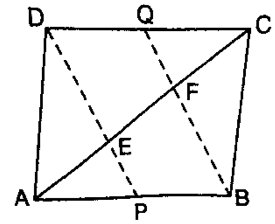
Class –IX	Topic – Midpoint Theorem



1. The adjoining figure shows a parallelogram ABCD in which P is mid-point of AB and Q is mid-point of CD. Prove that: AE=EF=FC

Solution:

Since $PB = \frac{1}{2}AB$ [Given, P is mid-point of AB]

$DQ = \frac{1}{2}DC$ [Given, Q is the mid-point of DC]

∴ PB=DQ [AB=DC; the opp. Sides of ||gm ABCD]

Also, PB||DQ [As AB||DC]

∴DPBQ is a parallelogram [Opp. sides are parallel and equal]

⇒ DP||QB [Opp. sides of the || gm DPBQ]

Now in Δ ABF:

P is the mid-point of AB [Given]

PE||BF [As DP||QB]

∴ PE bisects AF

i.e. AE=EF ...(1)

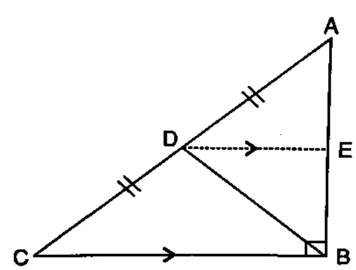
Similarly, in ΔCDE:

QF bisects CE [Q is the mid-point of CD and QF||DE]

∴ EF=FC ...(2)

∴ AE=EF=FC [From I and II]

Hence Proved



2. In a right-angled triangle ABC, $\angle ABC = 90°$ and D is the mid-point of AC

Prove that: BD=$\frac{1}{2}$AC

Solution:

According to the given statement, the figure will be as shown alongside:

Draw the line segment DE parallel to CB, which meets AB at point E.

Since, DE||CB and AB is transversal,

$\angle AED = \angle ABC$

$= 90° = \angle DEB$

Also, as D is the mid-point of AC and DE is parallel to CB; DE bisects side AB. i.e. AE=BE

In ΔAED and ΔBED

$\angle AED = \angle BED$ [Each 90^0]

AE=BE [Proved above]

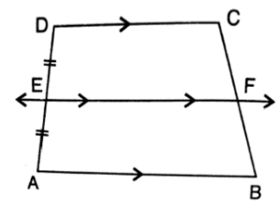
And, DE=DE [Common]

$\therefore \Delta AED \cong \Delta BED$ [By S.A.S]

$\Rightarrow$ BD=AD [C.P.C.T.C.]

$= \frac{1}{2}AC$ Hence Proved

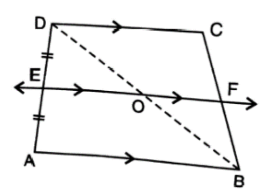
3. In a trapezium ABCD, AB||DC, E is the mid-point of AD. A line through E and parallel to intersect to AB intersects BC at point F. Show that:



(i) F is the mid-point of BC

(ii) 2EF=AB+DC

Solution:



(i) According to the given statement, the figure, will be shown alongside:

Draw diagonal BD which intersects EF at point O.

In triangle ABD, E is the mid-point of AD and EO||AB (as EF||AB)

∴ O is the mid-point of BD [By the converse of mid-point theorem]

In triangle ABD, E is mid-point of AD and EO||AB

∴ O is the mid-point of BD [By the converse of mid-point theorem]

In ΔBCD, O is the mid-point of BD [Proved above]

And OF||DC (as EF||AB||DC)

∴ F is the mid-point of BC [By the converse of mid-point theorem]

Hence Proved

(ii) In ΔABD,

E is the mid-point of AD and O is the mid-point of BD

$\therefore EO = \frac{1}{2}AB$ ....(1) [By mid-point theorem]

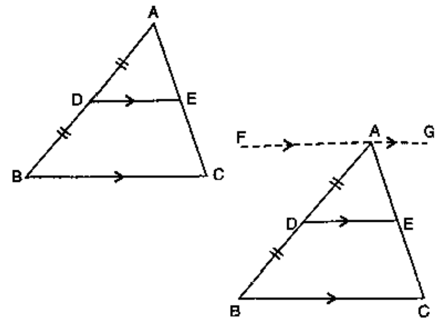
Also, O is the mid-point of BD and F is mid-point of BC

$\therefore OF = \frac{1}{2}DC$ ....(2) [By mid-point theorem]

$\Rightarrow EO + OF = \frac{1}{2}AB + \frac{1}{2}DC \Rightarrow EF = \frac{1}{2}(AB + DC)$

4. **Use the Intercept Theorem to prove the converse of the Mid-point Theorem.**

Solution:



Converse of Mid-point Theorem is: The straight line drawn through the Mid-point of one side of a triangle and parallel to another side bisects the Third side

Given: In triangle ABC, D is the mid-point of side AB and DE is parallel to BC

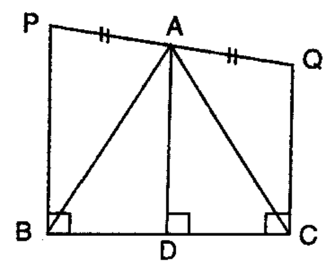
To Prove: DE bisects AC i.e. AE=CE

Construction: Through vertex A, draw FG parallel to BC so that FG||BC||DE

Proof:

Since, FG||DE||BC and the transversal AB makes equal intercepts on these three parallel lines i.e. AD=DB. Also, AC is an another transversal. According to Intercept Theorem, if a transversal makes equal intercepts on three or more parallel lines, then any other transversal, for the same parallel lines, will also make equal intercepts.

∴ AE=CE Hence proved



5. **Use the information, given in the adjoining figure, to show that: 0AB=AC.**

Solution:

Since PB, AD and QC are perpendicular to the same line BC, they are parallel to each other i.e. PB||AD||QC

Since, PB||AD||QC and PQ is a transversal making equal intercepts i.e. PA=AQ; therefore the other transversal BC will also make equal intercepts i.e. BD=CD.

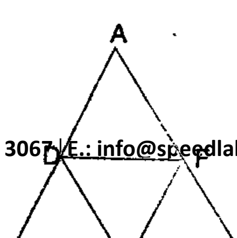
Now in ΔABD and ΔACD,

(i) BD=CD	[Proved above]
(ii) AD=AD	[Common]
(iii) $\angle ADB = \angle ADC = 90°$	[As, AD⊥BC]
∴ ΔABD ≡ ΔACD	[By SAS]
⇒ AB=AC	[By C.P.C.T.C]

Hence proved

6. **In the adjoining figure, D, E, F are the mid-points of the sides AB, BC and CA respectively.**

(i) If AC=7.4 cm, find DE.



(ii) If DF=4.1 cm, find BC.

(iii) If perimeter of ΔABC is 21.4 cm, find EF.

Solution:

(i) Since D and E are the mid-points of AB and BC respectively, so

$$\text{DE} = \frac{1}{2}\text{AC} = \left(\frac{1}{2} \times 7.4\right) \text{cm} = 3.7 \text{ cm}.$$

(ii) Since D and E are the mid-points of AB and AC respectively, so

$$\text{DF} = \frac{1}{2}\text{BC} \Rightarrow \text{BC} = 2\text{DF} = (2 \times 4.1)\text{cm} = 8.2 \text{ cm}$$

Hence, BC=8.2 cm

(iii) Now, AC=7.4cm, BC=8.2 cm and perimeter of ΔABC=21.4 cm.

∴ AB+AC+BC=21.4 cm⇒AB+7.4 cm+8.2 cm=21.4 cm

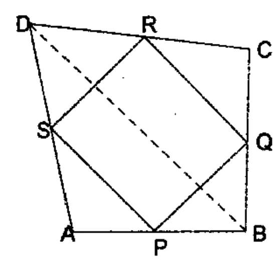
⇒AB=(21.4-15.6)cm=5.8 cm

Since E and F are the midpoints of BC and AC respectively, so

$$\text{EF} = \frac{1}{2}\text{AB} = \left(\frac{1}{2} \times 5.8\right) \text{cm} = 2.9 \text{ cm}.$$

7. Prove that the figure obtained by joining the mid-point of the adjacent Sides of a quadrilateral are a parallelogram.

Solution:



Given: P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively of ABCD

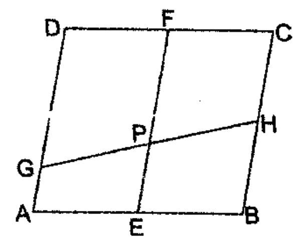
To prove: PQRS is a parallelogram.

Construction: Join BD.

Proof:

Statement	Reason
1. In ΔABD, PS||BD and PS=$\frac{1}{2}$ BD	By Mid-Point Theorem (P and S being mid-point of AB and AD resp.)
2. In ΔBCD, QR||BD and QR=$\frac{1}{2}$BD	By Mid-Point Theorem (Q and R being mid-points BC and CD resp)
3. PS||QR and PS=QR	From 1 and 2
4. PQRS is a parallelogram.	One pair of opposite sides are parallel are equal.

8. **In the adjoining figure, ABCD is a parallelogram in which E and F are the Mid-points of AB and CD respectively. GH is a line segment, intersecting AD, EF and BC in G, P and H respectively. Prove that GP=PH.**



Solution:

Proof:

Statement	Reason
1. AE=EB	E is the mid-point of AB
2. DF=FC	F is the mid-point of DC
3. AE=DF	AB=CD, being opposite, sides of a ||gm
4. AEFD is a ||gm	$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$
5. AD||EF	AE=DF(from 3)
6. AD||EF||BC and AEB and GPH	
7. GP=PH	