SpeedLabs

MATHS

ICSE 8th

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SpeedLabs
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1. A survey of 200families shows the results given below:

No. of girls in the family	2	1	0
No. of Families	32	154	14

Out of these families, one is chosen at random. What is the probability that the chosen family has 1 girl?

Ans. Total number of families = 200.

Number of families having 1 girl = 154.

Probability of getting a family having 1 girl

$$= \frac{\text{Number of families having 1 girl}}{\text{Total number of families}}$$

$$= \frac{154}{200} = \frac{77}{100}$$

2. Three dice are thrown together. Find the probability of:

(i) Getting a total of 5

(ii) Getting a total of atmost 5

(iii) Getting a total of at least 5.

(iv) Getting a total of 6.

(v) Getting a total of atmost 6.

(vi) Getting a total of at least 6.

Ans. Three different dice are thrown at the same time.

Therefore, total number of possible outcomes will be $63 = (6 \times 6 \times 6) = 216$.

(i) Getting a total of 5:

Number of events of getting a total of 5 = 6

i.e. (1, 1, 3), (1, 3, 1), (3, 1, 1), (2, 2, 1), (2, 1, 2) and (1, 2, 2)

Therefore, probability of getting a total of 5

$$P(E_1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{6}{216} = \frac{1}{36}$$

(ii) Getting a total of atmost 5:

Number of events of getting a total of atmost 5 = 10

i.e. (1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (2, 2, 1) and (1, 2, 2).

Therefore, probability of getting a total of atmost 5

$$P(E_2) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{10}{216} = \frac{5}{108}$$

(iii) Getting a total of at least 5:

Number of events of getting a total of less than 5 = 4

i.e. (1, 1, 1), (1, 1, 2), (1, 2, 1) and (2, 1, 1).

Therefore, probability of getting a total of less than 5

$$P(E_3) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{4}{216} = \frac{1}{54}$$

Therefore, probability of getting a total of at least 5 = 1 - P(getting a total of less than 5)

$$= 1 - \frac{1}{54} = \frac{(54-1)}{54} = \frac{53}{54}$$

(iv) Getting a total of 6:

Number of events of getting a total of 6 = 10

i.e. (1, 1, 4), (1, 4, 1), (4, 1, 1), (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1) and (2, 2, 2).

Therefore, probability of getting a total of 6

$$P(E_4) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{10}{216} = \frac{5}{108}$$

(v) Getting a total of atmost 6:

Number of events of getting a total of atmost 6 = 20

i.e. (1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (2, 2, 1), (1, 2, 2), (1, 1, 4), (1, 4, 1), (4, 1, 1), (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1) and (2, 2, 2).

Therefore, probability of getting a total of atmost 6

$$P(E_5) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{20}{216} = \frac{5}{54}$$

(vi) Getting a total of at least 6:

Number of events of getting a total of less than 6 (event of getting a total of 3, 4 or 5) = 10

i.e. (1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1) (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1).

Therefore, probability of getting a total of less than 6

$$P(E_6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{10}{216} = \frac{5}{108}$$

Therefore, probability of getting a total of at least 6 = 1 - P(getting a total of less than 6)

$= 1 - \frac{5}{108} = \frac{(108 - 5)}{108} = \frac{103}{108}$

These examples will help us to solve different types of problems based on probability for rolling three dice.

3. Suppose a fair coin is randomly tossed for 75 times and it is found that head turns up 45 times and tail 30 times. What is the probability of getting (i) a head and (ii) a tail?

Ans. Total number of trials = 75.

Number of times head turns up = 45

Number of times tail turns up = 30

(i) Let X be the event of getting a head.

P(getting a head)

$= \text{P (X)} = \frac{\text{Number of times head turns up}}{\text{Total number of trials}} = \frac{45}{75}$

$= 0.60$

(ii) Let Y be the event of getting a tail.

P(getting a tail)

$= \text{P (Y)} = \frac{\text{Number of times tail turns up}}{\text{Total number of trials}} = \frac{30}{75}$

$= 0.40$

Note: Remember when a fair coin is tossed and then X and Y are the only possible outcomes, and

P(X) + P(Y)

$= (0.60 + 0.40) = 1$

4. Two different coins are tossed randomly. Find the probability of:

(i) Getting two heads

(ii) Getting two tails

(iii) Getting one tail

(iv) Getting no head

(v) Getting no tail

(vi) Getting at least 1 head

(vii) Getting at least 1 tail

(viii)Getting atmost 1 tail

(ix) Getting 1 head and 1 tail

Ans. When two different coins are tossed randomly, the sample space is given by

S = {HH, HT, TH, TT}

Therefore, n(S) = 4.

(i) Getting two heads:

Let E_1 = event of getting 2 heads. Then,

$E_1 = \{HH\}$ and, therefore, $n(E_1) = 1$.

Therefore, P(getting 2 heads) = $P(E_1) = n(E_1)/n(S) = 1/4$.

(ii) Getting two tails:

Let E_2 = event of getting 2 tails. Then,

$E_2 = \{TT\}$ and, therefore, $n(E_2) = 1$.

Therefore, P(getting 2 tails) = $P(E_2) = n(E_2)/n(S) = 1/4$.

(iii) Getting one tail:

Let E_3 = event of getting 1 tail. Then,

$E_3 = \{TH, HT\}$ and, therefore, $n(E_3) = 2$.

Therefore, P(getting 1 tail) = $P(E_3) = n(E_3)/n(S) = 2/4 = 1/2$.

(iv) Getting no head:

Let E_4 = event of getting no head. Then,

$E_4 = \{TT\}$ and, therefore, $n(E_4) = 1$.

Therefore, P(getting no head) = $P(E_4) = n(E_4)/n(S) = \frac{1}{4}$.

(v) Getting no tail:

Let E_5 = event of getting no tail. Then,

$E_5 = \{HH\}$ and, therefore, $n(E_5) = 1$.

Therefore, P(getting no tail) = $P(E_5) = n(E_5)/n(S) = \frac{1}{4}$.

(vi) Getting at least 1 head:

Let E_6 = event of getting at least 1 head. Then,

$E_6 = \{HT, TH, HH\}$ and, therefore, $n(E_6) = 3$.

Therefore, P(getting at least 1 head) = $P(E_6) = n(E_6)/n(S) = \frac{3}{4}$.

(vii) Getting at least 1 tail:

Let E_7 = event of getting at least 1 tail. Then,

$E_7 = \{TH, HT, TT\}$ and, therefore, $n(E_7) = 3$.

Therefore, P(getting at least 1 tail) = $P(E_2) = n(E_2)/n(S) = \frac{3}{4}$.

(viii) Getting atmost 1 tail:

Let E_8 = event of getting atmost 1 tail. Then,

E_8 = {TH, HT, HH} and, therefore, $n(E_8) = 3$.

Therefore, P(getting atmost 1 tail) = $P(E_8) = n(E_8)/n(S) = \frac{3}{4}$.

(ix) Getting 1 head and 1 tail:

Let E_9= event of getting 1 head and 1 tail. Then,

E_9 = {HT, TH } and, therefore, $n(E_9) = 2$.

Therefore, P(getting 1 head and 1 tail) = $P(E_9) = n(E_9)/n(S) = 2/4 = 1/2$.

The solved examples involving probability of tossing two coins will help us to practice different questions provided in the sheets for flipping 2 coins.

5. When 3 coins are tossed randomly 250 times and it is found that three heads appeared 70 times, two heads appeared 55 times, one head appeared 75 times and no head appeared 50 times.

If three coins are tossed simultaneously at random, find the probability of:

(i) Getting three heads,

(ii) Getting two heads,

(iii) Getting one head,

(iv) Getting no head

Ans. Total number of trials = 250.

Number of times three heads appeared = 70.

Number of times two heads appeared = 55.

Number of times one head appeared = 75.

Number of times no head appeared = 50.

In a random toss of 3 coins, let E1, E2, E3 and E4 be the events of getting three heads, two heads, one head and 0 head respectively. Then,

(i) Getting three heads

P(getting three heads) = $P(E_1)$

$= \frac{\text{Number of times three heads appeared}}{\text{Total number of trials}}$

= 70/250

= 0.28

(ii) Getting two heads

P(getting two heads) = $P(E_2)$

$= \frac{\text{Number of times three heads appeared}}{\text{Total number of trials}}$

= 55/250

= 0.22

(iii) Getting one head

P(getting one head) = $P(E_3)$

$= \frac{\text{Number of times three heads appeared}}{\text{Total number of trials}}$

= 75/250

= 0.30

(iv) Getting no head

P(getting no head) = $P(E_4)$

$= \frac{\text{Number of times three heads appeared}}{\text{Total number of trials}}$

= 50/250

= 0.20

Note:

In tossing 3 coins simultaneously, the only possible outcomes are E_1, E_2, E_3, E_4 and $P(E_1) + P(E_2) + P(E_3) + P(E_4)$

= (0.28 + 0.22 + 0.30 + 0.20)

= 1

6. A bag contains red and what balls. The probability of getting a red ball from the bag of balls is 1/6. What is the probability of not getting a red ball?

Ans. The probability of getting a red ball from the bag of balls is $\frac{1}{6}$.

Therefore, the probability of not getting a red ball

P(ball is not red) $= 1 - \frac{1}{6} = \frac{5}{6}$

Therefore, the probability of not getting a red ball is $\frac{1}{6}$.

7. In a laptop shop there are 16 defective laptops out of 200 laptops. If one laptop is taken out at random from this laptop shop, what is the probability that it is a non defective laptop?

Ans. The total number of laptops in laptop shop = 200,

The number of defective laptops = 16,

Let E_1be the event of getting a defective laptops and

E_2 be the event of getting a non defective laptops

P(A) = The probability of getting a defective laptop

$= \frac{16}{200}$

= 0.08

Therefore, the probability of getting a non defective laptop = 1 – P(A) = 1 – 0.08 = 0.92.

8. The probability that it will rain in the evening 0.84. What is the probability that it will not rain in the evening?

Ans. Let E be the event that it will rain in the evening.

Then, (not E) is the event it will rain in the evening .

Then, P(E) = 0.84

Now, P(E) + P(not E) = 1

⇒ P(not E) = 1 - P(E)

⇒ P(not E) = 1 - 0.84

⇒ P(not E) = 0.16

Therefore, the probability that it will not rain in the evening = P(not E) = 0.16

9. A bag contains 8 black pens and 2 red pens and if a pen is drawn at random. What is the probability that it is black pen or red pen?

Ans. Let X be the event of ‘getting a black pen’ and,

Y be the event of ‘getting a red pen’.

We know that, there are 8 black pens and 2 red pens.

Therefore, probability of getting a black pen = P(X) = 8/10 = 4/5

Similarly, probability of getting a red pen = P(Y) = 2/10 = 1/5

According to the definition of mutually exclusive we know that, the event of ‘getting a black pen’ and ‘getting a red pen’ from a bag are known as mutually exclusive event.

We have to find out P(getting a black pen or getting a red pen).

So according to the addition theorem for mutually exclusive events, we get;

P(X U Y) = P(X) + P(Y)

Therefore, P(X U Y) $= \frac{4}{5} + \frac{1}{5} = \frac{5}{5} = 1$

Hence, probability of getting ‘a black pen’ or ‘a red pen’ = 1

10. What is the probability of getting a diamond or a queen from a well-shuffled deck of 52 cards?

Ans. Let X be the event of 'getting a diamond' and,

Y be the event of 'getting a queen'

We know that, in a well-shuffled deck of 52 cards there are 13 diamonds and 4 queens.

Therefore, probability of getting a diamond from well-shuffled deck of 52 cards = P(X) = 13/52 = 1/4

The probability of getting a queen from well-shuffled deck of 52 cards = P(Y) = 4/52 = 1/13

Similarly, the probability of getting a diamond queen from well-shuffled deck of 52 cards

$= P(X \cap Y) = 1/52$

According to the definition of mutually non-exclusive we know that, drawing of a well-shuffled deck of 52 cards

'getting a diamond' and 'getting a queen' are known as mutually non-exclusive events.

We have to find out Probability of X union Y.

So according to the addition theorem for mutually non- exclusive events, we get;

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

Therefore, P(X U Y)

$$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{(13 + 4 - 1)}{52} = \frac{16}{52} = \frac{4}{13}$$

Hence, probability of getting a diamond or a queen from a well-shuffled deck of 52 cards = 4/13

11. In a cricket match the Sachin hit a boundary 5 times out of 30 balls he plays. Find the probability that he

(i) hit a boundary

(ii) do not hit a boundary.

Ans. Total number of balls Sachin played = 30

Number of boundary hit = 5

Number of times he did not hit a boundary = 30 - 5 = 25

(i) Probability that he hit a boundary

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{5}{30} = \frac{1}{6}$$

(ii) Probability that he did not hit a boundary

$$P(B) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{25}{30} = \frac{5}{6}$$

12. What is the difference between odds and probability?

Ans. The difference between odds and probability are:

Odds of an event are the ratio of the success to the failure.

$$\text{odds} = \frac{\text{success}}{\text{Failures}}$$

Probability of an event is the ratio of the success to the sum of success and failure.

$$\text{odds} = \frac{\text{Success}}{(\text{Success} + \text{Failures})}$$

13. A card is drawn from a well shuffled pack of 52 cards. Find the probability of:

(i) '2' of spades

(ii) A jack

(iii) A king of red colour

(iv) A card of diamond

(v) A king or a queen

(vi) A non-face card

(vii) A black face card

(viii) A black card

(ix) A non-ace

(x) Non-face card of black colour

(xi) Neither a spade nor a jack

(xii) Neither a heart nor a red king

Ans. In a playing card there are 52 cards.

Therefore the total number of possible outcomes = 52

(i) '2' of spades:

Number of favorable outcomes i.e. '2' of spades is 1 out of 52 cards.

Therefore, probability of getting '2' of spade

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{1}{52}$$

(ii) A jack

Number of favourable outcomes i.e. 'a jack' is 4 out of 52 cards.

Therefore, probability of getting 'a jack'

$$P(B) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{4}{52} = \frac{1}{13}$$

(iii) A king of red colour

Number of favorable outcomes i.e. 'a king of red colour' is 2 out of 52 cards.

Therefore, probability of getting 'a king of red colour'

$$P(C) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{2}{52} = \frac{1}{26}$$

(iv) A card of diamond

Number of favorable outcomes i.e. 'a card of diamond' is 13 out of 52 cards.

Therefore, probability of getting 'a card of diamond'

$$P(D) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{13}{52} = \frac{1}{4}$$

(v) A king or a queen

Total number of king is 4 out of 52 cards.

Total number of queen is 4 out of 52 cards

Number of favorable outcomes i.e. 'a king or a queen' is $4 + 4 = 8$ out of 52 cards.

Therefore, probability of getting 'a king or a queen'

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{8}{52} = \frac{2}{13}$$

(vi) A non-face card

Total number of face card out of 52 cards = 3 times 4 = 12

Total number of non-face card out of 52 cards = 52 - 12 = 40

Therefore, probability of getting 'a non-face card'

$$P(F) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{40}{52} = \frac{10}{13}$$

(vii) A black face card:

Cards of Spades and Clubs are black cards.

Number of face card in spades (king, queen and jack or knaves) = 3

Number of face card in clubs (king, queen and jack or knaves) = 3

Therefore, total number of black face card out of 52 cards = $3 + 3 = 6$

Therefore, probability of getting 'a black face card'

$$P(G) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{6}{52} = \frac{3}{26}$$

(viii) A black card:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Therefore, probability of getting 'a black card'

$$P(H) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{26}{52} = \frac{1}{2}$$

(ix) A non-ace:

Number of ace cards in each of four suits namely spades, hearts, diamonds and clubs = 1

Therefore, total number of ace cards out of 52 cards = 4

Thus, total number of non-ace cards out of 52 cards = 52 - 4

= 48

Therefore, probability of getting 'a non-ace'

$$P(I) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{48}{52} = \frac{12}{13}$$

(x) Non-face card of black colour:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Number of face cards in each suits namely spades and clubs = 3 + 3 = 6

Therefore, total number of non-face card of black colour out of 52 cards = 26 - 6 = 20

Therefore, probability of getting 'non-face card of black colour'

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{20}{52} = \frac{5}{13}$$

(xi) Neither a spade nor a jack

Number of spades = 13

Total number of non-spades out of 52 cards = 52 - 13 = 39

Number of jack out of 52 cards = 4

Number of jack in each of three suits namely hearts, diamonds and clubs = 3

[Since, 1 jack is already included in the 13 spades so, here we will take number of jacks is 3]

Neither a spade nor a jack = 39 - 3 = 36

Therefore, probability of getting 'neither a spade nor a jack'

$$P(k) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{36}{52} = \frac{9}{13}$$

(xii) Neither a heart nor a red king

Number of hearts = 13

Total number of non-hearts out of 52 cards = 52 - 13 = 39

Therefore, spades, clubs and diamonds are the 39 cards.

Cards of hearts and diamonds are red cards.

Number of red kings in red cards = 2

Therefore, neither a heart nor a red king = 39 - 1 = 38

[Since, 1 red king is already included in the 13 hearts so, here we will take number of red kings is 1]

Therefore, probability of getting 'neither a heart nor a red king'

$$P(L) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{38}{52} = \frac{19}{26}$$

These are the basic problems on probability with playing cards.

14. The king, queen and jack of clubs are removed from a deck of 52 playing cards and then shuffled. A card is drawn from the remaining cards. Find the probability of getting:

(i) A heart

(ii) A queen

(iii) A club

(iv) '9' of red color

Ans. Total number of card in a deck = 52

Card removed king, queen and jack of clubs

Therefore, remaining cards = 52 - 3=49

Therefore, number of favorable outcomes = 49

(i) A heart

Number of hearts in a deck of 52 cards= 13

Therefore, the probability of getting 'a heart'

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{13}{49}$$

(ii) A queen

Number of queen = 3

[Since club's queen is already removed]

Therefore, the probability of getting 'a queen t'

$$P(B) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{3}{49}$$

(iii) a club

Number of clubs in a deck in a deck of 52 cards = 13

According to the question, the king, queen and jack of clubs are removed from a deck of 52 playing cards In this case, total number of clubs = 13 - 3 = 10

Therefore, the probability of getting 'a club'

$$P(C) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{10}{49}$$

(iv) '9' of red color

Cards of hearts and diamonds are red cards

The card 9 in each suit, hearts and diamonds = 1

Therefore, total number of '9' of red color = 2

Therefore, the probability of getting '9' of red color

$$P(D) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcome}} = \frac{2}{49}$$

15. Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive

Ans. Clearly, we have

A = {(1, 1)}, B = {(1, 2), (2, 1)} and C = {(1, 3), (3, 1), (2, 2)}.

(i) Since A consists of a single sample point, it is a simple event.

(ii) Since both B and C contain more than one sample point, each one of them is a compound event.

(iii) Since $A \cap B = \emptyset$, A and B are mutually exclusive.