\angle 2$

$\angle 2 = \angle 3 + \angle TPR$

Or $\angle 2 > \angle 3$

... $\angle 1 > \angle 3$...... ($\angle 1 = \angle 2$)

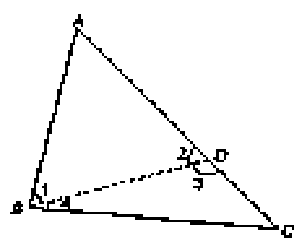
Now in ΔPQR, since

$\angle 1 > \angle 3$

$PR > PQ$ [Side opposite to greater angle]

11. Prove that the difference of any two sides of a triangle is less than the third side.

Solution:



To Prove: $AC - AB < BC$,

$BC - AB < AC$

And $AC - BC < AB$

Construction: Mark a point D on AC such that

$AB = AD$ and join BD.

Proof: In ΔABD, $AB = AD$

$\Rightarrow \angle 1 = \angle 2$ (i)

Now in ΔABD, AD is produced to C

$\Rightarrow \angle 3 = \angle 1 + \angle A$ (Exterior angle)

$\Rightarrow \angle 3 > \angle 1$ (ii)

Now in ΔBCD, CD is produced to A

$\Rightarrow \angle 2 = \angle 4 + C$

$\Rightarrow \angle 2 > \angle 4$ (iii)

From (i), (ii), and (iii), we get

$\angle 3 > \angle 4$

$BC > CD$

Or $BC > AC - AD$

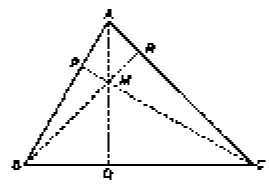
Or $BC > AC - AB$

Similarly we can prove other two inequalities

$BC - AB < AC$ and $AC - BC < AB$.

12. Show that the sum of the three altitudes of a triangle is less than the sum of the three sides of the triangle.

Solution:



ΔABC, in which altitudes AQ, BR and CP meet at M.

To Prove:

$AB + BC + CA > BR + CP + AQ$

Proof: In ΔABQ, we have $\angle AQB = 90°$

$\angle ABQ$ will be a acute angle

$\angle AQB > \angle ABQ$

$AB > AQ$ (i)

Similarly, we can prove that:

$BC > BR$ (ii)

And $CA > CP$ (iii)

Adding three inequalities, we get

$AB + BC + CA > AQ + BR + CP$

II Proof:

$AB > AQ, AC > AQ$ [Perpendicular is shortest]

$AB + AC > 2AQ$ (i)

Similarly $BC + CA > 2PC$ (ii)

And $AB + BC > 2BR$

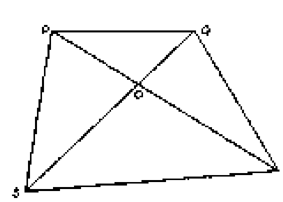
Adding (i), (ii) and (iii), we get

$2AB + 2BC + 2CA > 2AQ + 2BR + 2CP$

or $AB + BC + CA > AQ + BR + CP$

13. In a quadrilateral PQRS, the diagonals PR and QS intersect each other at O. Show that

(i) $PQ + QR + RS + SP > PR + QS$.



(ii) $PQ + QR + RS + SP < 2\ (PR + QS)$.

Solution:

We know that in a triangle the sum of two sides of a triangle is greater

Than the third side. Therefore in ΔPSR, we have

$PS + SR > PR$ (i)

In ΔQSR, $QR + RS > QS$ (ii)

In ΔPQS, $PQ + SP > QS$ (iii)

In ΔPQR, $PQ + QR > PR$ (iv)

Adding (i), (ii), (iii) and (iv), we get

$2\ (SP + PQ + QR + RS) > 2\ (QS + PR)$ $PQ + QR + RS + SP > PR + QS$

Similarly in ΔPQO, ΔQRO, ΔRSO and ΔSPO,

we have

$QO + PO > PQ$ (v)

$QO + RO > QR$ (vi)

$RO + SO > RS$. (vii)

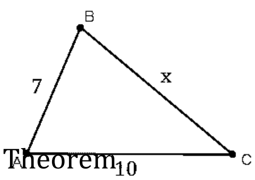
$SO + PO > SP$ (viii)

Adding (v), (vi), (vii) and (viii), we get

$2\ (QO + SO + PO + OR) > PQ + QR + RS + SP$ $2\ (QS + PR) > PQ + QR + RS + SP$

or $PQ + QR + RS + SP < 2\ (QS + PR)$.

14. In the figure below, what range of length is possible for the third side, x, to be.



Solution:

When considering the side lengths of a triangle, we want to use the Triangle Inequality Theorem

The sum of the two sides should always be greater than the length of one side in order for the figure to be a triangle. Let's write our first inequality

$AB + BC > CA$

$7 + x > 10$

$x > 3$

So, we know that x must be greater than 3. Let's see if our next inequality helps us narrow down the possible values of x.

$AB + CA > BC$

$7 + 10 > x$

$17 > x$

This inequality has shown us that the value of x can be no more than 17. Let's work out our final inequality.

$CA + BC > AB$

$10 + x > 7$

$x > -3$

This final inequality does not help us narrow down our options because we were already aware of the fact that x had to be greater than 3. Moreover, side lengths of triangles cannot be negative, so we can disregard this inequality.

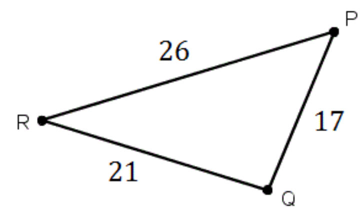
Combining our first two inequalities yields

$3 < x < 17$

So, using the Triangle Inequality Theorem shows us that x must have a length between 3 and 17.

15. List the angles in order from least to greatest measure.

Solution:



For this exercise, we want to use the information we know about angle-side relationships. Since all side lengths have been given to us, We just need to order them in order from least to greatest, and then look At the angles opposite those sides.

In order from least to greatest, our sides are PQ, QR, and RP.

This means that the angles opposite those sides will be ordered from least To greatest. So, in order from least to greatest angle measure, We have ∠R, ∠P, and then ∠Q.