Class –IX	Topic – Isosceles Triangle

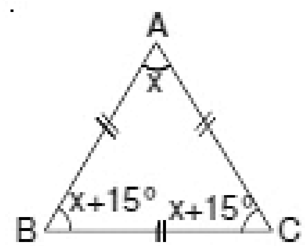
1. **Each base angles of an isosceles triangle is 15° more than its vertical angle. Find each angle of the triangle.**

Solution:

In an isosceles $\Delta ABC, AB = AC$

Let vertical angle, $\angle A = x$ then each angle $= x + 15$

$\therefore$ But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)



$\Rightarrow x + x + 15° + x + 15° = 180°$

$\Rightarrow 3x + 30° = 180°$

$\Rightarrow 3x = 180° - 30° = 150°$

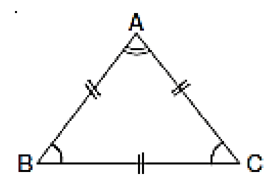
$\Rightarrow x = \dfrac{150°}{3} = 50°$

$\therefore \angle A = 50°, \angle B = 50° + 15° = 65°$ and $\angle C = 65°$

2. **The vertical angle of an isosceles triangle is twice the sum of its base angles. Find each angle of the triangle.**

Solution:

In an isosceles triangle $ABC, AB = AC$

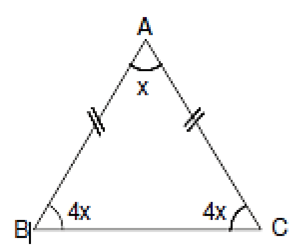


$\Rightarrow x = \dfrac{180°}{4} = 45°$

$\therefore \angle A = 2x = 2 \times 45° = 90°, \angle B = 45°, \angle C = 45°$

3. **In an isosceles triangle, triangle, each base angle is four times its vertical angle. Find each angle of the triangle.**

Solution:



In an isosceles $\Delta ABC, AB = AC$

Let vertical angle, $\angle A = x$ then each base angle $\angle B = \angle C = 4x$

But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\therefore x + 4x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow x = \dfrac{180°}{9} = 20°$

$\therefore \angle A = 20°, \angle B = 4x = 4 \times 20° = 80°, \angle C = 4x = 4 \times 20° = 80°$

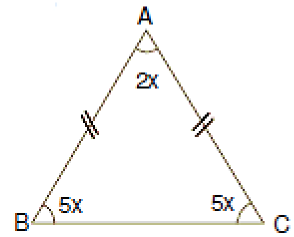
4. **The ratio between the base angle and the vertical angle of an isosceles triangle is 2 : 5. Find each angle of the triangle.**

Solution:

In an isosceles $\Delta ABC, AB = AC$

Ratio between vertical angle A and base angle $B = 2:5$

Let vertical angle $A = 2x$ then $\angle B = 5x$ and $\angle C = 5x$



But $\angle A + \angle B + \angle C = 180°$ (Sum of angles of a triangle)

$\therefore 2x + 5x + 5x = 180°$

$\Rightarrow 12x = 180° \Rightarrow x = \dfrac{180°}{12} = 15°$

$\therefore \angle A = 2x = 2 \times 15° = 30°, \angle B = 5x = 5 \times 15° = 75°$

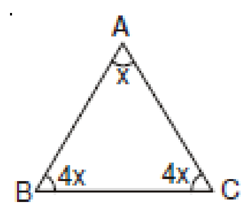
$\angle C = 5x = 5 \times 15° = 75°$

5. **In an isosceles triangle, a base angle is four times its vertical angle. Find all the angles of the triangle.**

Solution:

Let the vertical angle be $x°$, then

base angle $= 4x$



In an isosceles Δ, base angles are equal

$\therefore \angle A = x°, \angle B = 4x°$ and $\angle C = 4x°$

Sum of angles of a triangle is $180°$

$\therefore x + 4x + 4x = 180°$

$\Rightarrow x = \dfrac{180°}{9} = 20°$

$\therefore$ Vertical angle $= 20°$ and each base angle $= 4 \times 20° = 80°$

6. **One of the two equal angles of an isosceles triangle measure 65°. Find the measure of each angle of the triangle.**

Solution:

Let the unequal or third angle be $x°$.

In an isosceles triangle, $x + 65° + 65° = 180°$

$\Rightarrow x + 130° = 180°$

$\Rightarrow x = 180° - 130° = 50°$

Hence, measure of each angle of the triangle are 65°, 65° and 50°.

7. **The vertical angle of an isosceles triangle is three times the sum of its base angles. Find all angles of the triangle.**

Solution:

Let each base angle be x

$\therefore$ Vertical angle $= 3(x + x) = 3(2x) = 6x$

We know that,

Sum of angles of a traingle is 180°

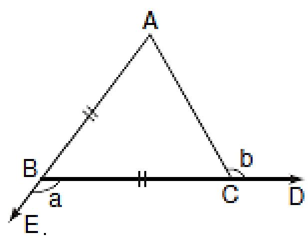
$\therefore x + x + 6x = 180°$

$\Rightarrow 8x = 180°$

$\therefore x = \frac{180°}{8} = 22.5°$

Hence, each base angle $= 22.5°$ and vertical angle $= 6x = 6 \times 22.5° = 135°$

8. **In the given figure express a in terms of b.**



Solution:

In ΔABC,

$BC = BA$

$\therefore \angle BCA = \angle BAC$ ... (1)

and exterior $\angle CBE = \angle BCA + \angle BAC$

$\Rightarrow a = \angle BCA + \angle BCA$ [by (1)]

$\Rightarrow a = 2\angle BCA$

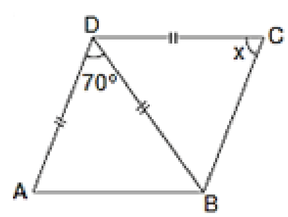
But $\angle ACB = 180° - b$ ($\because \angle ACD$ and $\angle ACB$ are linear pair)

$\Rightarrow \angle BCA = 180° - b$

$\because a = 2\angle BCA = 2(180° - b) = 360° - 2b$

9. **Find x in Figure**

Given: DA = DB = DC, BD bisects ∠ABC and ∠ADB = 70°.



Solution:

$DA = DB = DC$

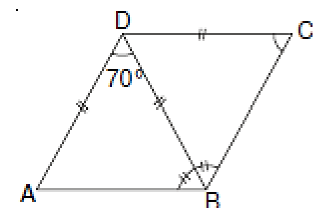
BD bisects $\angle ABC$ and $\angle ADB = 70°$

But $\angle ADB + \angle DAB + \angle DBA = 180°$ (Angle of a triangle)

$\Rightarrow 70° + \angle DBA + \angle DBA = 180°$ ($\because DA = DB$)

$\Rightarrow 70° + 2\angle DBA = 180°$

$\Rightarrow 2\angle DBA = 180° - 70° = 110°$



$\therefore \angle DBA = \dfrac{110°}{2} = 55°$

$\because$ BD is a bisector of $\angle ABC$, $\angle DBA = \angle DBC = 55°$

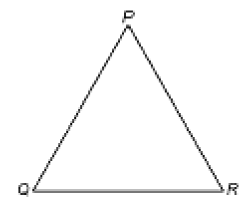
But in ΔDBC,

$DB = DC$

$\therefore \angle DCB = \angle DBC$

$\Rightarrow x = 55°$

10. In ΔPQR, $PQ = QR = RP = 7$ cm, then find each angle of ΔPQR.



Solution:

In ΔPQR, $PQ = PR$ (Given)

$\angle Q = \angle R$ (Angles opposite to equal sides) ... (1)

Again, $QR = RP$

$\angle Q = \angle P$ ...(2)

From equations (1) and (2), we get

$\angle P = \angle Q = \angle R$

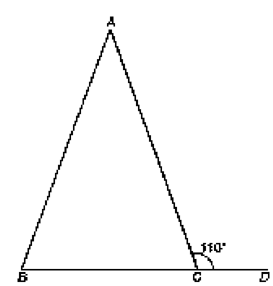
But in ΔPQR, we have

$\angle P + \angle Q + \angle R = 180°$

$3\angle P = 180° \Rightarrow \angle P = 60°$

Hence $\angle P = \angle Q = \angle R = 60°$

11. In the figure $AB = AC$ and $\angle ACD = 110°$ find A.



Solution:

$\angle ACB + \angle ACD = 180°$ (Linear pair)

$\angle ACB + 110° = 180°$

$\Rightarrow \angle ACB = 180° - 110° = 70°$

Since in ΔABC, $AB = AC$

$\angle B = \angle C = 70°$

Now, in ΔABC, we have

$\angle A + \angle B + \angle C = 180°$ (Sum angles)

$\angle A + 70° + 70° = 180°$

12. Prove that the medians of an equilateral triangle are equal.

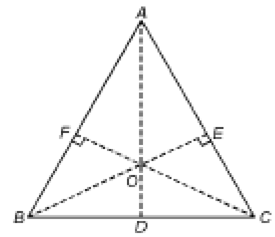
Solution:

Let ABC be an equilateral triangle and let AD, BE and CF be the medians.

Since ΔABC is an equilateral triangle $\angle A = \angle B = \angle C = 60°$

Now in ΔADC and ΔABE,

We have $BC = AC$



$DC = AE$

$\angle C = \angle A$ (Each 60°)

$AC = AB$

$\Delta ADC \cong \Delta ABE$

$AD = BE$ (c.p.c.t.)

Similarly, we can prove that $BE = CF$

$AD = BE = CF$

Hence medians of equilateral triangle are equal.