Class – 9

Topic – Logarithm

1. **Convert each of the following to logarithmic forms.**

   (i) $\mathbf{5^2 = 25}$

   (ii) $\mathbf{3^{-3} = \frac{1}{27}}$

   (iii) $\mathbf{(64)^{\frac{1}{3}} = 4}$

   (iv) $\mathbf{6^0 = 1}$

   (v) $\mathbf{10^{-2} = 0.01}$

   (vi) $\mathbf{4^{-1} = \frac{1}{4}}$

Solution:

We know that $a^b = x \Rightarrow b = \log_a x$

(i) $5^2 = 25$

$\therefore \log_5 25 = 2$

(ii) $3^{-3} = \frac{1}{27}$

$\therefore \log_3 \left(\frac{1}{27}\right) = -3$

(iii) $(64)^{\frac{1}{3}} = 4$

$\therefore \log_{64} 4 = \frac{1}{3}$

(iv) $6^0 = 1$

$\therefore \log_6 1 = 0$

(v) $10^{-2} = 0.01$

$\therefore \log_{10}(0.01) = -2$

(vi) $4^{-1} = \frac{1}{4}$

$\therefore \log_4 \left(\frac{1}{4}\right) = -1$

2. By converting to exponential form fin the value of each of the following.

(i) $\log_2 64$

(ii) $\log_8 32$

(iii) $\log_3 \frac{1}{9}$

(iv) $\log_{0.5}(16)$

(v) $\log_2(0.125)$

(vi) $\log_7 7$

Solution:

(i) Suppose $\log_2 64 = x$, then $2^x = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \Rightarrow 2^x = 2^6$

$\therefore x = 6$. Hence, $\log_2 64 = 6$

(ii) Suppose $\log_8 32 = x$, then $8^x = 32 \Rightarrow 2^{3x} = 2^5$

$\therefore 3x = 5 \Rightarrow x = \frac{5}{3}$

Hence, $\log_8 32 = \frac{5}{3}$

(iii) Suppose $\log_3 \frac{1}{9} = x$, then $3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$\therefore x = -2$. Hence, $\log_3 \left(\frac{1}{9}\right) = -2$

(iv) Suppose $\log_{0.5}(16) = x$, then $(0.5)^x = 16 \Rightarrow \left(\frac{1}{2}\right)^x = 2 \times 2 \times 2 \times 2$

(v) Suppose $\log_2(0.125) = x$, then $2^x = 0.125 = \frac{0.125}{1000} = \frac{1}{8} = \frac{1}{2^3}$

$\Rightarrow 2^x = 2^{-3}$

$\therefore x = -3$. Hence, $\log_2(0.125) = -3$

(vi) Suppose $\log_7 7 = x$, then $7^x = 7 = 7^1$

$\therefore x = 1$. Hence, $\log_7 7 = 1$

3. Find the value of x, when:

(i) $\log_2 x = -2$

(ii) $\log_x 9 = 1$

(iii) $\log_9 243 = x$

(iv) $\log_3 x = 0$

(v) $\mathbf{\log_4 32 = x - 4}$

(vi) $\mathbf{\log_{\sqrt{3}}(x - 1) = 2}$

Solution:

(i) $\log_2 x = -2$

$\therefore 2^{-2} = x \Rightarrow x = \frac{1}{2^2} = \frac{1}{2 \times 2} = \frac{1}{4}$

$\text{Hence}, x = \frac{1}{4}$

(ii) $\log_x 9 = 1$

$\therefore x^1 = 9 \Rightarrow x = 9$

$\text{Hence}, x = 9$

(iii) $\log_9 243 = x$

$\therefore 9^x = 243$

$\Rightarrow (3^2)^x = 3 \times 3 \times 3 \times 3 \times 3 \Rightarrow 3^{2x} = 3^5$

$\Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2} = 2.5.$

$\text{Hence}, x = 2.5$

(iv) $\log_3 x = 0$

$\Rightarrow 3^0 = x \Rightarrow x = 1.$

$\text{Hence}, x = 1$ $(\because x^0 = 1)$

(v) $\log_4 32 = x - 4$

$\therefore 4^{x-4} = 32 \Rightarrow (2 \times 2)^{x-4} = 2^5 \Rightarrow (2^2)^{x-4} = 2^5 \Rightarrow 2^{2x-8} = 2^5$

On comparing both sides $2x - 8 = 5$

$\Rightarrow 2x = 13 \Rightarrow x = \frac{13}{2}.$

$\text{Hence}, x = \frac{13}{2}$

(vi) $\log_{\sqrt{3}}(x - 1) = 2$

$\therefore \left(\sqrt{3}\right)^2 = x - 1 \Rightarrow \left(3^{\frac{1}{2}}\right)^2 = x - 1 \Rightarrow 3 = x - 1$

$\therefore x = 3 + 1 = 4.$

$\text{Hence}, x = 4$

4. Express each of the following as a single logarithm:

(i) $\mathbf{2\log_{10} 8 + \log_{10} 36 - \log_{10}(1.5) - 3\log_{10} 2}$

(ii) $\mathbf{1 - 2\log 5 + 3\log 2}$

(iii) $\mathbf{2\log_{102} 5 + 2\log_{10} 3 - \log_{10} 2 + 1}$

(iv) $\mathbf{2 + \frac{1}{2}\log_{10} 9 + 2\log_{10} 5}$

Solution:

(i) $2\log_{10} 8 + \log_{10} 36 - \log_{10}(1.5) - 3\log_{10} 2$

$= \log_{10}(8)^2 + \log_{10} 36 - \log_{10}(1.5) - \log_{10} 2^3$

$= \log_{10} 64 + \log_{10} 36 - \log_{10} 1.5 - \log_{10} 8$

$= \log_{10} \frac{64 \times 36}{1.5 \times 8} = \log_{10}\left(\frac{64 \times 36 \times 10}{15 \times 8}\right) = \log_{10} 192$

(ii) $1 - 2\log 5 + 3\log 2 = \log 10 - \log 5^2 + \log 2^3$ $[\because \log_a 10 = 1]$

$= \log 10 - \log 25 + \log 8$

$= (\log 10 - \log 25) + \log 8$

$= \log \frac{10}{25} - \log 8 = \log \frac{10}{25} \times 8 = \log \frac{16}{5}$

(iii) $2\log_{10} 5 + 2\log_{10} 3 - \log_{10} 2 + 1$

$= \log_{10}(5)^2 + \log_{10}(3)^2 - \log_{10} 2 + \log_{10} 10$ $[\log_{10} 10 = 1]$

$= \log_{10} 25 + \log_{10} 9 - \log_{10} 2 + \log_{10} 10 = \log_{10} \frac{25 \times 9 \times 10}{2} = \log_{10} 1125$

(iv) $2 + \frac{1}{2}\log_{10} 9 - 2\log_{10} 5$

$= \log_{10} 100 + \log_{10}(9)^{\frac{1}{2}} - \log_{10}(5)^2$ $[\because \log_{10} 100 = 2]$

$= \log_{10} 100 + \log_{10} 3 - \log_{10} 25$

$= \log_{10} \frac{100 \times 3}{25} = \log_{10} 12$

5. **Solve for x:**

(i) $\mathbf{\log_{10}(x - 10) = 1}$

(ii) $\mathbf{\log(x^2 - 21) = 2}$

(iii) $\mathbf{\log(x - 2) + \log(x + 2) = \log 5}$

(iv) $\mathbf{\log(x + 5) + \log(x - 5) = 4\log 2 + \log 3}$

(v) $\mathbf{\log(x + 4) - \log(x - 4) = \log 2}$

Solution:

(i) $\log_{10}(x - 10) = 1$

$\Rightarrow \log_{10}(x - 10) = \log_{10} 10$ $(\because \log 10 = 1)$

$\Rightarrow x - 10 = 10 \Rightarrow x = 10 + 10 \Rightarrow x = 20$

(ii) $\log(x^2 - 21) = 2$

$\Rightarrow \log_{10}(x^2 - 21) = \log_{10} 10$ $\quad (\because \log_{100} 2)$

$\Rightarrow x^2 - 21 = 100$

$\Rightarrow x^2 = 100 + 21 \Rightarrow x^2 = 121$

$\Rightarrow x = 11$

(iii) $\log(x - 2) + \log(x + 2) = \log 5$

$\Rightarrow \log(x - 2) \times (x + 2) = \log 5$ $\quad [\because \log_a m + \log_a n \log_a mn]$

$\Rightarrow \log(x^2 - 4) = \log 5$

$\Rightarrow x^2 - 4 = 5$

$\Rightarrow x^2 = 5 + 4$

$\Rightarrow x^2 = 9$

$\Rightarrow (x)^2 = (3)^2$

$\Rightarrow x = 3$

(iv) $\log(x + 5) + \log(x - 5) = 4\log 2 + 2\log 3$

$\Rightarrow \log(x + 5) \times (x - 5) = 4\log 2 + 2\log 3$ $\quad [\because \log_a m + \log_a n = \log_a mn]$

$\Rightarrow \log[(x)^2 - (5)^2] = \log 2^4 + \log 3^2$

$\Rightarrow \log(x^2 - 25) = \log 16 + \log 9$

$\Rightarrow \log(x^2 - 25) = \log(16 \times 9)$ $\quad [\because \log_a m + \log_a n = \log_a mn]$

$\Rightarrow \log(x^2 - 25) = \log 144$

$\Rightarrow x^2 - 25 = 144$

$\Rightarrow x^2 = 144 + 25$

$\Rightarrow x^2 = 169 \Rightarrow (x^2) = (13)^2$

$\Rightarrow x = 13$

(v) $\log(x + 4) - \log(x - 4) = \log 2 \Rightarrow \log\dfrac{(x + 4)}{(x - 4)} = \log 2$

$\Rightarrow \dfrac{x + 4}{x - 4} = 2 \Rightarrow x + 4 = 2(x - 4)$

$\Rightarrow x + 4 = 2x - 8$

$\Rightarrow x - 2x = -8 - 4$

$\Rightarrow -x = -12$

$\Rightarrow x = 12$

6. **Given $\log x = m + n$ and $\log y = m - n$, express the value of $\log \frac{10x}{y^2}$ in terms of m and n.**

Solution:

$\log x = m + n$ (Given) ... (1)

$\log y = m - n$ (Given) ... (2)

$\log \frac{10x}{y^2} = \log 10x - \log y^2$ $\left(\log_a \frac{m}{n} = \log_a m - \log_a n\right)$

$= \log 10 + \log x - \log y^2$

$= \log 10 + \log x - 2 \log y$

$1 + \log x - 2 \log y$ $[\because \log 10 = 1]$

Putting the value of $\log x = m + n$ and $\log y = m - n$, we get

$= 1 + (m + n) - 2(m - n) = 1 + m + n - 2m + 2n = 1 - m + 3n$

7. **If $\log 9 = 0.9030$, find the value of:**
   (i) $\log 4$
   (ii) $\log \sqrt{32}$
   (iii) $\log(0.125)$

Solution:

$\because \log 8 = 0.9030 \Rightarrow \log(2^3) = 0.9030 \Rightarrow 3 \log 2 = 0.9030$

$\therefore \log 2 = \frac{0.9030}{3} = 0.3010$

(i) $\log 4 = \log 2^2 = 2 \log 2 = 2(0.3010) = 0.6020$

(ii) $\log \sqrt{32} = \log(32)^{\frac{1}{2}} = \frac{1}{2} \log 2^5$

$= \frac{5}{2} \log 2 = \frac{5}{2}(0.3010) = 5 \times 0.1505 = 0.7525$

(iii) $\log(0.125) = \log \frac{125}{1000} = \log\left(\frac{1}{8}\right)$

$= \log\left(\frac{1}{2}\right)^3 = \log(2)^{-3} = -3 \log 2 = -3(0.3010) = -0.9030$

8. **If $\log 27 = 1.4313$, find the value of :**
   (i) $\log 9$

(ii) $\log 30$

Solution:

$\log 27 = 1.4313 \Rightarrow \log 3^3 = 1.4313 \Rightarrow 3\log 3 = 1.4313$

$\Rightarrow \log 3 = \frac{1.4313}{3} = 0.477$

(i) $\log 9 = \log 3^2 = 2\log 3 = 2(0.4771) = 0.9542$

(ii) $\log 30 = \log(3 \times 10) = \log 3 + \log 10 = 0.4771 + 1 = 1.4771$

9. Show that $\log(1+2+3) = \log 1 + \log 2 + \log 3$

$\mathbf{l = \log\frac{a^2}{bc}, m = \log\frac{b^2}{ca} \text{ and } n = \log\frac{c^2}{ab}}$

Solution:

$\log(1+2+3) = \log 6 = \log(1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$.

Hence proved.

10. If $I = g\frac{a^2}{bc}$; $m = \log\frac{b^2}{ca}$ and $n = \log\frac{c^2}{ab}$ find the value of $l + m + n$.

Solution:

$l = \log\frac{a^2}{bc}, m = \log\frac{b^2}{ca}, n = \log\frac{c^2}{ab}$

$l + m + n = \log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = \log\left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab}\right)$

$= \log\frac{a^2b^2c^2}{a^2b^2c^2} = \log 1 = \log 1 \times \frac{10}{10} = \log\frac{10}{10}$

$= \log 10 - \log 10 = 1 - 1 = 0$

11. Solve:

(i) if $\log(a+1) = \log(4a-3) - \log 3$; find a

(ii) if $2\log y - \log x - 3 = 0$, express x in terms of y.

(iii) **Prove that: $\log_{10} 125 = 3(1 - \log_{10} 2)$**

Solution:

(i) $\log(a+1) = \log(4a-3) - \log 3$

$\Rightarrow \log(a+1) = \log\frac{(4a-3)}{3}$

$\Rightarrow (a+1) = \frac{(4a-3)}{3} \Rightarrow 3(a+1) = 4a - 3 \Rightarrow 3a + 3 = 4a - 3$

$\Rightarrow 3a - 4a = -3 - 3$

$\Rightarrow -a = -6$

$\Rightarrow a = 6$

(ii) $2\log y - \log x - 3 = 0$

$\Rightarrow 2\log y - \log x = 3$

$\Rightarrow \log y^2 - \log x = 3$

$\Rightarrow \log\frac{y^2}{x} = 3$

$\Rightarrow \log\frac{y^2}{x} = \log 1000 \Rightarrow \frac{y^2}{x} = 1000 \Rightarrow y^2 = 1000x$

$\Rightarrow 1000x = y^2 \Rightarrow x = \frac{y^2}{1000}$

(iii) $\log_{10} 125 = 3(1 - \log_{10} 2)$

$\text{LHS} = \log_{10} 125 = \log_{10} 5^3 = 3\log_{10} 5$

$\text{RHS} = 3(1 - \log_{10} 2) = 3\log_{10}\left(\frac{10}{2}\right) = 3\log_{10}(5) = 3\log_{10} 5$

$\therefore \text{LHS} = \text{RHS}$

12. If $a^2 = \log x, b^3 = \log y$ and $3a^2 - 2b^2 = 6\log z$, express y in terms of x and z.

Solution:

$a^2 = \log x, b^3 = \log y \Rightarrow 3a^2 - 2b^3 = 6\log z$

$\Rightarrow 3\log x - 2\log y = 6\log z \Rightarrow \log x^3 - \log y^2 = \log z^6$

$\Rightarrow \log\frac{x^3}{y^2} = \log z^6 \Rightarrow \frac{x^3}{y^2} = z^6 \Rightarrow y^2 z^6 = x^3 \Rightarrow y^2 = \frac{x^3}{z^6}$

$\Rightarrow y = \left(\frac{x^3}{y^6}\right)^{\frac{1}{2}} \Rightarrow y = \frac{x^{\frac{3}{2}}}{z^3} = x^{\frac{3}{2}} \div z^3.$

Hence, $y = x^{\frac{3}{2}} \div z^3$

13. If $a^2 + b^2 = 23ab$, show that : $\log\frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$

Solution:

$a^2 + b^2 = 23ab$

$\Rightarrow a^2 + b^2 + 2ab = 23ab + 2ab$ (Adding 2ab both sides, we get)

$\Rightarrow a^2 + b^2 + 2ab = 25ab \Rightarrow (a+b)^2 = 25ab$

$\Rightarrow \frac{(a+b)^2}{25} = ab \Rightarrow \left(\frac{a+b}{5}\right)^2 = ab$

Taking log both sides, we get

$\log\left(\frac{a+b}{5}\right)^2 = \log(ab)$ $\quad [\because \log_a m^n = n\log_a m]$

$\Rightarrow 2\log\frac{a+b}{5} = \log(ab) \Rightarrow \log\frac{a+b}{5} = \frac{1}{2}\log(ab)$

$\Rightarrow \log\frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$

14. If $m = \log 20$ and $n = \log 25$, find the value of x, if: $2\log(x+1) = 2m - n$.

Solution:

$m = \log 20\,, n = \log 25$

$2\log(x+1) = 2m - n$

$\Rightarrow \log(x+1)^2 = 2\log 20 - \log 25 \Rightarrow \log(x+1)^2 = \log(20)^2 - \log 25$

$\Rightarrow \log(x+1)^2 = \log 400 - \log 25 \Rightarrow \log(x+1)^2 = \log\frac{400}{25}$

$\therefore (x+1)^2 = 16 = (4)^2$

$\therefore x + 1 = 4$

$\Rightarrow x = 4 - 1 = 3$

$\therefore x = 3$

15. If $\left[\log 7 - \log 2 + \log 16 - 2\log 3 - \log\frac{7}{45}\right] = 1 + \log n$, find the value of n.

Solution:

$\log 7 - \log 2 + \log 16 - 2\log 3 - \log\frac{7}{45} = 1 + \log n$

$\Rightarrow \log 7 - \log 2 + \log 16 - \log 3^2 - \log\frac{7}{45} = \log 10 + \log n$ $\quad (\because \log 10 = 1)$

$\Rightarrow \log 7 - \log 2 + \log 16 - \log 9 - \log\frac{7}{45} = \log 10 + \log n$

$\Rightarrow \log\left(\frac{7\times16\times45}{2\times9\times7}\right) = \log(10\times n) = \log(10\,n)$ $\quad \left(\therefore 10n = \frac{7\times16\times45}{2\times9\times7}\right)$

$\Rightarrow n = \frac{7\times16\times45}{10\times2\times9\times7} = 4$

Hence, $n = 4$